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Message From the Program Chairs

Following the traditions on organizing international forums in St. Petersburg, the cradle of the Russian Academy of Sciences, the First International Conference on Fuzzy Sets and Soft Computing in Economics and Finance, FSSCEF 2004, was held this year in Northern Palmira. The conference is organized as a platform for exchange of ideas, experiences, and opinions among the academicians, professional engineers and financial community's practitioners on the applications of soft computing methods and techniques to economics and finance. Conference papers were carefully selected in accordance with best quality international standards. All papers were reviewed by international Program Committee. Finally, the International Program Committee accepted 64 papers from 24 countries. Also Bernard De Baets, Hans De Meyer, Alexander Yazenin, Arkady Borisov, Kaouru Hirota and Toshihiro Kaino accepted the invitation as keynote speakers to present their work.

Soft computing techniques have been applied to a number of systems in economics and finance showing in many cases better performance than competing approaches. At this conference, promising areas such as fuzzy data mining, fuzzy game theory, multi-agent systems, fuzzy and neural modeling have been studied for macro-economic analysis, investment and risk management, time series analysis, portfolio optimization and other applications.

Conference Program is composed of 6 sections reflecting the main problem areas and application domains studied in the conference papers: "Fuzzy Data Mining in Economics and Finance", "Fuzzy Games, Decisions and Expert Systems", "Fuzzy Mathematical Structures and Graph Theory", "Multi-Agent Systems and Soft Computing Applications in Economical and Financial Systems", "Soft Computing Methods in Investment and Risk Analysis and in Portfolio Optimization", and "Fuzzy Economic and Information Systems".

Many persons contributed numerous hours to organize the Conference. First of all, we would like to thank all the authors and participants for contributing to the Conference and stimulating the technical discussions. Special thanks to the Program and Organizing Committees members and all the institutions supporting this event. We hope that all participants enjoyed the Conference and their stay in St. Petersburg.

Alexey Averkin, Ildar Batyrshin, Janusz Kacprzyk
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MULTI-AGENT SYSTEMS AND SOFT COMPUTING APPLICATIONS IN ECONOMICAL AND FINANCIAL SYSTEMS

Combining multi-agent approach with intelligent simulation in resources flow management

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Keywords: *Enterprise Modeling, Intelligent Simulation, Artificial Agent, Multi-Agent System, Computer Integrated Management*

ABSTRACT
The aim of the paper is to develop a new approach to designing decentralized management systems, which perform the exchanges of industrial resources based on the model of collective behavior of intelligent agents. The results of constructing multi-agent architecture for computer-integrated management system are represented. Here various activities and interactions of intelligent agents specify a required intelligent behavior of the whole multi-agent system. These agents create some required types of resources and interchange by them for reaching global purpose of the production system. The intelligent simulation tools were used to investigate the agent-based management strategies

1. The Multi-Agent Approach to Decentralized Management Systems

Decentralization of management in computer integrated manufacturing (CIM) and networked enterprise (NE) expresses the important fact that all local management systems (LMS) make decisions under uncertainty and vagueness, because any available information about other LMS is incomplete, fuzzy and ambiguous. Here some LMS ensures a real-time control of all elements in the enterprise cell and searches for additional resources located in other cells. For this purpose the LMS should frequently make decisions about using its own resources and other needed resources. The missing resources are at the disposal of others LMS and can be borrowed to implement the necessary industrial operations. The local management system in CIM and NE is linked to all other LMS. Here the separate LMS possess complete information about their own resources and purposes, but they have only partial information about purposes and effectiveness of the whole system.

In particular, the LMS also should select the best plan among the set of all possible plans. Therefore, the set of LMS should exchange the information intensively for organizing a purposeful CIM/ NE system.

Therefore, among the most significant functions of decentralized management system, we point out the following:

- Connection with management systems of higher level. In spite of the fact that we create our system as a decentralized one, the links with management systems of higher level can not be completely lost. Our task is the elimination of a governing role of higher level system during the real-time management. The role of higher level management system should be reduced to the definition of the production strategy and the plan distribution between LMS at the beginning of changes.
- Monitoring of resources and services. The process of providing enterprise cells with resources has two aspects: A – definition of necessary resources, and as a consequence, ordering of missing resources; B – mechanism of searching for and delivery of necessary resources. When the LMS orders required resources, it should not take care of their location and state.
- Detection and resolution of conflicts. During the LMS interactions, various conflict situations may occur (for example, conflicts concerning the ordering of resources use). To detect and face such problems, it is necessary to introduce some additional functions into the system.
- Distributed database creation for the accumulation of both knowledge and data concerning the state of the system.

Thus, every LMS should work independently with analyzing its own state and making decisions; so it needs to be autonomous and active. Everyone LMS should work in an optimal way to attain the goals, such that the global CIM/NE purpose would be achieved (in the more efficient way, if possible). To prevent conflicts, unavoidable in the autonomous work case, LMS should perceive the external world state and react operatively to its modification; it also should be connected with other LMS and accepted by them. The autonomous mode of functioning also needs learning capacities.

Therefore, the LMS may be viewed as an artificial agent possessing such basic properties as autonomy, reactivity, activity, social behavior, life in the network, flexibility, adaptability etc. [2, 3, 4, 7, 9, 10, 15].

The use of multi-agent architecture enables distributed or decentralized decision-making. The same can be argued about problem solving. For instance, the problem may be shared between agents in a hierarchical manner. Often the multi-agent approach allows us to solve sub-problems locally with specific agents, and to propose some global solution as a result of interactions between different agents [8]. Unfortunately this structure presents some disadvantages, due mainly to possible contradictory solutions made by specific agents that can lead to a global breakdown of the system.

Nowadays, researchers pay a serious attention to distributed, hybrid, heterogeneous systems in all the computer-related fields [1, 11, 13]. In particular, multi-agent and simulation approaches to management problems are viewed as complementary. It leads to a growing interest in building hybrid systems involving the cooperation of both approaches. Apart from the usual reasons for building a multi-agent architecture, some conditions were specially taken into account, such as modularity and ease of integration [11].

2. Types of the Agents in the Decentralized Management System

Agents may be introduced to the CIM/NE system by using technological, functional, topological or organizational criteria. In our case, an intelligent agent (IA) is the CIM/NE unit. Each IA is an entity able to modify its environment and, if necessary, interact with other agents. To do it, the IA uses its knowledge base and inference mechanism. The goal of each artificial IA is to simulate the decision that can be made by a human for a given CIM system. Here the IA uses heuristic methods and takes into account what products have already been manufactured, what resources (its own or borrowed) are available or ordered, what data depend on the decisions made by other IA, what due-dates are planned etc.

In the basic workflow model [14] in the CIM/NE system the interaction between two agents, the Customer and the Performer, is considered. Both of them are instances of the same IA class, called resource USER agent. The USER agents function independently and pursue their own goals (performing prescribed basic operations), with using their knowledge (heuristic methods), data and criteria. So the USER agents are really autonomous agents because nothing initiates their operations from outside.

As a Customer, the USER agent sends requests to all other USER agents; it demands that they either get some resources from it or send resources to it. Another USER agent receives such a request and, after analyzing it, can either accept it (putting it into the queue of its basic operations and hence becoming Performer) or reject it. After receiving a request IA sends a relevant message to other USER agents. As a result of such information exchange, the interaction is realized between two IA, the Customer and the Performer. Any USER agent may be at the same time both Customer and Performer with respect to other USER agents. Here Customer and Performer are autonomous IA in CIM system: they can initiate their own operations and the operations of other IA.

To deliver requests and responses, a Channel agent (or agents) is used. Its purpose is to provide information exchange for all other IA. The Channel agent delivers a request or a message to some addressee (if there is any) or to the all IA in the system (if there is no specific address). So the Channel agent takes functions of a various packages (information, controlling, introducing) routing. Using knowledge models the Channel agent can optimize various retrieval operations and process the distributed information in a network.

In addition, some transportation and storage systems are necessary in the CIM. Here Service agents are introduced for the resources storage and re-distribution. The USER agents negotiate with the Service agents about resources receipt, sending or moving. The Service agents receive requests for transporting or storing resources. These Service agents are not quite autonomous; they don't initiate other IA's actions and don't seek them.

Agents coordination, as well as the detection and solution of conflict situations is made by Supervisor agent. His task also consists in correcting the system while some new agents emerge. To connect agent's cyberspace with an external environment an Interface agent is introduced. He plays the role of mediator between various agents and operator (Fig.1).

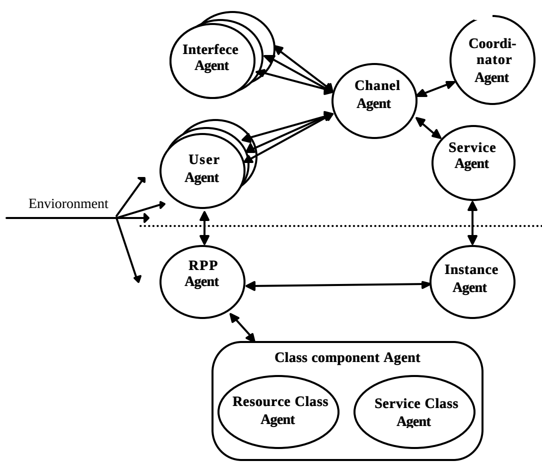

Figure 1. Architecture of multi-agent distributed management system

Thus, a basic model of multi-agent system seen as a functional- structural unit of the decentralized management system is suggested. It includes the interactions between five basic IA classes (User, Channel, Supervisor, Service, Interface), which have the connections of three types (Coordination, Subordination and Information) (see also [12]).

3. Multi-Agent Model of the Resource Exchange Process

We shall consider the following types of resources: a) finite activity products, if we deal with production cell of the CIM; b) services, if the deal with the transport cell or some other auxiliary cell; c) parts or free places for them, if the storage system is considered, etc. The resources have two basic parameters: the site and the state. To obtain or consider the opportunity of receiving a required resource, the basic IA called the Customer should know the meaning of these parameters. However, if the given parameters constantly change and the set of resources (even for identical resources) is very large, then the accumulation of such information amount becomes practically impossible.

Therefore it is necessary to create a flexible mechanism that could initialize the resources exchange. It is also necessary to have the mechanism of dynamic parameters adjustment for the resources exchange process when the CIM system works.

First of all, it is necessary to deliver the knowledge about the resources structure and state from the user (Customer) to the resources themselves (or to places of their generation). Furthermore, it is necessary to organize the resources interaction and finally to entrust supplying the resource components to the resources themselves. So we convert the resources into active objects, which can be naturally represented as IA. Thus, both the resource itself (material, operation, equipment), and its virtual counterpart are present in the system.

To realize the multi-agent resources supply system, the following new agents classes can be introduced (Fig.1):

- Resource provision process agent (RPP agent) that initiates and coordinates the process of resource maintenance for the User agent, playing the Customer role. It is generated by the User agent, which appears as the resource Customer.
- Class component agent (CC agent) that is the agent containing the knowledge on the resource components and parameters. Because the system possesses multi-component resources and there can be some different ways of combining these components, we obtain the resources class.

Let us introduce the resource A owned by an IA, as well as the resources, owned by others IA. These resources are required to supply the resource R (Fig.2). Some combinations of the resources {B, C} and {D, E} are possible, each of them allowing the construction of the resource R. Depending on a selected combination, some various instances of the resource class can be obtained.

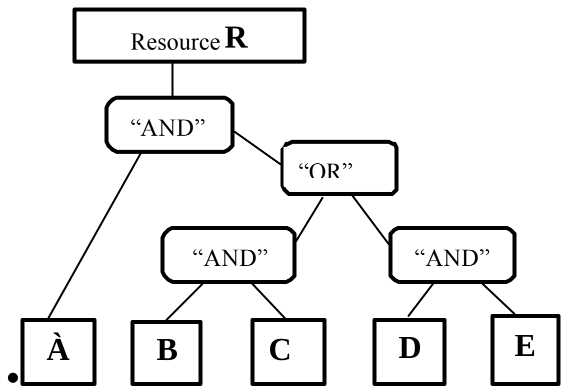

Figure 2. On the structure of the resource class components

Each agent belonging to the class of component agents "is linked" to a User agent or Service agent. Therefore it is possible to select two types of CC agents:

1. Resource Class agent (RC agent) that contains knowledge on resource current state and its technological structure. This agent is linked to the User agent, which creates or stores given resource.
2. Service Class agent (SC agent) that includes knowledge about current state and technology of service operation. This agent is linked to the User or Service agent, which implements the given service operation.

An Instance agent selects the components of a required resource. It is formed by the decomposition of the Class component agent through the digitalization of resource parameters. The Instance agent is the direct initiator of some operations in the system. It addresses the call to make up the operation to the User or Service agents. The operations should be feasible by the reference moment. The Instance agent is supported by the Class agents during the previous phase of resource maintenance process.

The set of all class attributes may be represented in a natural way by a frame. It is suggested to introduce the set of the frames, which will reflect the structure of a class. This set contains real values of the class parameters at various stages of work. The top level frame will be referred as the "Frame of resource class" (Table 1).

Table 1. Prototype frame of resource class

Prototype frame of resource class			
Slot	**Slot name**	**Meaning**	**Description**
1	Provider name		Name of User or Service agent
2	Type of resource		Name of resource type, which supports its class
3	Resource parameters		Reference to frame of basic type of class resource
4	List of sets class component resources		Names of sets
5	Name of active set		Name of active set of components in class instance
6	Class accessible		YES or NOT

The following frame contains the information about a component of the resources set. This frame is called “Frame of resource components set". Each class component is a basic resource for any other class. Moreover, there can be a variety of such classes. They are unified by their membership to the same resource type, and distinguished by the name of User or Service agents which are providers of a given class.

4. Agents Interaction

It was already mentioned that the process of resources exchanging in multi-agent system generally can be founded on the workflow model [14]. In this model the basic cycle links the Customer agent with the Performer agent. This cycle consists of four phases called workflows: "preparation – negotiation – performance – acceptance".

At the preparation stage the Customer addresses a demand (or an order for work realization) to the Performer. At the following stage the negotiation takes place which is directed to the development of mutually acceptable conditions of the order satisfaction. Then the Performer performs the job and by ending it, reports to the Customer on its completion, and at the last phase the Customer accepts or rejects the job.

Here the engagement network appears that forms an organizational structure, and the feasible agents communication acts collected in the communication protocol are specified as possible speech acts.

Thus, the following operational cycle is suggested. The User agent makes the decision about the initialization of some industrial activity on the basis of its knowledge or behavior model. To do this work he needs some additional resources. In this case he will order such absent resources

Each of RC agents, after receiving the request, specifies its correspondence to own resources. When the correspondence takes place, the class becomes active and produces the operations necessary for a concrete definition of all parameters of the basic resource. If these concrete parameters correspond to the specification, then the RC agent forms an answer to the RPP agent and generates the Instance agent, which, in its turn, will initiate and co-ordinate the User or Service agents operations to provide a required resource.

5. RAO-Model of Interaction Class Agents

Here we consider the design of simulation model of multi-agent management system and implement various elements of model in RAO-environment [1, 4, 5]. All the IA, inquiries, messages, tasks etc. are represented as resources in RAO-model. The type of resource determines structure of its attributes, their possible meanings, meaning by default. There are 7 resource types in the model: *Resource_Class, Service_Class, Instance, User, Service, System* and *Node.*

For example, the description of *Resource_Class* as resource type, which is permanent looks like:

```
$Resource_type Resource_Class: permanent
  $Parameters
     Number      : (A, B, C, D, E, F, Z)
     Owner       : (Ma, Mb, Mc, St)
     Graph_no    : integer
     State       : integer = -1
     UsC         : integer = 0
     Ct          :integer =0
$End
```

All actions performed by resources and over resources are described in object patterns of operations. Each pattern represents usual or modified production rule. All operations in the model should concern some pattern and differ one from another only by used resources and parameters.

So the pattern of decomposition *Resource_Class* looks like:

```
$Pattern          R_ClassDecPr: rule trace
$Relevant_resources
        Tr: Resource_Class      Keep
        Sys:Sys1                Keep
$Body
Tr
```

```
          Choice from
          Tr.State = -1
first
Convert_rule
          State    set      -2
          { Selection of class for decomposition}
Sys
          Choice from
          Sys.T_f  = 0  {System is free}
first
Convert_rule
          T_f      set      3        {Set of the counter flag}
          Pz_c     set      1
$End
```

Here the following simulation performance measures were used: the average time of request processing by IA, the Channel IA average load, the Service agent average load etc.

Conclusion

The suggested approach to multi-agent modeling of resources exchanges ensures the decentralized production management in CIM/ NE. It is applicable to various management systems incorporating specially developed LMS, which are capable to realize the inference mechanism, to receive, to accumulate and to treat uncertain and fuzzy knowledge.

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Agent-based Collective Intelligence for Inter-bank Payment Systems

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1 Introduction

The liberalization of financial markets experienced around the world in the last three decades coupled with the advances in computing and telecommunication technology, have been the driving forces behind fundamental changes experienced by the world financial system. Among these changes perhaps the most important are the growing integration of international markets, the innovation in financial instruments, and the resulting increase in the volume of funds transferred between agents on a daily basis.

This increase in the volume of funds transferred has raised concern among central banks around the world regarding the risks associated with the systems used to handle these transfers. Issues regarding payment and settlement are of particular importance to central banks for three reasons. First, the main policy of the central bank's objective is to promote macroeconomic stability through the instruments of monetary policy, which is effectively carried out through the payment system. Second, aside from monetary policy considerations, payment systems are the means by which transactions in the real economy take place; thus the smooth functioning of payment and settlement systems is fundamental to economic stability. Finally the liquidity transformation activity of banks makes them vulnerable to runs which, due to the interdependence created by a payment system, gives rise to the possibility of systemic risk.

Recent contributions to the study of various issues regarding inter-bank payment systems (PS) include: [1], [2],[3], [4], [5],[6], [7] and [8].

One of the main assumptions of these models is complete knowledge of the environment, which may not be the case in many real scenarios. That is why, in this work the net settlement payment system problem (NSPSP) is addressed within the context of the COllective INtelligence (COIN) theory [9, 10, 11]. Being part of the soft computing technologies, COIN framework permits to capture the main properties of distributed problem domains in general and the NSPSP in particular, like following:

- Distributed: The different entities in the NSPS that might be distributed across different geographical locations must be considered as a whole by using dis-

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tributed knowledge-based system. For example, the banks that might be located in different cities need to know all the customers.
- Dynamic: The NSPS is changing, there are no obligations to be part of the bank for a certain period and the elements may join or leave the bank based on their own interest. Moreover, the information can change continuously (*e.g.* returns, risk, etc.).
- Cooperation: Greedy behavior of the consumer-agents conduces to the degradation of all the system, therefore the agents in the PS must cooperate in order to achieve the global objective (*e.g.* global utility function (GUF) optimization).
- Online adaptation: The NSPS optimization process must be oriented towards reducing the time to be able to respond to customer demands in real time.

According to our conceptualization of the NSPSP within the framework of the COIN theory, a payment system is represented as a Multi-Agent System (MAS) where:

- One of the main objectives is the decentralization of the control and communication.
- Each agent of the PS (consumer, bank) has autonomous behavior and a local utility function (LUF).
- The learning process consists of adapting the "local behavior" of each agent with the aim of optimizing a given "global behavior".
- The agents execute Reinforcement Learning (RL) algorithms at the local level.

In this work, we developed an agent-based Collective Intelligence framework to analyze the consumer-agents' behavior in the NSPSP. We show how the consumer-agents are able to learn the best action that maximize the global utility function. The question addressed is: how can it be determined if agents' local decisions are contributing to optimize the global utility function? More precisely, how much time the consumer-agents must wait before consuming their goods?

The paper is organized as follows: section 2 discusses the institutional aspects of the payment systems. Section 3 describes the model proposed in this work within the framework of the net settlement payment system (NSPS). Section 4 describes the COIN theory as a framework to address the payment system problem in a proposed net settlement scenario. The algorithm 'Q-net' developed in this work based on the RL methods and COIN techniques is adapted to the scenario of the net settlement payment system in Section 5. Section 6 presents a case study description and the simulation results. Section 7 concludes discussing some of the ways in which our model can be improved upon, and some directions in which it could be extended.

2 Aspects of the Payment Systems

An inter-bank payment system can be defined as the set of rules, institutions and technical mechanisms by which the transfer of funds between banks is carried out for the purpose of settling obligations assumed by economic agents, which may be the banks themselves or their clients . In general we can divide the participants of the payment system into four categories: the payees, the payors, the banks and the regulators [1].

Payment systems are important given the value of transfers that are made through them, the speed with which they happen, and because they are a major channel through which endogenous and exogenous shocks are transmitted across

[1] For a detailed definition of payment systems see [12].

financial systems . In general a potentially large amount of resources must be dedicated to reducing the effect of the shocks being transmitted to the system. Next we discuss the most important risks and costs in a payment system.

2.1 Risks in the Payment System

Although the risks involved arise primarily from the explicit or implicit extension of credit made among agents , for completeness the risks include [2]:

a. Credit Risk, which refers to the possibility that a transaction will not be realized at full value due to the failure of an agent to meet its financial obligations.
b. Liquidity Risk, which refers to the possibility of a transaction not taking place at the desired time, due to a party having insufficient funds to meet its financial obligations at a given time.
c. Operational Risk, which refers to the possibility of delays or failures to settle obligations due to breakdowns in computer or telecommunications systems.
d. Legal Risk, which refers to the possibility that an inadequate legal framework will exacerbate the effects of the other risks.
e. Systemic Risk, which in its broadest sense refers to the possibility that the failure of a single agent in the system to meet its obligations, or that a shock to the system itself, could result in the inability of other agents in the system to meet their obligations.

Systemic risk is the one to be explicitly considered in the model defined below. However the definition above is very broad, thus it is necessary to discuss the different aspects that the notion of systemic risk encompasses. The most widely used aspect of systemic risk regarding payment systems refers to the possibility of a propagation of liquidity problems due to coordination problems regarding how to deal with failures or liquidity shortages in the system [3]. The second aspect to which systemic risk refers to the way risk is shared among agents when the system itself is subject to exogenous shocks, such as a generalized fall in asset prices. A third aspect to which systemic risk refers, is what Rochet and Tirole [6] refer to as learning related contagion. The basic idea is that agents can observe signals about the solvency of banks, which may induce depositors to find it optimal to withdraw their deposits early, causing a run. These aspects of the systemic risk are taking into account by the coefficient of risk aversion of the agents (γ in our model).

2.2 Costs in the Payment System

Regarding the costs associated with payment systems these include [4]:

- The costs associated with the resources used in processing payments; such as the costs of setting up and maintaining communication and safeguard systems, as well as the explicit cost of processing and settling payment orders.
- The financial costs associated with maintaining an investment portfolio different to the optimal portfolio that would be kept under the classical assumption that there exists a perfect payment system that implies negligible transaction costs. In particular these are the costs of holding more liquid portfolios than optimal, in order to be able to face liquidity needs as they arise, and not having to liquidate assets at disadvantageous prices to generate liquidity.

[2] See [6] and [8] for a detailed discussion of the risks associated with payment systems.
[3] See [13] for a discussion of the transmission of shocks through the payment system.
[4] See [14] for a discussion of the costs in payment systems.

- The costs associated with identifying and reducing risks in payment systems. These include for example the cost of assessing credit and liquidity risks, and the costs of holding collateral.
- The explicit costs associated with delays or failures in the settlement procedure, such as the opportunity cost implied by any time lags between the time a transaction is agreed and the time it is finally settled.

These costs are not necessarily independent of each other, as some of them are consequences of the others. However listing in this manner is useful for exposition purposes. In our model below, we will mainly be concerned with the financial costs (denoted by L) associated with maintaining an investment portfolio different from the optimal portfolio, given excessive early withdrawals.

2.3 The Time Dimension

There are a number of ways in which payment system designs can be categorized, for example it can be done depending (i) on whether they handle large value or low value payments, (ii) on whether they are electronic or paper based, or (iii) on the way settlement is carried out [15]. For the purposes of this work we will classify them on the basis of the way settlement takes place.

In reality there is a continuum of ways in which the settlement of obligations can take place in a payment system, with a pure Real Time Gross Settlement (RTGS) System at one end of this continuum, and a pure Deferred Net Settlement (DNS) system at the other end. The discussion that follows will focus on the polar cases, although in our model we will examine the interaction of agents under of a pure net settlement payment system.

With respect to the settlement of obligations, when they are settled on a gross basis, it means that each transaction is settled individually, under an RTGS system, transmission and settlement of obligations occurs simultaneously as soon as orders are accepted by the system. Under a system without intra day credit, the payments of a bank with insufficient funds to cover their obligations are rejected by the system or queued until funds become available. Without intra day credit, funds can either come from incoming payments or by the liquidation of illiquid assets.

Under a multilateral net settlement system, banks send their payment orders to a central location, usually the central bank in the case of large value payment systems. However these orders are not settled continuously but at specific points in time, when the net position of each bank with respect to the central location is calculated. Banks only transfer their net obligation to the central location at the end of the netting process, which usually occurs at the end of the day.

3 Model of the net settlement payment system problem

In this section, the model proposed within the framework of the net settlement payment system problem is described.

We have n island economies, and there is a single consumption good. In each island there is a large number m of identical risk averse agents with a coefficient of risk aversion γ affecting the decision-making of every agent (which for simplicity we normalize one) and a single mutual bank . Agents live for three periods $t = 0, 1, 2$. At $t = 0$, the planning period, each agent is endowed with one unit of the good. The agents face uncertainty about the time when they must consume. With probability α they must consume at t=1, we will call this type of agents the impatient consumers, while with probability $(1 - \alpha)$ they must consume at t=2, we will call this type of agents the patient consumers. Agents learn privately their types at t=1.

In order to move the good across the time, agents and their banks can store it from one period to the next at no cost, the storage technology yields the riskless rate, which we will assume is equal to zero. In the context of inter-bank payment systems, we can think of the storage technology as being analogous to maintaining liquid funds in the form of reserves.

Alternatively the good can be invested, however agents cannot invest by themselves but must do so by depositing their endowment at time zero at their bank, which has exclusive access to the investment technology. The investment technology is illiquid in the following sense, the technology yields a return $Re > 1$, per unit of investment invested at $t = 0$ and liquidated at $t = 2$, however if a proportion of investment is liquidated at $t = 1$ it only yields a return $L \leq 1$. In this context the investment technology is analogous to the overnight investment opportunities available in money markets.

In order to motivate the existence of an inter-bank payment system, we assume all patient consumers must consume abroad at time two, thus the source of uncertainty about when to consume is also a source of uncertainty about where consumption takes place. In this sense our model is a simplification of the framework proposed by [2] where the proportion of patient consumers who may consume in the island where they were originally located is arbitrarily close to zero.

At $t = 1$ when the agents learn their type, patient consumers are faced with two choices. On the one hand they can withdraw their deposit at $t = 1$, and carry the good themselves to the other island and store it there until $t = 2$ when it will be consumed (Running action completed by the proportion of consumer-agents μ), the implicit cost of transporting the good themselves is the foregone investment. Alternatively they can leave their deposit, and have the bank transfer their claims to the other island where it will be available at $t = 2$ for consumption (Wait action completed by the proportion of consumer-agents $(1-\mu)$). We assume that all patient consumers travel at $t = 1$ to the island where their consumption will take place at $t = 2$.

In the following section, the COIN theory framework for the NSPSP is explained.

4 COllective INtelligence as a framework for the Net Settlement Payment System Problem

The COllective INtelligence (COIN) theory addresses *the question of knowing how to modify a local behavior to contribute to the optimization of all the system.* A COIN can be defined as a large MAS [11, 16] with the following properties [9, 10]:

a. There is a global *task* to perform.
b. *Many agents exist*, functioning simultaneously, which carry out local actions that affect one another's behavior.
c. One of the objectives in the design of a COIN is the *decentralization* of communication and/or control.
d. There is a *global utility function (GUF)* which makes it possible to measure the overall performance of the system by observing the local behaviors of each agent.
e. The individual agents carry out algorithms of "Reinforcement Learning" (RL) [17] and have a local utility function (LUF). The use of RL enables the agent to adapt and modify its behavior.

f. One of the objectives is to simplify the algorithms used by the agents (or processors) for utilization in real-time applications.

The problem of an individual 'learning-agent' is to know what action must be performed to maximize its local utility function. The problem becomes more complex when several agents act and must make their decisions with the purpose of maximizing a GUF. The central question is: *how to fix the individual local utility functions (LUF) of the agents in order to maximize the global utility function (GUF)?*

Each resource has a limited *capacity*, which is the maximum usage that it can support. Once this limited capacity is exceeded, an additional use will degrade the benefit to all users who, in their turn, will enter into a cycle of competition where they will strive to take the use of the resource from other users. This excessive use will lead to damage of the resource. This phenomenon is known as the 'Tragedy of the Commons (TOC)' (see [18],[19]).

Limitation of the individual freedom is the only means that, according to Hardin [18], can help to avoid the TOC. In MAS, the manner of imposing this type of punishment on the agents is not obvious.

In the proposed net settlement payment system framework, each agent will execute its actions taking into account its own interests. So, a punishment for an action that does not contribute to overall PS performance must be attributed. This punishment is attributed to each agent thanks to a mechanism of punishment developed in this work.

This work proposes a model of the PS within the framework of the COIN theory. In our approach, an agent can represent any entity of the PS like banks, customers, etc. The goods in the PS are represented as objects that are part of the environment. Therefore, every agent can change or influence these environment objects. The framework proposed is composed of the following elements:

- Set of n island economies: $\mathbb{I} = \{I_1, I_2, I_3, \ldots, I_n\}$
- For each island a mutual bank with exclusive access to the investment technology exists.
- Set of m consumer-agents by island I_i: $\mathbb{C}_i = \{c_1, \ldots, c_m\}$
  The consumer-agents' preferences are defined over consumption, with utility function:
  $$\boldsymbol{U}(C(t)) = (C(t)^{1-\gamma} - 1)/(1-\gamma)$$
  where $C(t)$ is period t's consumption, and γ is the coefficient of relative risk aversion.
  This particular form of the utility function was chosen because it captures several characteristics of economic behavior. Utility levels are increasing in consumption levels, and yet marginally decreasing, that is additional consumption yields progressively less utility in absolute terms. Moreover this utility function allows for the modification of agents preferences, through the modification of the relative risk aversion coefficient γ.
- At $t = 1$ a proportion α of consumers' in each island are revealed to be impatient consumers and must consume in their home island at $t = 1$, while the remaining proportion $(1 - \alpha)$ are revealed to be patient consumers and must consumer in the other island at $t = 2$.
- The patient consumer-agents' strategy set is S $=\{$W,R$\}$, where W stands for waiting and having its claims transferred to the other bank, while R stands for running, that is claiming its deposit early and transferring it to the other island itself.
- The local utility function (LUF) is represented as follows:

$$LUF = \boldsymbol{Q}_{(\mathbf{x}(t),a_{\mathbf{x}(t)})}(t+1) = \boldsymbol{Q}_{(\mathbf{x}(t),a_{\mathbf{x}(t)})}(t) + \beta\Big[r(t+1) + \rho \max_{a_{\mathbf{x}(t+1)}} \boldsymbol{Q}_{(\mathbf{x}(t+1),a_{\mathbf{x}(t+1)})}(t+1) - \boldsymbol{Q}_{(\mathbf{x}(t),a_{\mathbf{x}(t)})}(t)\Big] \quad (1)$$

where:
- β is the learning rate; ρ is the discount rate.
- The Q-values $\boldsymbol{Q}_{(\mathbf{x}(t),a_{\mathbf{x}(t)})}$ give an estimation of the PS environment. The way in which the Q-values update is done can be considered as one of the most important problems to resolve in our framework.
- The reinforcement for the action performed is represented by $r(t+1) = U(C(t))$.

This equation represents the Q-learning (see [17] for more details) equation used in the Reinforcement Learning field.

- The global utility function (GUF) is represented as follows:

$$GUF = \alpha\, \boldsymbol{U}(L) + (1-\alpha)\mu\, \boldsymbol{U}(L) + (1-\alpha)(1-\mu)\, \boldsymbol{U}\left(\frac{[1-(\alpha+(1-\alpha)\mu)]Re}{(1-\alpha)(1-\mu)}\right)$$

5 Q-NET ALGORITHM for the Net Settlement Payment System

Algorithm 1: Q-net parameters' initialization

-Initialize at $t=-1$:
-All the Q-values with random values $0 < \boldsymbol{Q}_{\mathbf{x}(t),a_{\mathbf{x}(t)}} < 1$
-The RL parameters: $\beta,\ \rho$
-The number of n islands in the set $\mathbb{I} = \{I_1, I_2, I_3, \ldots, I_n\}$
-The number of m consumer-agents by island I_i in the set $\mathbb{C}_i = \{c_1, \ldots, c_m\}$
-The coefficient or relative risk aversion: $0 < \gamma < 1$
-The reward $Re > 1$; -The punishment $L \leq 1$
-The number of agent $0 \leq \mu \leq 1$ that take the running R action
-The money in each bank: M_j
-The individual reinforcement function of consumer-agents: $r(t) = U(C(t)) = (C(t)^{1-\gamma} - 1)/(1-\gamma)$
-The withdrawals: $C(t)$
-For impatient consumer-agents: At $t=1$ $C(1) = \alpha L/\alpha = L$
-For patient consumer-agents:
At $t=2$ $C(2)_R = (1-\alpha)\mu L/(1-\alpha)\mu = L$
At $t=2$ $C(2)_W = \frac{[1-(\alpha+(1-\alpha)\mu)]Re}{(1-\alpha)(1-\mu)}$
-The global utility function:
$GUF = \alpha\, \boldsymbol{U}(L) + (1-\alpha)\mu\, \boldsymbol{U}(L) + (1-\alpha)(1-\mu)\, \boldsymbol{U}\left(\frac{[1-(\alpha+(1-\alpha)\mu)]Re}{(1-\alpha)(1-\mu)}\right)$

To optimize the GUF within the NSPSP framework, the 'Q-net' algorithm is proposed. The first step of the algorithm is the initialization of the system payment parameters described in the algorithm 1. The algorithm 2 describes the operation of Q-net repeated for n episodes each one consisting of three time periods $t = 0, 1, 2$. Finally, the algorithm 3 describes the method to choose the actions at the local level of each consumer-agent allowing the GUF optimization.

Algorithm 2: Q-net operation

REPEAT Update the episode n

At $t = 0$
- -All deposits are made: $M_j = 1$
- -Bank invest deposits

At $t = 1$
- -Consumer-agents learn their types (impatient α / patient $(1 - \alpha)$)
- -Patient consumer-agents execute an action according with the action-selection-method (Algorithm 3)
- -Impatient consumer-agents consume: αL
- -Patient Running consumer-agents consume: $(1 - \alpha)\mu L$
- -Banks must liquidate a proportion of their investments to meet their consumer-agents' needs: $z_* = \alpha + (1 - \alpha)\mu$
- -Patient consumer-agents travel

At $t = 2$
- -Claims under net settlement are transferred and received
- -Remaining deposits are claimed to the bank: $C(2)_W(1 - \alpha)(1 - \mu)$
- -Consumption takes place

Algorithm 3: Action-selection-method

-Read the input vector $\mathbf{x}$ from the environment state
-Choose an action $a_{\mathbf{x}(t)} \in \{W(ait), R(unning)\}$ from the input vector $\mathbf{x}$ by using the strategy ϵ-greedy derived from $\boldsymbol{Q}_{(\mathbf{x}(t),a_{\mathbf{x}(t)})}(t)$
$\arg\max\{\boldsymbol{Q}_{(\mathbf{x}(t),W)}(t), \boldsymbol{Q}_{(\mathbf{x}(t),R)}(t)\}$
-Compute the reinforcement function taking into account the punishement L and the reward Re:$r(t+1) = (C(t)^{1-\gamma} - 1)/(1 - \gamma)$
-After reinforcement signal reception, apply Q-learning update rule:

$$\boldsymbol{Q}_{(\mathbf{x}(t),a_{\mathbf{x}(t)})}(t+1) = \boldsymbol{Q}_{(\mathbf{x}(t),a_{\mathbf{x}(t)})}(t) + \beta\left[r(t+1) + \rho \max_{a_{\mathbf{x}(t+1)}} \boldsymbol{Q}_{(\mathbf{x}(t+1),a_{\mathbf{x}(t+1)})}(t+1) - \boldsymbol{Q}_{(\mathbf{x}(t),a_{\mathbf{x}(t)})}(t)\right]$$

6 Case study description and performance results

In this section, we present a case study to demonstrate the learning ability of the consumer-agents to optimize the GUF. Suppose we have two similar islands I_1 and I_2 where the NSPS (see Fig. 1) exists with the parameters defined in Table 1.

At the beginning of the every experiment, we fixed arbitrarily the number of impatient consumer-agents α. We modified the γ value in three different experiments to study the effects of the risk aversion in the learning ability of the agents. As we show in the Fig. 2 for every variation of the γ parameter, the agents learn to complete the action Wait instead of Running. As γ increases, the relative risk aversion of agents increases making them more averse to execute the Wait action, and therefore the learning is more slow.

In the Fig. 3 we show the global utility function optimization. As γ increases, the relative risk aversion of agents increases making them more averse to proportional looses as their wealth increases. In other words, the agents would be willing to pay a higher premium to insure themselves against looses, in this case due to bank runs. This is why the global utility function increases as γ decreases.

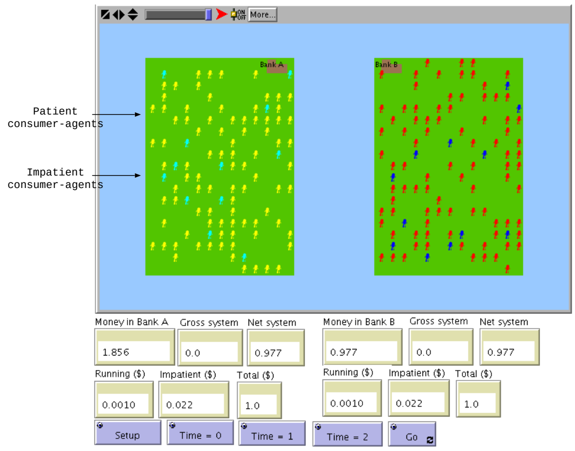

Fig. 1. Net Settlement Payment System Scenario (a screen-shot of the Netlogo environment)

Table 1. Net settlement payment system scenario parameters

Parameter	Description	Value
α	Proportion of impatient consumer-agents	0.1
μ	Proportion of patient consumer-agents taking the Running action at the beginning	0.9
ρ	Discount rate	0.5
β	RL Learning rate	0.6
Re	Reward (or return)	1.9
L	Punishment	0.1
γ	Coefficient of relative risk aversion	0.01; 0.5; 0.9

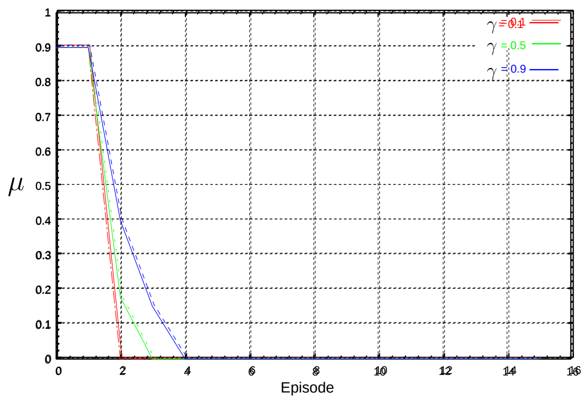

Fig. 2. Learning of the action Wait (1- μ) by the patient consumer-agents for $\gamma = 0.1; 0.5; 0.9$

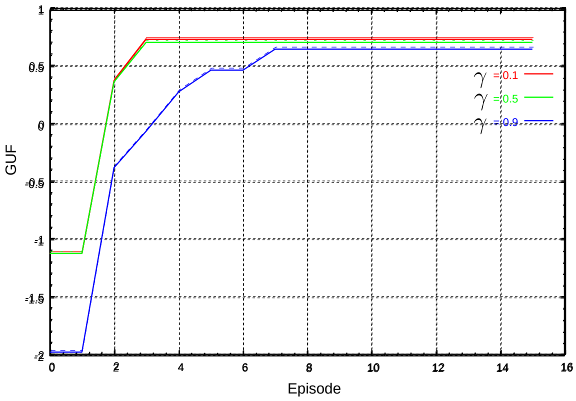

Fig. 3. Global Utility Function Optimization

As [1] showed in a similar environment, two Nash equilibria can be supported: one where all the patient consumer-agents decide to wait and one where all patient consumers decide to run even-though it is inefficient to do so. It is obvious that the optimal solution is the one that maximizes the global utility function: the state where all patient consumer-agents decide to wait.

7 Conclusions and future work

In this paper, the NSPSP was addressed within the framework of the COIN theory. In order to optimize the global utility function, an algorithm (Q-net) based on learning processes using RL algorithms and COIN techniques was developed. A model of the net settlement payment system problem was proposed and implemented in the agent-based parallel modeling and simulation environment over the Netlogo platform. The algorithm developed can easily be implemented into the real-world thanks to the learning abilities of the agents without needing the knowledge of all environment model.

We conclude that the net settlement payment system problem is well suited for the application of the COIN theory. In addition, the adaptive algorithm presented, Q-net, provides a GUF optimization avoiding problems like the 'Tragedy of the Commons'.

Possible extensions of this work include: introducing stochastic returns across islands to evaluate the effect on the learning mechanism when another source of uncertainty is considered. Another interesting extension would be the use of the COIN framework to an economy where net and gross settlement payment systems co-exist, a common feature of modern economies. Within this framework the issue of the tradeoff between costs and systemic risk could be addressed. In addition, more complicated punishment algorithm will be developed to adjust the local utility functions. We also pretend to compare our algorithms with other classic optimization methods.

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Macroeconomic fuzzy model of Azerbaijan

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Abstract

In this report we present fuzzy macroeconomic model of Azerbaijan. For this purpose Mamdani system that take into account the fuzziness of information is used. We propose comparative analysis efficiency and accuracy of the fuzzy model with statistical and neural networks forecasting.

Introduction

The economy of oil exporting countries, including Azerbaijan, depends on not only the world prices of petroleum but also on such economic indicators as the volume of investments, a rate of inflation and others.

The macroeconomic analysis is always accompanied by different types of uncertainties. For instance, when determining the correlation between various indicators using econometric models and choosing well investigated simple functions whose parameters are based on statistical data, we impose rigid restrictions on forms of this correlation. On the other hand, when using classical mathematical means it happens to be difficult, and sometimes impossible to formalize verbal expressions of experts' perceptions.

Wide application of fuzzy logic in economic researches in the last few years is stipulated by need of solving the above mentioned and similar problems, originating in conditions of uncertainty.

It is well known that every continual function with highly enough accuracy can be approximated with fuzzy rules so that the peak of accuracy is achieved by more detailed gradation of variables. This in its turn depends on the expert's level of knowledge about correlation characteristics of indicators reflected by the given function.

Macroeconomic fuzzy model

Macroeconomic fuzzy model of Azerbaijan constructed using fuzzy logic and reflecting effect of growth rate of volumes of the investments (INV) in economy, growth rate of volumes of crude oil production (NP), growth rates of volumes of export (EXP) and import (IMP), oil price in the global markets (PN - dollar / barrel) and inflation rate (CPI), on growth rate of a gross domestic product (GDP).

With the purpose of gradation of these indicators, the following terms are defined for the corresponding linguistic variables: "very low", "low", "average", "high", "very high".

For semantics of introduced linguistic variables, we use fuzzy numbers of trapezoidal form (a_1,a_2,a_3,a_4), where $[a_1,a_4]$, $[a_2,a_3]$ are, accordingly, lower and upper basis of a trapezoid.

Relations between the corresponding linguistic variables are determined by the semantic rules as shown at the tables 1 and 2.

The rules of correlation for GDP, PN, CPI shown at the table 1 are obvious.

Table 1. Fuzzy rules

indicators		Numbers of rules									
		1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
		Expert estimated									
GDP		1	2	3	4	5	2	2	5	4	2
PN	$\wedge$	1	2	3	4	5	1	2	5	5	3
CPI	$\wedge$	1	2	3	4	5	2	3	4	3	2

At this table we use following designations: 1 - very low, 2 - low, 3 - middle, 4 - high, 5 - very high.

Based on statistical data, the following fuzzy rules describing correlation of chosen indicators for a period 1991-2002 (tab.2) are added to the specified rules:

Table 2. Fuzzy rules corresponding to 1991-2002

Indicators		***Numbers of rules (continued)***											
		11	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***	***21***	***22***
		1991	***1992***	***1993***	***1994***	***1995***	***1996***	***1997***	***1998***	***1999***	***2000***	***2001***	***2002***
GDP		2	1	1	1	1	3	5	5	5	5	5	5
INV	$\wedge$	1	1	1	5	1	5	5	5	1	1	5	5
NP	$\wedge$	1	1	1	1	1	2	2	5	5	3	4	4
EXP	$\wedge$	3	5	1	1	1	1	5	1	5	5	5	5
IMP	$\wedge$	2	3	2	5	5	2	2	1	5	1	1	1
PN	$\wedge$	4	4	2	2	3	5	4	1	3	5	5	5
CPI	$\wedge$	1	1	1	1	1	1	1	5	5	1	3	3

For example, it is necessary to read 11-th rule from table 2 as follows: If INV is *very low* and NP is *very low* and EXP is *middle* and IMP is *low* and PN is *high* and CPI is *very low*, then GDP is *very low*.

When constructing table 2 the fact that regularity existing between indicators is also preserved for some range of their values has been taken into consideration.

Mamdani algorithm, in which operation of a minimum is chosen for an implication, operation of a maxima for a composition and the method of centroid for defuzzyfication, is used to derive endogenous values of GDP.

This list fuzzy rules can be continued on the basis of knowledge of the expert of interrelation of the chosen indicators.

Based on the fuzzy model the values of GDP were forecasted based on the statistical values of other exogenous variables using FUZZY LOGIC TOOLBOX package.

The following table 3 shows the results of calculations for values of GDP under different hypotheses concerning values PN and CPI using Mamdani [6] system types. The values INV, NH, EXP, IMP are kept constant and equal to their values in 2002.

Table 3 shows calculated (GDP_C) and actual (GDP_A) values of GDP.

Table 3. The outcomes obtained using Mamdani algorithm

Indicators	***Years***					
	1991	***1992***	***1993***	***1994***	***1995***	***1996***
INV	-27,4	-7,7	-20,4	41,6	-71,0	123,5
NP	-6,4	-5,1	-7,2	-6,8	-4,2	-1,1
EXP	3,3	46,2	-48,8	-10,6	-55,2	-22,3
IMP	4,3	2,6	7,0	-16,3	-40,5	6,0
PN	19,1	18,7	16,7	15,7	17,1	20,4
CPI	83,6	1065	747,6	1384	545,8	26,5
GDP_A	-0,7	-22,6	-23,1	-19,7	-11,8	1,2
GDP_F	-7,86	-7, 97	-7,87	-8,07	-8,07	2,3
GDP_A - GDP_F	7,16	-14,63	-15,23	-11,63	-3,73	-1,1

continued

Indicators	***Years***					
	1997	***1998***	***1999***	***2000***	***2001***	***2002***
INV	33,5	32,1	-5,6	-1,8	19,2	75,1
NP	0,0	25,3	21,1	1,4	6,4	2,7
EXP	27,2	-7,1	51,6	69,2	14,7	14,3
IMP	3,6	22,1	-5,6	8,0	7,9	48,8
PN	18,9	12,5	17,4	27,6	21,8	23,8
CPI	5,8	-8,3	-10,8	6,1	1,5	2,8
GDP_A	9,2	18,8	23,0	17,8	11,0	8,3
GDP_C	11	11,3	11,3	11,1	8,72	8,88
GDP_A - GDP_C	-1,8	7,5	11,7	6,7	2,28	-0,58

The penultimate line in the table 3 shows the values of GDP calculated using our model for real statistical values of indicators INV, NP, EXP, IMP, PN, CPI from 1991 to 2002. The last line reflects the difference between calculated and actual values of GDP.

Apparently for a period after 1995 calculated values of GDP are in line with their actual values, which is explained by stabilization of economy in this period. Rather big deviations observed in 1998-1999, are related to the number of well known crisis phenomena that took place in the world in the considered period. Calculated and actual values of GDP in 1996, 1997, 2001 and 2002, show the adequacy of the created fuzzy model for the real stable condition of Azerbaijani economy in the last years.

Thus, data from table 3 confirm that utilized fuzzy model has allowed for simulating the macroeconomic processes happening in the Republic in a retrospective period (1991-2002) and to realize their forecast on prospect.

Table 4 shows outcomes of calculations for GDP values under various hypotheses of values of PN and CPI. Thus, values of INV, NP, EXP, IMP are held constant and equal to their values in 2002.

As it is shown in the table 4, at price cut of petroleum in the global markets up to 17 dollars for a barrel and at normal level of inflation (1.2%) negative growth rate of GDP and corresponding worsening of the economic situation is observed.

Table 4. Results of the forecast

Indicators	***Hypotheses***							
	1	2	3	4	5	6	7	8
INV	75,1	75,1	75,1	75,1	75,1	75,1	75,1	75,1
NH	2,7	2,7	2,7	2,7	2,7	2,7	2,7	2,7
EXP	14,3	14,3	14,3	14,3	14,3	14,3	14,3	14,3
IMP	48,8	48,8	48,8	48,8	48,8	48,8	48,8	48,8
PN	25	24	23	22	21	20	19	18
CPI	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2
GDP	9.44	9.44	9.44	9.44	9.44	9.44	5.5	2.54

continued

Indicators	***Hypotheses***							
	9	10	11	12	13	14	15	16
INV	75,1	75,1	75,1	75,1	75,1	75,1	75,1	75,1
NH	2,7	2,7	2,7	2,7	2,7	2,7	2,7	2,7

EXP	14,3	14,3	14,3	14,3	14,3	14,3	14,3	14,3
IMP	48,8	48,8	48,8	48,8	48,8	48,8	48,8	48,8
PN	17	16	23	22	21	20	19	18
CPI	1,2	1,2	1,5	2	2	3	3	3
GDP	-0.29	8.88	8.88	8.88	8.88	8.88	0	0

From the above table it follows that upon decline in oil price in the world markets up to 17 dollars for barrel and at a normal rate of inflation 1.2% (Hypothesis 10**)**, the negative growth rate of GDP and corresponding aggravation of the economic situation is observed.

Based on information in table 4, the linear regression is found to be in the following form:

$$GDP = -7.50 + 0 \cdot INV + 0.6 \cdot NH - 0.1 \cdot EXP +$$
$$+0.1 \cdot IMP + 0.8 \cdot PN + 0 \cdot CPI$$

where, the coefficient of determinacy is r2=1 and the observed F-statistics, which is used to determine whether the observed correlation between dependent and independent variables is random or not, is equal FN=20.1. Under confidence probability of 0.05 and 6 and 5 degrees of freedom the critical value for F-statistic is equal FK=4.95 and is significantly less than the observed value of F-statistic. Consequently, the obtained regression equation is effective for predicting values of GDP. However, zero coefficient of *INV* seems to be illogical.

Table 5 below shows the results of estimations for the values of GDP using the obtained linear regression under above-mentioned hypotheses.

Table 5. Forecast results with linear regression

Indica-tor	Hypotheses															
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***	***16***
GDP	10,2	9,5	8,9	8,6	7,6	7	6,3	5,7	5,1	4,4	10	10	9,5	6,9	6,3	5,7

As it is shown in the table 5, at oil price cut in the global markets up to 16 dollars for a barrel and at normal level of inflation of 1.2 % (Hypothesis 10**)** positive growth rate of GDP is observed which is not corresponding to reality.

The problem under consideration has been also solved using neural networks where data for 2002 has been used for testing and the remaining sets for training. Estimation results for actual values are given in table 6.

Table 6. Neural Networks results for actual values

Neuro net	Years					
	1991	**1992**	**1993**	**1994**	**1995**	**1996**
GDP	-2,7	-17,2	-16,7	-17,5	-14,8	1,4
Deviation from actual	-2,0	5,5	6,4	2,2	-3,0	0,2

continued

Neuro net	Years					
	1997	**1998**	**1999**	**2000**	**2001**	**2002**
GDP	9,7	18,0	22,1	18,3	15,5	5,6
Deviation form actual	0,5	-0,8	-0,9	0,5	4,5	-2,67

Table 7 shows estimation results for values of GDP using this neural network model under above-mentioned hypotheses.

As it is shown in the table 7, at oil price cut in the global markets up to 16 dollars for a barrel and at normal level of inflation of 1.2 % (Hypothesis 10**)** positive growth rate of GDP is observed which is also not corresponding to reality.

Table 7. Forecast Results using neural networks

Neuro net	1	2	3	4	5	6	7	8
GDP	14,8	14,5	14,2	13,9	13,5	13,1	12,8	12,4

continued

Neuro net	9	10	11	12	13	14	15	16
GDP	12,0	11,5	14,7	14,5	14,1	12,1	11,7	11,2

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Application of Fuzzy Cognitive Maps to the Political-Economic Problem of Cyprus

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Abstract

The Cyprus problem exists since the early 1960s, but has been amplified after the Turkish invasion of the island in 1974. The politico-economic dynamics involved are related to the interests of both the Greek Cypriot community and the Turkish Cypriot community. Furthermore, there are antagonistic and cooperative interests among other countries involved in the conflict. These are primarily Greece, Turkey and the United Kingdom. The expected entry of Cyprus into the European Union, as well as the recent terrorist events, has made the involvement of Europe, USA, Russia and Israel important influential-dynamical factors.

An attempt to model the dynamics of the problem, using Fuzzy Cognitive Maps has been made. The main parameters involved in the interrelated dynamics are nationalism, religiousness, knowledge of history, level of educational development, tourism, prospect of finding crude oil, other natural resources, Anatolian settlers, and the general interests of the countries involved, as well of course the general interests of two communities.

The system that has been developed can de used to study the effects of a change in any parameter, or a combination of parameters, on the stability and growth of the remaining parameters. Different scenarios may be implemented, observed and appraised.

Keywords

Fuzzy cognitive map, Cyprus politico-economic problem.

1. Introduction

Cyprus is a small island in the eastern Mediterranean sea. It has about 660,000 inhabitants. The population consists of about 78% Greek-Cypriots (GC), 18% Turkish-Cypriots (TC) and the remaining 4% of Armenian, Maronite and Latin-Cypriots. Turkish and Greek Cypriots lived together on the island for almost five centuries. The Cyprus problem exists since the early 1960s, but has been amplified after the Turkish invasion of the island on July 20, 1974. Turkey launched a second offensive in August 1974 and as a result occupies about one third of the territory of the Republic of Cyprus. Since 1974 there has been a systematic, planned and steady inflow of Anatolian settlers in the occupied northern part of the Republic. This has been identified as a systematic process of changing the ethnographic characteristics of the country. Presently, these Turkish settlers are estimated at about 130,000. These people have changed the social fabric of the Turkish-Cypriot community, and introduced a new set of parameters and dynamics into the Cyprus problem.

The dynamics involved in characterizing the security and stability of the island, as well as its economic development, are related to the interests of both the Greek-Cypriot and the Turkish-Cypriot community. Due to the historical roots of the two communities and the strategic position of Cyprus, there are antagonistic and cooperative interests among other countries involved in the problem. These are primarily Greece, Turkey and the United Kingdom. The expected entry of Cyprus into the European Union, as well as the recent terrorist events (USA on September 11 2001 and Spain on March 11 2004), has made the involvement of Europe, USA, Russia and Israel to be important influential-dynamical factors.

An attempt to model the dynamics of the problem, using a fuzzy cognitive map-like approach has been made. The main parameters involved in the interrelated dynamics are: Nationalism, religiousness, knowledge of history, level of educational development, tourism, oil, other natural resources, settlers, and the general interests of the above countries and the two communities.

Recently there have been intensive efforts to solve the problem based on a series of UN plans that have been prepared by a team appointed by the UN Secretary General Kofi Annan. The final plan (version 5) provides for a new state of affairs, whereby the Republic of Cyprus is to be evolved into the United Cyprus Republic, based on bi-zonality, bi-community and on a federal structure composed of two constituent federal states. The plan incorporates strong mechanisms for cooperative advancement, as well as mechanisms that maintain the dividing lines. The proposed plan (version 5) for the settlement of the problem will be voted in referenda during April 2004, to decide whether it is accepted or not.

A cognitive map consists of a set of cause-effect *relationships* that are presumed to exist among specific *concepts* (factors, parameters, activations, attributes, …). Using a suitable and systematic process, it is possible to establish the effect of a change in the state of any other concept (or set of concepts), or on the entire system. Different fuzzy cognitive map (FCM) paradigms have been proposed [Craiger et al 1996, Hagiwara 1992, Kosko 1986, Kosko 1992, Schneider et al. 1998] as well as different application areas [Dickerson and Kosko 1994, Pelaez and Bowles 1996, Stylios et al. 1997]. Each concept may be related to the other concepts through appropriate degrees of effectiveness (causality, sensitivity).

The present work is an effort to apply an FCM variant to the dynamics of the Cyprus problem, aiming at getting some useful insight on the effectiveness and interactivity of the most important parameters that govern the politico-economic dynamics of the system.

2. Dynamic simulation

A growing dynamical system may be modeled as a quasi-statically and interactively evolving set of components, so that changes in the various parameters (concepts in the established FCM terminology) affect either directly or indirectly all other parameters of the system. The basic algorithm that may be followed is depicted in Figure 1.

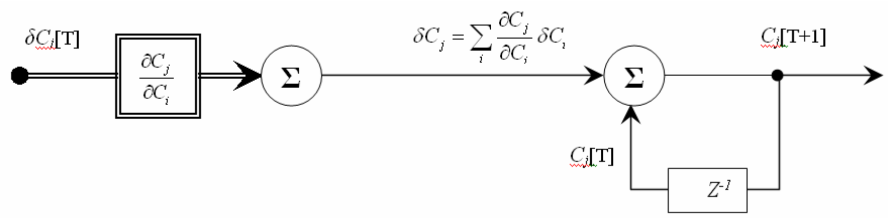

Fig. 1. The basic unit adopted for the evolution of the system

Where C's are the concepts, Z^{-1} the unit delay, and T the discrete integration counter. The partial derivatives $\frac{\partial C_j}{\partial C_i}$ are considered to be the sensitivities (or influencing factors, or weights, or degrees of effectiveness, or causality). These sensitivities S_{ij} characterize the degree by which a change in concept i may affect (either increasing or decreasing) another concept j. The FCM we adopted is forced to evolve as per the equations shown in Figure 1.

3. The FCM concepts

For this study, 30 influencing parameters (concepts) have been explored. These are presented in Table 1.

Table 1. The various influencing parameters that have been studied

	INFLUENCING PARAMETERS (Concepts)
C1	Welfare of the Federal State of Cyprus
C2	Welfare of the Greek Cypriot State
C3	Welfare of the Turkish Cypriot State
C4	Greek Cypriot nationalism
C5	Christian religiousness
C6	Knowledge of Turkish language by the Greek Cypriots
C7	Knowledge of Turkish history by the Greek Cypriots
C8	Educational level of the Greek Cypriots
C9	Turkish Cypriot nationalism
C10	Islamic religiousness
C11	Knowledge of Greek language by the Turkish Cypriots
C12	Knowledge of Greek history by the Turkish Cypriots
C13	Educational level of the Turkish Cypriots
C14	Political interests of Europe
C15	Political interests of USA
C16	Political interests of Russia
C17	Political interests of UK
C18	Political interests of Israel
C19	Political interests of Greece
C20	Political interests of Turkey
C21	Military interests of Israel
C22	Military interests of Greece
C23	Military interests of Turkey
C24	Military interests of UK
C25	Military interests of USA

C26	Interests of Anatolian settlers
C27	Level of tourism in the federated state
C28	Oil fields
C29	Quality of environment
C30	Other natural resources

It should be emphasized that these factors are difficult to quantify. They are not agreed by every person involved in the analysis and decision. Their initial values and the interrelated causalities are not universally agreed. Therefore, some form of fuzzy approximation may be employed by combining the knowledgeable opinions of various experts and the extensive experience as well as the cumulative wisdom of politicians. In the present study, the decision on which parameters to use, what initial values and what sensitivities to employ, has been based on feedback from such persons.

Furthermore, each parameter embodies a general state (and associated trend), as it is commonly employed and understood. For instance by the term "*Welfare of the Federal State of Cyprus*" we mean a state that has characteristics of prosperity, political stability, social development, high quality of life, etc.

Some parameters are included, even though they are non-existent presently – e.g. the presence of oil fields and other natural resources – because of the prospects of finding such, and as an input for a possible systematic scenario. Of course such developments, as well as a much awaited political solution of the problem, necessitate a drastic re-evaluation of the parameters, their interactivities, sensitivities and initial values.

The initial values of these parameters as well as the sensitivities themselves are fuzzy in the sense that they can not be known with certainty, and they represent a general state as being defined by some expert.

4. The parameter sensitivities

The sensitivities S_{ij} have been used to update the various concepts by implementing the general equation 3. The sensitivities that have been used are as shown in Table 2. They were suggested by few knowledgeable persons that have been inquired to give their expert advice.

$$\delta C_j = \sum_{i=1}^{30} \frac{\partial C_j}{\partial C_i} \delta C_i = \sum_{i=1}^{30} S_{ij} \delta C_i \tag{3}$$

Obviously, different experts, or more opinions, would result in possibly different values. They express the accumulated wisdom of knowledgeable persons and thus are fuzzy.

5. Results of simulations

Due to the constraint of having to be confined to a reasonably short paper, only three cases will be presented. These are:

- The effect on all other parameters in the case of a significant increase in the Islamic religiousness
- The effect on all other parameters in the case of a significant increase in the political interests of Europe
- The effect on all other parameters in the case of a significant increase in the political interests of USA

In order to study these effects, the system was forced to grow from a set of starting values for each of the influencing parameters, setting the parameter under study to zero value. The system, after few iterations (no more than 20 were needed), gradually settles to new parameter values. The same was done when setting the parameter under study to its maximum possible value (100%). Table 3 shows results for each of the three parameters studied. The results shown are for changes in the influenced parameters, and are also depicted graphically in Figure 2.

With this model, it is noted that a significant increase in the Islamic religiousness results in significant increase in the Greek Cypriot nationalism (+36%) and most importantly a very high increase in Christian religiousness (+283%). Also, this will result in an increase in the military interests of Turkey, Greece and Israel, as depicted in the table. Of significance is the observation that the welfare of the Federal State of Cyprus will reduce by 7% and the welfare of the Turkish Cypriot community by 4%. Some reduction in the level of tourism and the quality of the environment is also observed.

As for the case of significant increase in the European political interests in Cyprus, it may be observed that the model predicts a significant increase of the knowledge of Greek language and history by the Turkish Cypriots and vice versa. This is more pronounced in the case of knowledge of the Turkish language and history by the Greek Cypriots. It is however very interesting that the model suggests a significant reduction in Christian religiousness (-28%) and somewhat less (-5%) in the Islamic religiousness. Most interesting though, is the suggestion that the interests of the Anatolian settlers will reduce by 20%.

For the case of significant increase in the political interests of USA in Cyprus, the model predicts a 12% increase in the political interests of Israel and a 12% increase in the welfare of the Turkish Cypriots. What is most interesting though, and somewhat unusual, is the reduction of the Christian religiousness by 16%, of the Turkish Cypriot nationalism by 18%, of the Islamic religiousness by 8%, of the political interests of Greece and Turkey by 11% and 16% respectively, and of the military interests of UK by 18%.

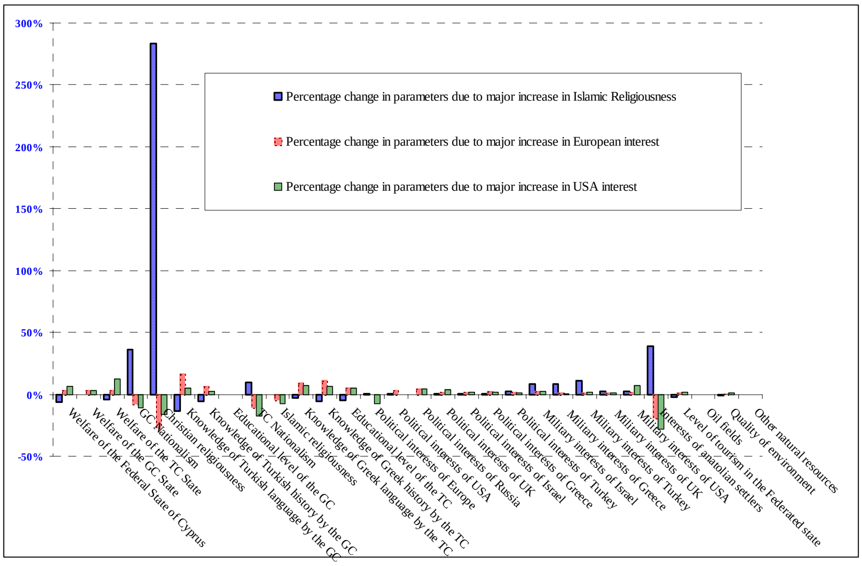

Fig. 2. Effects of significant changes in Islamic religiousness, European interests and USA interests

6. Concluding remarks

The FCM system that has been developed has shown reasonable trends and effects in relation to the studied scenarios, but also suggested some unexpected trends. It is an interesting tool to study the trends and the dynamics involved. One should not look at the absolute values of the parameters involved, which are highly fuzzy in the first place, but rather the relative changes. The system is easily expandable to include more influencing factors, focusing in more specific effects such as the overall economy or sectors of it. Further explorations are needed to verify its credibility and to fine-tune the various parameters and processes.

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Table 2. The sensitivities that have been used in the FCM. Concept *i* (row) influences concept *j* (column)

0	0.7	0.8	-0.1	0	0.2	0.1	0.6	-0.6	-0.7	0.8	0.2	0.5	0.1	0.4	0	0.1	0.4	0.8	0.8	0	0.1	-0.1	-0.2	0.5	0.2	0.5	0	0.1	0
0.6	0	0.3	-0.3	-0.1	0.3	0.1	0.3	0.1	0.1	0.6	0.2	0	0.2	0.2	0.2	0	0.2	0.7	0.1	0	0.2	-0.2	-0.2	0.3	-0.1	0.2	0	0.3	0
0.2	0.1	0	0.1	0.1	0.2	0.1	0	-0.1	-0.3	0.8	0.2	0.6	0.1	0.6	-0.1	0	0	-0.1	0.8	0	-0.2	0.5	0	0.4	0.2	0.1	0	0.2	0
-0.2	-0.1	-0.9	0	0.2	-0.2	-0.1	-0.1	0.8	0.8	-0.5	-0.7	0	-0.1	-0.1	0	-0.2	0	0.1	-0.7	-0.1	0.2	-0.6	-0.1	0	-0.3	0	0	0	0
-0.3	0	-0.5	0.6	0	-0.1	-0.1	-0.1	0.6	0.8	-0.2	-0.1	-0.1	-0.2	0	0.1	0	0	0.1	-0.1	0	0.1	-0.1	0	0	-0.2	-0.1	0	0	0
0.3	0.1	0.4	-0.1	-0.1	0	0.4	0.2	-0.2	-0.2	0.1	0.1	0.1	0.2	0	0	0	0	0.1	0.4	0	0	0	0	0	0.1	0.2	0	0	0
0.1	0	0.3	0	0	0.2	0	0.2	-0.1	-0.1	0.1	0.1	0.1	0.1	0	0	0	0	0.1	0.2	0	0	0	0	0	0	0	0	0	0
0.5	0.5	0.2	-0.2	-0.1	0.2	0.1	0	0	0	0	0	0.2	0.4	0.1	0.1	0.1	0.1	0.4	-0.1	0.1	0.4	-0.1	0.1	0.2	-0.1	0.4	0	0.6	0
-0.4	-0.5	-0.1	0.4	0.3	-0.1	-0.1	0	0	0.3	-0.1	-0.1	-0.5	-0.4	-0.7	-0.2	-0.1	-0.7	-0.4	0.2	-0.1	-0.3	0.2	-0.1	-0.3	0.1	-0.3	0	-0.3	0
-0.3	-0.3	-0.1	0.2	0.3	-0.1	-0.1	0	0.8	0	-0.1	-0.3	-0.2	-0.2	-0.3	-0.1	-0.1	-0.4	-0.1	0	-0.2	-0.1	0	0	-0.1	0.2	-0.4	0	-0.2	0
0.2	0.1	0.1	-0.1	-0.1	0.2	0.1	0	-0.1	-0.1	0	0.2	0.2	0.3	0.2	0.1	0.1	0	0.3	0	0.1	0.1	0.2	0	0.1	-0.1	0.2	0	0.1	0
0.1	0.1	0.1	-0.1	-0.1	0.1	0.1	0	0.1	-0.1	0.1	0	0.2	0.2	0.1	0.1	0	0	0.2	0	0	0	0	0	0	0	0.1	0	0	0
0.6	0.2	0.8	-0.1	-0.1	0.2	0.1	0.1	-0.4	-0.3	0.2	0.1	0	0.3	0.2	0.1	0.1	0	0.3	0.1	0	0.1	0	0	0	-0.1	0.5	0	0.4	0
0.6	0.4	0.4	-0.1	0	0.1	0.1	0.3	-0.3	0.1	0.2	0.1	0.1	0	-0.7	0.1	0.1	0	0.4	0.3	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0	0.1	0.1	0.3	0.1	0.1	0.1	0	0	0	0.1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0.2	0	0.1	0	0.1	-0.1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0	0	0	0	0	0	0	0	0.2	0.1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.1	0	0.1	0	0	0	0.2	0	0.1	0	0.1	0	0	0	0	0
0.1	0.2	0	0.1	0	0	0	0.1	0.1	0	0	0	0	0.2	0.1	0.1	0.1	0	0	0.8	0	0.2	0.3	0.1	0.1	-0.3	0	0	0	0
0.1	0.1	0.2	0.2	0.1	0	0	0	0.2	0.1	0	0	0	0.2	0.1	0.1	0.1	0.1	0.4	0	0.2	0.2	0.6	0.1	0.2	0.5	0	0	-0.1	0
-0.1	-0.1	-0.1	0.1	0.2	0	0	0	0	0.5	0	0	0	0.2	0.2	0.1	0.1	0.8	0.2	0.2	0	0.3	0.4	0.2	0.2	0	-0.4	0	0	0
-0.1	0	-0.3	0.5	0.2	-0.2	-0.1	0	0.6	0.5	-0.1	0	0	0.1	0.1	0.1	0.1	0	0.5	0.8	0	0	1	0.2	0.2	-0.3	0	0	0	0
-0.3	-0.5	-0.1	0.8	0.4	-0.1	0	0	0.9	0.6	0	0	0	0.2	0.3	0.2	0.1	0.1	0.4	0.7	0.1	0.9	0	0.1	0.4	0.2	-0.6	0	0	0
-0.2	-0.1	-0.1	0.3	0	0	0	0	0.3	0.1	0	0	0	0.1	0.1	0	0.5	0.1	0.1	0.1	0.2	0	0.1	0	0.3	0	0.4	0	0.1	0
-0.1	-0.1	-0.1	0	0	0	0	0	0.1	0.1	0	0	0	0.1	0.5	0.4	0.1	0.2	0.1	0.1	0.8	0.2	0.2	0.5	0	0	0	0	0	0
-0.4	-0.7	-0.1	0.3	0.1	0	0	0	0.4	0.8	0	0	-0.2	-0.5	-0.7	-0.3	-0.1	-0.6	-0.6	0.2	-0.1	-0.1	0.2	0	-0.2	0	-0.4	0	-0.3	0
0.6	0.6	0.7	-0.2	0	0	0	0.1	-0.3	-0.1	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0	0	-0.1	0
0.8	0.8	0.8	0	0	0.1	0	0.1	0	0	0.1	0	0.1	0.8	0.8	0.4	0.4	0.4	0.7	0.7	0.2	0.8	0.8	0.6	0.8	0.2	-0.4	0	-0.4	0
0.2	0.2	0.2	0	0	0	0	0.1	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0.4	0	0	0
0.5	0.5	0.5	0	0	0	0	0	0	0	0	0	0	0.4	0.4	0.1	0.2	0.1	0.5	0.5	0	0	0	0	0	0.2	-0.1	0	-0.2	0

Table 3. Effects of significant changes in Islamic religiousness, European interests and USA interests

	Welfare of the Federal State of Cyprus	Welfare of the GC State	Welfare of the TC State	GC Nationalism	Christian religiousness	Knowledge of Turkish language by the GC	Knowledge of Turkish history by the GC	Educational level of the GC	TC Nationalism	Islamic religiousness	Knowledge of Greek language by the TC	Knowledge of Greek history by the TC	Educational level of the TC	Political interests of Europe	Political interests of USA
Percentage change in parameters due to a major increase in Islamic Religiousness	-7	0	-4	36	28 3	-13	-6	0	9	---	-3	-6	-5	0	0
Percentage change in parameters due to a major increase in European interest	3	3	3	-8	-28	16	7	0	-11	-5	9	11	5	---	3
Percentage change in parameters due to a major increase in USA interest	7	3	12	-11	-16	5	2	0	-18	-8	7	6	5	-8	---

	Political interests of Russia	Political interests of	Political interests of Israel	Political interests of Greece	Political interests of Turkey	Military interests of Israel	Military interests of Greece	Military interests of Turkey	Military interests of UK	Military interests of USA	Interests of Anatolian settlers	Level of tourism in the	Oil fields	Quality of environment	Other natural resources
Percentage change in parameters due to a major increase in Islamic Religiousness	0	0	0	0	3	8	8	11	2	3	39	-2	---	-1	---
Percentage change in parameters due to a major increase in European interest	4	2	2	3	2	3	1	1	1	2	-20	1	---	0	---
Percentage change in parameters due to a major increase in USA interest	7	3	12	-11	-16	5	2	0	-18	-8	7	6	5	-8	---

Fuzzy linear regression
Application to the estimation of air transport demand

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Abstract:
The demand for an air transport market is very sensitive to many factors. The vagueness of the impacts of all these factors makes the task of prediction of this demand by classical methods very hazardous, especially when this estimation is used afterwards for critical decisions such as those related with the definition of supply (frequency of flights, number of seats put on the market, trip price..). Then it appears that crisp methods are not able to take fully into account all the uncertainty making up the demand while possibilistic reasoning could be a way to catch it. Following this idea, it is shown in this communication how regressions based on fuzzy logic which combine statistics and experts' attitudes can be used to improve the estimation for air transport demand.
In the first section of the communication, following Tanaka's model, fuzzy linear regression is introduced. Then in the second part an extension using trapezoidal fuzzy numbers is displayed. Finally, in the last section, the application of the proposed fuzzy linear regression to the estimation of air transport demand is considered.

Keywords: Fuzzy Logic, Fuzzy Regression Analysis, Demand Estimation

1. Introduction:

The purpose of regression analysis is to relate analytically the variation of a dependent variable Y in terms of explanatory variables $x_1,...,x_N$. An estimation of Y denoted $\hat{Y}$ in terms of $X(={}^t[1\ x_1\ ...\ x_N])$ can be obtained from data samples (see table 1) through a linear statistical regression. The analysis of this latter has been much considered [2], where *f* was *naturally* taken as a crisp linear function such as:

$$f(X) = a_0 x_0 + a_1 x_1 + ... + a_N x_N \quad \text{with} \quad x_0 = 1 \tag{1}$$

where $a_0, a_1,..., a_N$ are real values. Defining $A={}^t(a_0, a_1,..., a_N)$, we can write: $f(X)={}^tAX$.

Since in general the relationship between the input and the output cannot be known exactly, a random variable u which represents the disturbance or the error term can be added to the right side of *(1)*:

$$y_i = f(X_i) + u_i \tag{2}$$

This disturbance term is a surrogate for the uncertainty due not only to the *a priori* affine form chosen for function *f*, but also to the omitted variables that affect the output. The vector of the parameters a_j is then estimated through a least square regression as: $\hat{A} = ({}^tTT)^{-1}\,{}^tT\Upsilon$ where T is the matrix composed by the inputs samples and Y is the vector of the output samples:

$$T = \begin{bmatrix} 1 & x_{11} & .. & x_{1N} \\ 1 & x_{21} & .. & x_{2N} \\ . & . & .. & . \\ . & . & .. & . \\ 1 & x_{M1} & .. & x_{MN} \end{bmatrix} \quad \text{and} \quad \Upsilon = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_M \end{bmatrix}$$

with the condition that the matrix ${}^tTT$ is non singular.

sample	Output	Inputs
1	y_1	$x_{11},...,x_{1N}$
.	.	.
.	.	.
i	y_i	$x_{i1},...,x_{iN}$
.	.	.
M	y_M	$x_{M1},...,x_{MN}$

Table 1: Observed input output data

Then, given a set of predefined inputs X, a crisp estimation of Y will be given by:

$$\hat{Y} = {}^t\hat{A}X \qquad (3)$$

and to get some insight into the estimation error, strong assumptions related with the distribution of the data must be made (for example the values of the error terms can be supposed mutually independent and identically distributed [2] along a centred normal distribution $N(0,\sigma)$). In the following, fuzzy sets are used to contain the uncertainty related with the inputs-output relationship.

2. Tanaka's model:

2.1. Model's exposition:

In fuzzy linear regression (FLR) analysis [1], some of the assumptions of the classical statistical approach are relaxed and the uncertainty is traduced by a fuzzy relationship between the input and the output. Such a relationship is given by a fuzzy function $\tilde{f}$. The present paper considers first the model of Tanaka [5] which is a pioneer for such models.

The basic Tanaka's model assumes a linear fuzzy function:

$$\tilde{f}(X) = \tilde{A}_0 x_0 + \tilde{A}_1 x_1 + \ldots + \tilde{A}_N x_N = {}^t\tilde{A}X \qquad \text{With} \quad x_0 = 1$$

(4)

Where $\tilde{A}$ is the fuzzy vector of the model's parameters.

For every $j \in \{0,1,\ldots,N\}$, $\tilde{A}_j$ is a symmetric fuzzy number presented by $(\mathrm{c_j}, \mathrm{w_j})$ where c_j and w_j are respectively its centre and its width. The reference membership function of these numbers is denoted L and is such as:

- $L(x) = L(-x)$
- $L(0) = 1$
- L is decreasing on $[0,1[$
- $L(x) = 0$ when $x \in [1,+\infty[$
- L is concave on $]-1,1[$

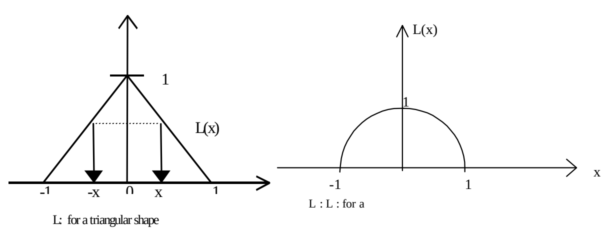


Fig.1: Examples of reference membership functions

The membership function $\mu_{\tilde{A}_j}$ is deduced from L as $\mu_{\tilde{A}_j}(a_j) = L((a_j - c_j)/w_j)$ when $w_j > 0$.

An interesting case is when the $\tilde{A}_j$ are triangular. It is the case the most developed in the literature; where:

$$L(x) = \begin{cases} 1-|x| & \text{if } -1 \le x \le 1 \\ 0 & \text{if not} \end{cases} \qquad (5)$$

and

$$\mu_{\tilde{A}_j}(a_j) = \begin{cases} 1-|c_j - a_j|/w_j & \text{if } c_j - w_j \le a_j \le c_j + w_j \\ 0 & \text{otherwise.} \end{cases} \qquad (6)$$

It can be shown (see [5]) that when the $\tilde{A}_j$ are triangular fuzzy numbers, then the resulting Y *(7)* is a triangular fuzzy number as well. The centre of Y is then ${}^tCX$ and its width is the sum of the widths of all the terms: ${}^tW|X|$, where C is the vector of the centres of the $\tilde{A}_j$ and W is the one of their widths . The membership function of Y is then given by:

$$\mu_Y(y) = \begin{cases} \text{Max}\,(0, 1-\dfrac{|y-{}^tCX|}{{}^tW|X|}), & \text{if } X \ne 0 \\ 1 & \text{if } X = 0, y \ne 0 \\ 0 & \text{if } X = 0, y = 0 \end{cases} \qquad (7)$$

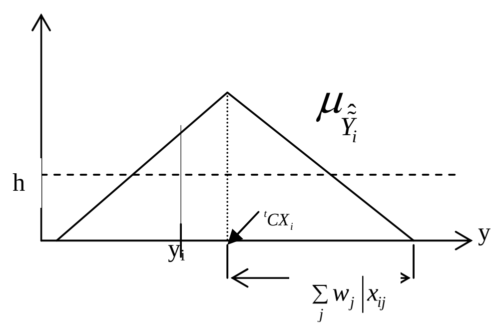


Fig.2. Membership function of $\tilde{Y}_i$

Then the uncertainty about Y is illustrated by the width of the membership function of the resulting fuzzy number. Given a set of data samples *D*, it appears to be of interest to minimise the total vagueness resulting from the fuzzy regression through the tuning of its parameters.
Given a threshold number h ($0 \le h \le 1$), let us define a reduced data set D_h where the sample i is retained if y_i has a membership degree greater than h:

$$\forall i \in \{1, 2, ..., M_h\}, \mu_{\tilde{Y}_i}(y_i) \ge h$$

(8)

where M_h is the size of D_h

This can be written: $L(\left|y_i - {}^tCX_i\right| / {}^tW\left|X_i\right|) \ge h$ and since L is decreasing over $[0,1[$ then:

$$\left|y_i - {}^tCX_i\right| \le L^{-1}(h).{}^tW\left|X_i\right| \qquad (9)$$

Observe that in the case of triangular fuzzy numbers, $L^{-1}(h) = 1 - h$. Let us estimate the total vagueness associated to D_h and W:

$$\sum_{i=1}^{M} (\sum_{j=0}^{N} w_j \left|x_{ij}\right|) = \sum_{j=0}^{N} (\sum_{i=1}^{M} \left|x_{ij}\right|) w_j \qquad (10)$$

Then a linear program can be formulated to minimise the total vagueness under an h-degree membership constraints over D_h :

$$\begin{cases} \delta_L^h = \underset{W,C}{Min} \quad \sum_{j=0}^{N} (\sum_{i=1}^{M} \left|x_{ij}\right|) w_j & (a) \\ \text{st} \quad \sum_{j=0}^{N} c_j x_{ij} + \left|L^{-1}(h)\right| \sum_{j=0}^{N} w_j \left|x_{ij}\right| \ge y_i \quad \forall i = 1, \dots M_h & (b) \\ \sum_{j=0}^{N} c_j x_{ij} - \left|L^{-1}(h)\right| \sum_{j=0}^{N} w_j \left|x_{ij}\right| \le y_i \quad \forall i = 1, \dots M_h & (c) \\ W \ge 0, C \in \Re^N, x_{i0} = 1; i = 1, \dots, M_h. & (d) \end{cases} \qquad (11)$$

The resulting linear fuzzy regression model will be denoted F_L^h .

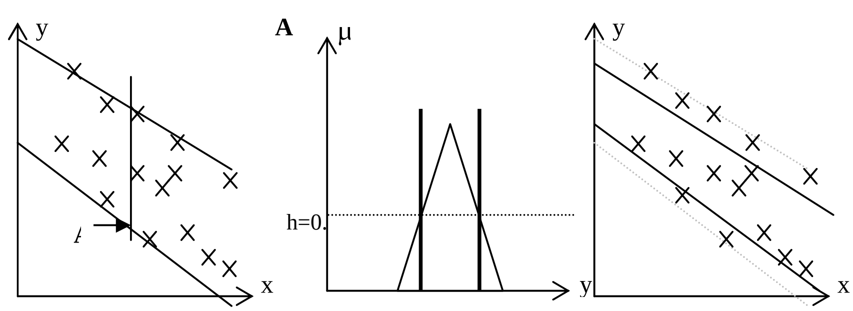


Fig.4.2 .Interval estimation and membership

2.2. Analysis of model:

If $C_h^* = {}^t(c_0^*, c_1^*, \ldots, c_N^*)$ and $W_h^* = {}^t(w_0^*, w_1^*, \ldots, w_N^*)$ compose the optimal solution of problem *(11)*, then the vector of estimated parameters resulting from the regression F_L^h is:

$$\hat{\tilde{A}}_L^h = (C_h^*, W_h^*)_L . \qquad (12)$$

When another membership degree *h'* ($h' \neq h$) is considered, it is easy to show that the resulting linear fuzzy regression $F_L^{h'}$ is given by: $\hat{\tilde{A}}_L^{h'} = (C_h^*, [L^{-1}(h)/L^{-1}(h')]W_h^*)_L$. Then, once a given reference function L is adopted, the LFR associated to a threshold h can be deduced from the one corresponding to $h = 0$.

This model can be interpreted as an estimation of the interval of the dependent variable Y. At the beginning ($h = 0$) an interval containing all the observations is defined and when an effective threshold h is chosen a resulting narrower interval is defined for the estimation. As some data samples located near the bounds of the current interval become outliers, they are removed from the refined data set. Some observations can be made here about this method: it can be instructive to interpret the detected outliers samples instead of merely removing them. This method does not take fully into consideration the effective dispersion of the data samples within the learning interval. When rather large uncertainties are involved, L may be not a strictly decreasing function on $[0,1[$ (trapezoidal numbers can be of interest in this case) and the above approach is no more applicable.

3. Extension of the Tanaka's model:

The proposed extension makes use of *level fuzzy functions* in the sense of Zimmermann[6]. A level fuzzy function $\tilde{f}$ is given by

- four level crisp functions: f_a, f_b, f_c, f_d .
- f_b, f_c provide the curves for which the degree of membership reaches 1.
- f_a, f_d provide the curves for which the grade of membership starts from zero.

For consistency reasons, these four functions cannot intersect on the input domain given by $[X_{\min}, X_{\max}]$ $(= [(x_1)_{\min}, (x_1)_{\max}] \times \ldots \times [(x_N)_{\min}, (x_N)_{\max}]$):

$$\forall x \in [X_{\min}, X_{\max}], f_a(x) \leq f_b(x) \leq f_c(x) \leq f_d(x)$$

Then a membership function can be attached to this level fuzzy function:

$$\mu_{\tilde{f}}(f(x)) = \begin{cases} (f(x)-f_a(x))/(f_b(x)-f_a(x)) & \text{if } f_a(x) \le f(x) \le f_b(x) \\ 1 & \text{if } f_b(x) \le f(x) \le f_c(x) \\ (f_d(x)-f(x))/(f_d(x)-f_c(x)) & \text{if } f_c(x) \le f(x) \le f_d(x) \\ 0 & \text{otherwise} \end{cases} \qquad (13)$$

A simple way to determine the extreme level functions f_a and f_d is to use the Tanaka's model considering the resolution of *(11)* for $h=0$:

$$\begin{cases} Min \quad \sum_{j=0}^{N} w_j \sum_{i=1}^{M} |x_{ij}| & (a) \\ \text{st} \quad \sum_{j=0}^{N} c_j x_{ij} + \sum_{j=0}^{N} w_j |x_{ij}| \ge y_i \quad \forall i = 1,\dots M & (b) \\ \quad \sum_{j=0}^{N} c_j x_{ij} - \sum_{j=0}^{N} w_j |x_{ij}| \le y_i \quad \forall i = 1,\dots M & (c\ ) \\ W \ge 0\ , C \in \Re^N, x_{i0} = 1; i = 1,\dots,M. & (d) \end{cases} \qquad (14)$$

giving: $f_a(X) = \sum_{j=0}^{N} c_j^* x_j - \sum_{j=0}^{N} w_j^* |x_j|$ and $f_d(X) = \sum_{j=0}^{N} c_j^* x_j + \sum_{j=0}^{N} w_j^* |x_j|$

The determination of the central functions f_b and f_c is not so straightforward. They provide the bounds of the certainty domain. There are many ways to define them, in the following two methods are considered.

a) Method using an h-cut:

The h-cut considered in the Tanaka's model can be used here to define the bounds of the set of possibilities that will correspond to the certainty domain. It is assumed that any output value having a membership level higher than a given level h_1 ($h_1 \in]0,1]$) is in the certainty domain. So f_b and f_c are here defined by the resolution of *(11)* where L is the triangular reference membership function and h is a chosen number in $[0,1[$.

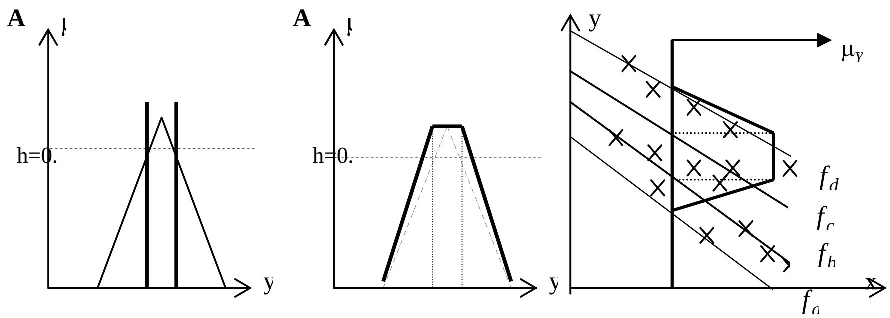


Fig.5 . Interval estimation and construction of trapezoidal

b) Mixed method

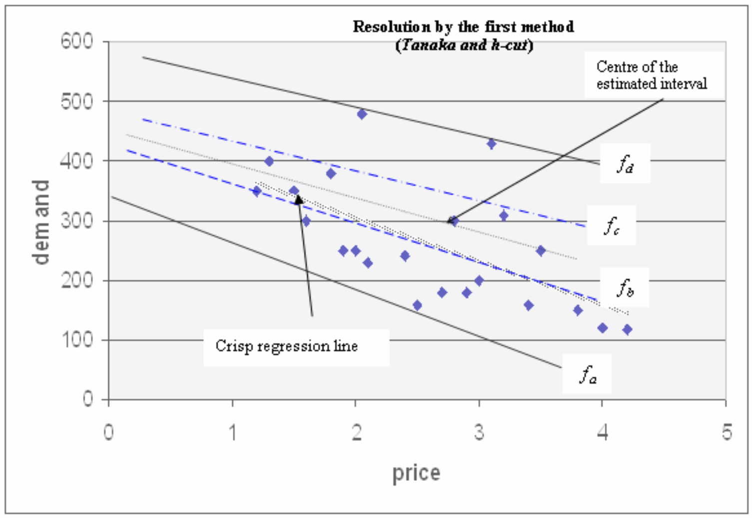
Since the methods presented above do not pay a direct attention to how the data samples are dispersed in the interval, with the mixed method, the level functions f_a and f_d are obtained through a 0-cut using Tanaka's model while a least square regression is used to determine the central level functions f_b and f_c . From the resulting statistical regression model $\hat{f}(X)$ and standard deviation σ , f_b and f_c are given by $f_b = \hat{f} - \lambda\sigma$ and $f_c = \hat{f} + \lambda\sigma$ where λ is a positive constant chosen by an expert depending about his opinion about the representativeness of the proposed samples. A large λ means that he has a poor opinion about their representativeness.

These two methods define trapezoidal fuzzy numbers taking into account all the data samples for the definition of their limits since they are effective realisations. Besides that, experts can choose directly the criteria used to determine the central functions f_b and f_c . The possibilities above $f_c(X)$ can be interpreted as corresponding to optimistic scenarios and the ones under $f_b(X)$ can be associated to pessimistic conditions.

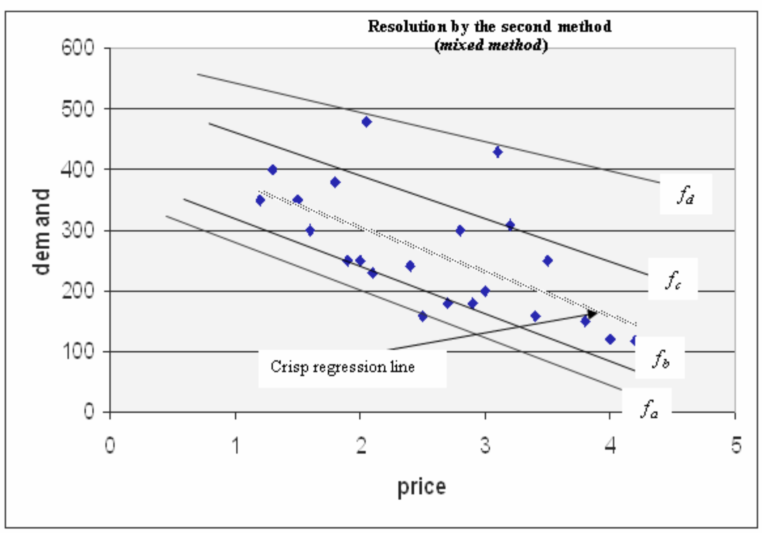
4. Example

In the air transport market uncertainty is very frequent so let us consider a simple example of air transport demand estimation as function of ticket price. The table below provides a set of data samples:

price(u.m)	2	1,5	3	4	2,5	1,8	2,4	3,5	2,8	3.1	3,2
Demand	250	350	200	120	160	380	240	250	300	430	310
price(u.m)	3,8	1,6	1,3	2,1	1,9	3,4	2,7	1,2	4,2	2.05	2,9
Demand	150	300	400	230	250	160	180	350	118	480	180



Tanaka's method in this example provides an estimate of the demand function centred in the interval (h=0), a large number of observations is then excluded, contrarily to the proposed method to build trapezoidal fuzzy numbers which keeps all these data samples. In this case, it can be observed that the concentration of the data is rather bellow the centre of the fuzzy estimation.



With the second trapezoidal fuzzy numbers method, the crisp linear regression estimate remains in the centre of the base of the trapezoidal fuzzy estimation.

5. Conclusion

In this paper, a new approach for fuzzy linear regression analysis has been introduced. This approach is inspired from the Tanaka's method. The target of this approach is to build trapezoidal fuzzy sets for the estimated variable. It tries also to take into account all the data samples and sometimes the dispersion of these latter. A simple example has been treated to compare these methods.

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Methodology of The Estimation of Quality of Objects with Complex Structure Under Conditions of Non-Stochastic Uncertainty

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Abstract

In this article methodology of the quality estimation of objects with complex structure is generalized, based on the analysis of the quantitative and qualitative imperfect and changeable data. In order to assess decision support systems (DSS) quality an example of this methodology using is given. Ratings of objects are compared, obtained by means of two variants of the methodology.

1. Introduction

The transitive transformations in economics of the Republic of Belarus are characterized by unstable dynamics of macroeconomic conditions. Statistic information is subject to sharp conjunctural fluctuations and has weak comparability in time. Such conditions, described by structural variability of the data and irregular situations presence, are defined as the data non-stochastic uncertainty. In case of the assumption that economic processes have a stochastic character casual approach is used for the data analysis. However, methods of probability theory and mathematical statistics are inefficient under conditions of non-stochastic uncertainty. Therefore, the perfection of approaches to construction of the methodology of economic information processing, which corresponds to such conditions, is actual.

2. Generalized methodology of quality evaluation of the objects with complex structure

Processes which take place in economic environment are characterized by complexity, uncertainty and have weak structurization. Therefore, they could not be described with classic mathematical formulations. A number of researches, concerning the assessment of the enterprise financial statement, investment attractiveness [2, 4] and production processes [1] quality were carried out by the authors. Bank rating, insurance organizations rating and sovereign credit rating [3] were obtained on the base of evaluating indices fuzzy classification and aggregate complex index calculation. In general case the methodology of estimation of quality of the objects with difficult structure consists of the following stages.

1. Determination the indices influencing on the object quality. The indices may have both quantitative and qualitative character. If the number of indices is large, indices are united in several separate groups.
2. Assignment the values to the indices. Values of some indices could be taken from the statistic reports about analyzing object functioning. However, qualitative indices, such as accessibility of necessary resources for the investment project realization, perspectives of the branch development and other, may be estimated only by means of the examination by experts.
3. Estimation of significance degrees of the indices and of the significance degrees indices groups.
4. Indices normalization and complex aggregate index calculation which characterize the object statement.

At stages 3 and 4 the examinations by experts are used as a rule. Specialists competent in the appropriate sphere are involved into examination as the experts. Results of the examination may be represented in the form of fuzzy triangle or trapezoidal numbers. Fuzzy numbers are defuzzyficated during the examination results processing. Methods of data processing depend on the researched object essence. Quality assessment of the object with difficult structure may also be based on the theoretic-number approach. The main tools of such methodology are discrete description and analyzing object representation by means of the cognitive graphics. Methods of discrete mathematics may be used in all cases of the analysis of complex problems of management, and also in situations where systematization and convenient visual representation of the data concerning with analyzing problem and its structure are required.

3. Requirements to objects with complex structure

Quality parameters of the object are determined by a number of object properties. Quality management should account all factors influencing on these properties. A set of requirements is formed on the base of coordination of interconnected and reciprocally varied in time requirements to the object by target groups concerning to the object functioning. Target groups (and their quantity) may vary. It depends on the essence of analyzing object and on conditions of its activity. For example, producers and customers of some productions, creditors and share holders of the bank may be regarded as target groups. Thus, management of quantitative requirements to the object will allow to determine its "excesses" and "shortcomings", i. e. reserves and directions of its development. "Excesses" production leads to the resources loss, as "excess" does not bring benefits to target groups. For example, software product could have a number of functions unavailing for users, but programmer's work time was wasted on their realization, and presence of such functions led to program file size increase. "Shortcomings" lead to the competitiveness loss.

4. Theoretic-number approach to the quality assessment of objects with complex structure

Object quality may be represented by relative index characterizing this object accordance to a "perfect" sample. A set of parameter's values for the "perfect" object is determined by choice of the best characteristics of similar items, and on some parameters it is done by expert inquest. Suppose that 8 characteristics of some object are allocated. "Perfect" object characteristics after the processing (normalization, defuzzyfication, convolution the values in groups) are comparable and represented by $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$ set, and for analyzing object characteristics after some processing are represented by $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$ set. Elements of these sets should be ordered on decrease of the significance degrees of factors, and at equal significance degrees, they are ordered in both sets on decrease of absolute values of elements for the "perfect" sample. Define the measure of analyzing object competitiveness relatively to "perfect" object. Suggest defining it as an area of the figure obtained by crossing of the regions restricted by segments which connect adjacent endings of the element value vectors (see Figure 1). The ratio of this area to the area restricted by $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ and a_8 represents an index of accordance of the analyzing object to the "perfect" sample. An example of evaluation the competitiveness of production which includes welding constructions [1] is represented on the graph. A part of restricted by $b_1, b_2, b_3, b_4, b_5, b_6, b_7$ and b_8 points area, which is not crossed with restricted by $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ and a_8 points area, characterizes the “excesses” of analyzing object. A part of restricted by $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ and a_8 points area, which is not crossed with restricted by $b_1, b_2, b_3, b_4, b_5, b_6, b_7$ and b_8 points area, characterizes the “shortcomings” of analyzing object.

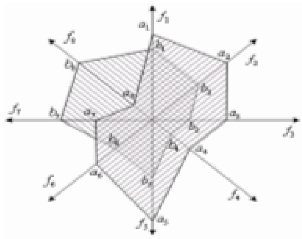

Fig. 1. Cognitive model of competitiveness parameter

In general case quality assessment of the object with difficult structure may be estimated by the following formula:

$$Q = \frac{S_{fact} \cap S_{best}}{S_{best}} = \frac{S_{fact} \cap S_{best}}{\sum_{i=1}^{n-1}\left(\frac{1}{2} \cdot a_i \cdot a_{i+1} \cdot \sin\left(a_i \cdot a_{i+1}\right)\right)} = \frac{S_{fact} \cap S_{best}}{\sum_{i=1}^{n-1}\left(\frac{1}{2} \cdot a_i \cdot a_{i+1} \cdot \sin\left(\frac{360^\circ}{n}\right)\right)} \qquad (1)$$

where S_{fact} – the area of region describing quality of the analyzing object; S_{best} – the area of region describing the “perfect” object; i – number of the “perfect” sample parameter (or a group of parameters); a_i – value of the “perfect” sample parameter after some processing (or intermediate assessment of a group of parameters); n – quantity of the evaluating parameters (or groups of parameters).

Information systems may serve as an example of objects with difficult structure. Below in the given article quality assessment of decision support systems (DSS) is expounded, as the special class of information systems.

5. Design and development of DSS and evaluation of their quality

Decision support system is a software product, which allows to solve professional work problems with the use of knowledge bases, databases, model bases, and to work out conclusions, recommendations, possible alternative variants of decisions. During information systems design and development (and DSS design and development as a class of information systems) investigation of requirement of target groups concerning to its functioning is recommended to make. Such groups are designers, producers (programmers), sellers (marketologists, specialist in technical support) and customers (users). Each of these groups has its own requirements to the software (DSS in our case), on the base of which a set of intercoordinated requirements (DPSC-requirements) is formed. The most successful project from several alternatives is that one, which provides the fullest satisfaction of requirements of target groups. Quality index Q_{DSS} may be determined as specific weight of satisfied requirements in the total set of requirements with taking in account significances of target group opinions, significances of requirements and degrees of satisfaction of requirements:

$$Q_{DSS} = \frac{\sum_{u=1}^{4}\left(r_u \sum_{j=1}^{n_u}\left(x_j(u)\cdot k_j\right)\right)}{\sum_{u=1}^{4}\left(r_u \sum_{j=1}^{n_u} x_j(u)\right)}\cdot 100\%, \qquad (2)$$

where u – number of target group; r_u – coefficients of significance of target groups opinions, $r_u \in [0;1]$; j – number of target group requirement; n_u – quantity of requirements of u-th target group; $x_j(u)$ – significance of j-th requirement; k_j - degree of satisfaction of j-th requirement.

The sum of coefficients of target group significances must be equal 1. Degree of satisfaction of requirement $k_j \in [0;1]$, and $k_j = 1$, if j-th requirement is fully satisfied, $k_j = 0$, if j-th requirement does not satisfied, $0<k_j<1$, if j-th requirement is partially satisfied. DSS quality will be the better, when Q_{DSS} value is the nearer to 100%.

A set of values of characteristics for "perfect" DSS may be represented as:

$$A=\{a_u | a_u = r_u \sum_{j=1}^{n_u} x_j(u)\}, \qquad (3)$$

and a set of values characteristics for analyzing sample may be represented as:

$$B=\{b_u | b_u = r_u \sum_{j=1}^{n_u}\left(x_j(u)\cdot k_j\right)\}. \qquad (4)$$

On the base of theoretical methods of the best decision choice DSS prototypes were created at the Department of Information Technologies of

BSEU. In Table 1 several DSSs are described and spheres of their application are shown.

Table 1.

DSS	Sphere of application	Using methodology	Main functions
Assistant Choice	The best alternative choice from several variants	Modification of Saaty's Analytic Hierarchy Process	Alternative evaluation by criteria in hierarchy; evaluation result – complex index for each alternative
Stock Exchange DSS	Analysis of securities quality and security rating building	Methodology of scoring the securities	Building the securities rating; getting the recommendations on selling, buying or holding the securities; result of evaluation – complex index for each security and broker's recommendation
Daina	Evaluation of financial statement of economic objects	Methodology "DAINA" for evaluation of financial statement of economic objects	Defuzzyfication of complex index of financial statement of economic object; graphical representation of results; export of report in different text formats
Study Expert	Expert examination for the best alternative choice from several variants	Saaty's Analytic Hierarchy Process; expert examination methodology	Expert examination in Intranet; demarcation of user access to expertise information and system functions; processing of expert data and generalization of expert inquest results.
Fuzzy$^+$ – AHP System	The best alternative choice from several variants	Fuzzy$^+$ – AHP method	Alternative evaluation by criteria in hierarchy; estimations is represented as fuzzy numbers; system is located in Internet (www.eco.brsm.by).

By means of the expert inquest, requirements of target groups were determined. Significance degrees of customers' requirements and their satisfaction degrees for *Stock Exchange DSS* [4] are represented in table 2. A part of estimations is represented as real numbers; another part is represented as fuzzy numbers. Requirements of other target groups and for other DSSs were estimated similarly. Elements of *B*-set (see Formula (4)) for enumerated DSS and elements of *A*-set (see Formula (3)) for "perfect" DSS (after defuzzyfication of fuzzy numbers) are represented in Table 3. Defuzzyfication of fuzzy numbers was made by the following formula, which also used in [2]:

$$\overline{Z} = \frac{z_1 + 2(z_2 + z_3) + z_4}{6}, \qquad (5)$$

where z_1, z_2, z_3, z_4 – elements of fuzzy number Z.

Table 2.

Customers' requirements	Significance degree of requirement, $x_j(u)$	Satisfaction degree of requirement, k_j
interface convenience, simplicity in work	1.00	(0.50; 0.60; 0.90; 1.00)
efficiency of practical use	(0.21; 0.26; 0.36; 0.41)	(0.70; 0.80; 0.90; 1.00)
universality (possibility of various problem solving)	(0.21; 0.26; 0.36; 0.41)	1.00
possibility of viewing the procedure of getting results	(0.15; 0.20; 0.30; 0.35)	0.00
speed of work	(0.15; 0.20; 0.30; 0.35)	1.00
low technical requirements	(0.15; 0.20; 0.30; 0.35)	1.00
possibility of initial information processing	(0.09; 0.14; 0.24; 0.29)	(0.30; 0.50; 0.60; 0.70)
visualization the results of estimation	(0.09; 0.14; 0.24; 0.29)	(0.70; 0.80; 0.90; 1.00)
detailed help-system	(0.09; 0.14; 0.24; 0.29)	(0.50; 0.60; 0.70; 0.80)

Graphical interpretation of DSS competitiveness includes 4 cross perpendicular vectors. Areas, characterizing DSS quality, are the quadrangles with cross perpendicular diagonals. Diagonals' lengths are equal to:

– for analyzing DSS - b_1+b_3 and b_2+b_4;

– for "perfect" DSS - accordingly a_1+a_3 and a_2+a_4.

As an area of such figure is equal to a half of product of its diagonal lengths, then relative coefficient of DSS competitiveness is calculated by the following formula:

$$Q = \frac{(b_1+b_3)\cdot(b_2+b_4)}{(a_1+a_3)\cdot(a_2+a_4)} \tag{6}$$

In Table 3 final results of DSS quality estimation are shown. Calculations were made by the formulae (6) and (2). Ranks were conferred to each DSS in accordance with coefficient of quality assessment.

Table 3.

Target groups and significance degrees of their opinions	Decision Support Systems					
	Assistant Choice	Stock Exchange DSS	Daina	Study Expert	Fuzzy$^+$ – AHP System	"perfect" DSS
b_1 - customers' requirements, 0.5	1.135	1.119	1.116	0.965	1.185	a_1=1.470
b_2 - producers' requirements, 0.2	0.325	0.332	0.318	0.370	0.317	a_2=0.414
b_3 - designers' requirements, 0.2	0.162	0.154	0.145	0.162	0.162	a_3=0.188
b_4 - sellers' requirements, 0.1	0.184	0.184	0.178	0.175	0.184	a_4=0.188
Q (formula (6))	0.799	0.795	0.757	0.743	0.766	
Rank by Q	1	2	4	5	3	
Q_{DSS} (formula (2))	91.4	90.6	88.8	84.7	89.3	
Rank by Q_{DSS}	1	2	4	5	3	

Results of DSS assessment show that all the systems have enough high quality (degrees of their accordance to the “perfect” DSS have values from 0,743 to 0,799). Ranks conferred to systems coincide with the ones, obtained by Q_{DSS} calculations by means of the formula (2) with the same data usage. Identical ratings were received by two variants of methodology.

6. Conclusion

Suggested methodology of estimation of quality of objects with complex structure allows to analyze data, which specifies by structural variability and incompleteness, therefore it is applicable for conditions of the data non-stochastic uncertainty. During description of objects and their characteristics experts’ fuzzy judgments may be used, and cognitive graphics may be used for estimation results visualization. Perspective direction of the given research is the validation of different variants of usage of this evaluation methodology in dependence on objects’ essence and environmental conditions.

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Decision Making in Stock Market with Neural Networks and Fuzzy Logic

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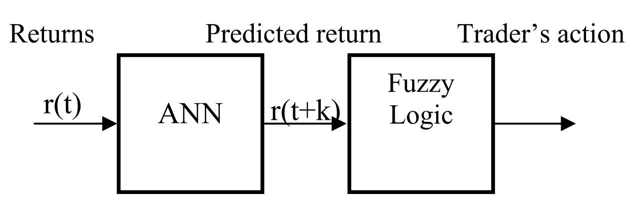
Abstract - The use of intelligent systems for stock market has been widely established. In this paper we have considered how a stock market trading strategy could be well generated using the components of soft computing technologies. We consider prediction of one-day ahead stock return using Artificial Neural Networks (ANN) trained by Levenberg-Marquardt training algorithm and generating set of trading rules from predicted behavior using fuzzy logic approach. The proposed approach provides a smoother decision surface with much a finer trading strategy than the simple buy/hold/sell strategy.

1 Introduction

Prediction of stock return is generally considered to be a complex task due to nonlinear behavior of the process (Yochanan 2003; Lebeling and Milton 1996; Sadig 2003) and hence decision-making process becomes challenging. In this paper we describe an original approach for generating profitable trading strategies based on combination of ANN and fuzzy logic as shown in Figure 1. We provide an alternative form of reasoning to assist on agent's decision-making strategies.



In the paper we combine fuzzy reasoning with highly accurate ANN predictor into a neuro-fuzzy model. The proposed model provides qualitative

Figure 1. Neuro Fuzzy model

(how to trade) and quantitative (how much to trade) approach to the trading decision. Such a model gives advantages to simple buy/hold/sell strategies. Replacing discrete decision making rules based on buy/hold/sell strategies by the finer discrete decision making rules allows for better decision making and benefits risk averse trader.

2 ANN Using Levenberg-Marquardt Optimization Algorithm

Statistical methods and ANN are commonly used methods for time series prediction. Empirical results have shown that ANN outperform linear regression. ANN make very little assumption about the nature of the process as opposed to statistical methods. From a statistical point of view ANN are analogous to nonparametric, nonlinear regression methods.
The backpropagation ANN (Hagan et al. 1996) is reliable for modeling nonlinear, dynamic market signals. Classical BPNN uses 1^{st} order steepest descent techniques based on estimation of the gradient of the sum squared error for each layer.

In practice a mean-squared error E(n) surface has the local minimum, flat sections, saddle interval (Demuth and Raele 1998). A simple steepest descent minimization algorithm based on first-order minimization is slow (requires many repetitions of the training epoch to achieve a satisfactory minimization of error) due to computing of error serially on the layer-by-layer basis. To increase speed of training and to avoid oscillation around a local minimum two groups of algorithm are used. In the first group algorithms the weight update is performed using combination techniques such as adaptive learning rate and momentum term, efficient initialization of NN using Nguen-Widrow method or smoothing a zigzag direction on error surface applying conjugate gradient algorithm. A second group algorithm for minimization of error E(n) is based on optimization techniques that basically use 2^{nd} derivative of performance index or cost function J(w). A Taylor series expansion of J (w):

$$J(w_{n+1})=J(w)+\Delta w\nabla J(w)+\frac{1}{2}\Delta w\nabla^2 J(w)+...$$

where $\frac{\partial E(n)}{\partial (w(n))} = \nabla J(w) = g$ is gradient of performance index;

$\frac{\partial^2 E(n)}{\partial^2 (w(n))} = \nabla^2 J(w) = H$ is a Hessian matrix.

The Levenberg-Marquardt algorithm (Hagan and Menhaj 1994; Saw 2003) was designed to approach second-order training speed without having to compute the Hessian matrix. When the performance function has the form of a sum of squares (which is typical in training feedforward networks), then the Hessian matrix can be approximated as

$$H(n) = J^T(n)J(n)$$

and the gradient can be computed as

$$g(n) = J^T(n)E(n)$$

where ***J*** is the Jacobian matrix that contains first derivatives of the network errors with respect to the weights and biases. The Jacobian matrix can be computed through a standard backpropagation technique that is much less complex than computing the Hessian matrix. The Levenberg-Marquardt algorithm uses this approximation to the Hessian matrix (Hagan and Menhaj 1994; Saw 2003) in the following Newton-like update formula

$$W(n+1) = W(n) - \left[J(n)^T J(n) + \mu I\right]^{-1} J^T(n)E(n)$$

Where μ is a nonnegative scalar that controls both magnitude and direction; I is identity matrix. When the scalar μ is zero, this is just Newton's method. When μ is large, this becomes gradient descent with a small step size. Newton's method is faster and more accurate near an error minimum, so the aim is to shift towards Newton's method. Thus, μ is decreased after each successful step (reduction in performance function) and is increased only when a tentative step would increase the performance function. In this way, the performance function will always be reduced at each iteration of the algorithm. This algorithm appears to be the fastest method for training moderate-sized feed forward neural networks (up to several hundred weights). It also has a very efficient MATLAB implementation, since the solution of the matrix equation is a built-in function in a MATLAB setting.

3 Design of ANN

Stock return prediction *r(t+p)* of close price at time *(t+p)*is estimated using open- r_o, high- r_h, low- r_l, volume returns- r_v and RSI index r_i the *k*-time steps back time from *t* time, defined by the following equation:

$$r(t+p) = f\,(r_o(t-k),\, r_h(t-k),\, r_l(t-k),),\, r_v(t-k),),\, r_i(t-k))$$

For simplicity in modeling we have used around 1000 daily data of only one stock (ticker-MSFT) listed at NYSE from 30 May 2003 to 27 February 2004.

Two-layer (hidden and output) feedforward ANN based on Levenberg-Marquardt algorithm was developed with five inputs: ***p*=** $[r_o(t), r_h(t), r_l(t), r_v(t), r_i(t)]$ and single output. Target is chosen to be return $r_c(t+1)$ of close price. ANN consists of *50 neurons* in hidden and *one neuron* in output layer. We used the tangential sigmoid activation functions in hidden and outputs layers. As shown in Figure 2 the goal (10^{-7}) is meet after n =1529 epoch.

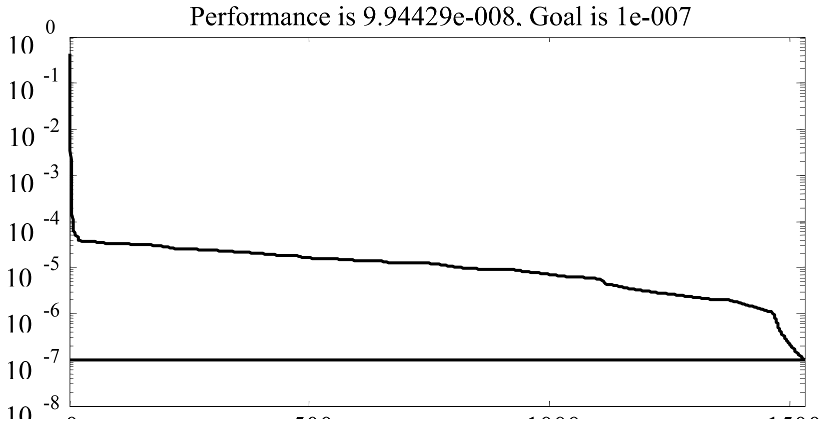


Figure 2. Training of ANN to meet goal

ANN architecture is shown in Figure 3. ANN involve N=(50x5) +(50x1)=300 updated weights.

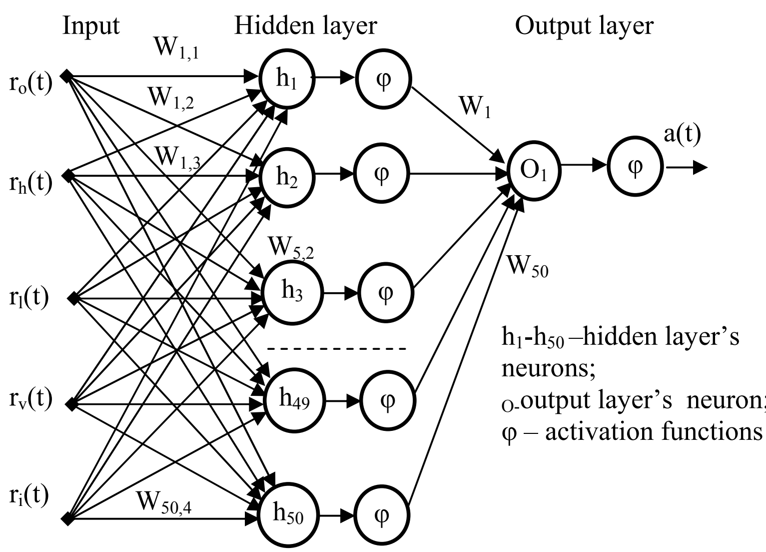


Figure 3. Architecture of ANN

Figure 4 shows a fragment of target return d(t) and ANN output a(t).

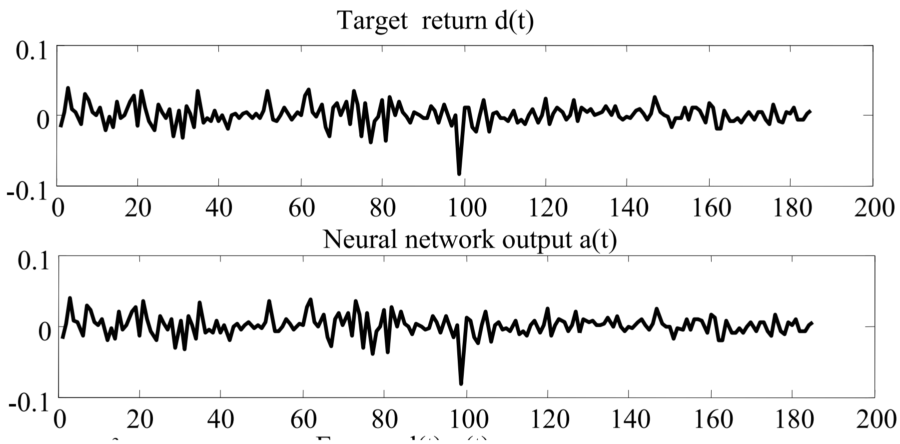


Figure 4. Actual returns, ANN predicted output and prediction error

Absolute error of prediction is e(t) = d(t)-a(t). A relative error prediction is defined by

$$\beta,\% = \frac{d(t)-a(t)}{|\,d(t)\,|_{max}}100\%$$

where $|d(t)|_{max}$ is a maximal absolute value of d(t). It is less than 5 % in our case. Fragment of numerical results is shown in Table 1. The analysis shows that a relative error of prediction $\beta \leq 1$ %.

Table 1. Numerical Results

Date	d(t)	a(t)	β, %
23-Feв-04	0.0101	0.0101	-0.0113
24-Feb-04	-0.0067	-0.0064	-0.7645
25-Feb-04	-0.0075	-0.0074	-0.1845
26-Feb-04	0.0011	0.0009	0.6544
27-Feb-04	0.0064	0.0063	0.2117

4 Fuzzy Logic Inference and Decision Making

Fuzzy logic inference performs the input (predicted return PR) and output (Trader's action TA) mapping using fuzzy logic (Nikola et al. 2001). Fuzzy inference consists of the following steps: fuzzification of inputs, applications of the fuzzy operators AND ,OR on the antecedents, implication from antecedent to consequent, aggregations of the consequents across rules and defuzzification. In our study we have utilized *FuzzyTECH* software that is widely used in business and finance (Constantin 1997).

4. 1 Fuzzification of ANN output signal

Using approach described in Nikola et al. (2001) the range of predicted return (a(t)) is decomposed into 9 set of fuzzy regions (sets): "Very Negative (VN)", "Negative (N)", "Medium Negative (MN)", "Weakly Negative (WN)", "Stable(S)", "Weakly Positive (WP)", "Medium Positive (MP)", "Positive (P)" and "Very Positive (VP)". For fuzzification each fuzzy set is represented by its membership function. Each point N in the input space is mapped to a degree of membership value between 0 and 1. Mathematically, a fuzzy set PR is defined by ordered pairs:

$$PR = \{a, \mu_{PR}(a)\} \qquad a \in N, \quad \mu_{PR} \in [0,1]$$

where $\mu_{PR}(a)$ is an input membership function.

For input fuzzification model we use Gaussian bell membership function producing a relatively smoother input/output mapping (Aliev et al. 2000; Demuth et al. 1998).

The fuzzification of predicted return PR is shown in Figure 5.

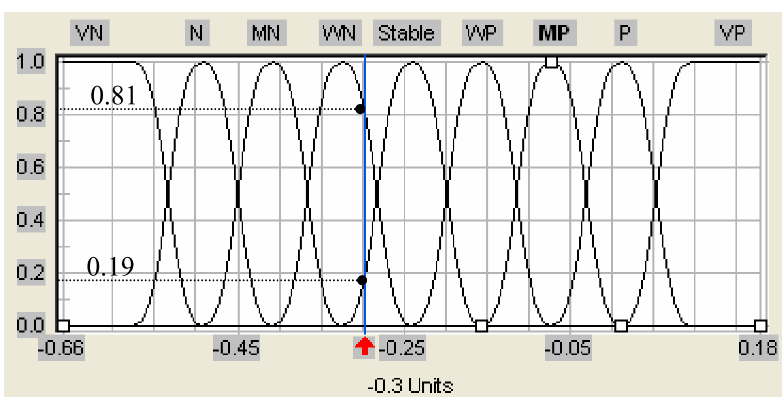


Figure 5. Fuzzification of ANN output

The small arrow (↑)under the horizontal axis shows how input a(t)=-0.3 (chosen for illustrative purpose) maps in the membership functions of the term "WN" at 0.81 and of the term "Stable" at 0.19.

The traders action is also fuzzy variable with the triangular fuzzy membership functions that represent 9 fuzzy sets beween -1 and 1: "Very Strong Sell(VSS)", "Strong Sell(VSS)", "Medium Sell(MS)", "Weak Sell(WS)", "Hold (H)", "Weak Buy (WB)", "Medium Buy (MB)", "Strong Buy (SB)", "Very Strong Buy (VSB)".

4.2 Composition of the rules

The Rule block consists of "If-Then" operators that link input linguistic terms (fuzzified PR) with the Trader's actions TA in the following manner:

- *If Predicted Return is **VN**, then Trader's action is **VSS***
- *If Predicted Return is **N**, then Trader's action is **SS***
- *If Predicted Return is **MN**, then Trader's action is **MS***
- *If Predicted Return is **WN**, then Trader's action is **WS***
- *If Predicted Return is **Stable**, then Trader's action is **Hold***
- *If Predicted Return is **WP**, then Trader's action is **WB***
- *If Predicted Return is **MP**, then Trader's action is **MB***
- *If Predicted Return is **P**, then Trader's action is **SB***
- *If Predicted Return is **VP**, then Trader's action is **VSB***

In this paper we apply the most commonly used fuzzy methodology – Mamdani type fuzzy inference model (Aliev et al. 2000; Demuth et al., 1998). This fuzzy inference model is well suited to human decision. Mamdani Inference System applies min and max operators for fuzzy AND (intersection) and OR (union) functions. It also requires that output membership function is a fuzzy set

$$\mu_{yi}(y) = \min(\mu_{PRi}(u_0)\mu_{TAi}(y_0))$$

where $\mu_{PR}(a_0)$; $a_0 \in A$ is an input membership function;

$\mu_{TA}(y_0)$; $y_0 \in Y$ is an output membership function.

Aggregation of all membership functions

$$\mu(y)=\sum_{i=1}^{j} \mu_{yi}(y_0)$$

Defuzzification is realised using centre of gravity method

$$y = \sum_{j=1}^{k} \mu(y_j)p_j \Big/ \sum_{j=1}^{k} \mu(y_j)$$

where k is the number of elements of y.

The spreadsheet Rule editor generated by *FuzzyTECH* is shown in Table 2.

Table 2. Spreadsheet Rule Editor

	IF	THEN	
	in1	DoS	out1
1	VN	1.00	VSS
2	N	1.00	SS
3	MN	1.00	MS
4	WN	1.00	WS
5	Stable	1.00	Hold
6	WP	1.00	
7	MP	1.00	
8	P	1.00	SB
9	VP	1.00	VSB

Aggregation: 0.1878
DoS: 1.0000
Composition: 0.1878

As shown in Table 2 the rule 4 has the result "Trader's action =Weakly sell" to the degree 0.81 and Rule5 has the result "Trader's action =Hold" to the degree 0.19. A composition of the "Then-part" rule would be:

TA ={VSS=0, SS=0, MS=0, WS=0.81, Hold=0.19, WB=0, MB=0, SB=0, VSB=0}

4.3 Defuzzification

As shown in Figure 6 the Trader's action is both fuzzy and ambiguous, because two different actions ("WS" and "Hold") have non zero truth degree 0.81 and 0.19 respectively.

For defuzzifications we use Centre-of-Area (CoA) method (Aliev et al., 2000; Constantin, 1997). The resulting area under corresponding membership functions gives the best compromise crisp value (-0.16) for Trader's action.

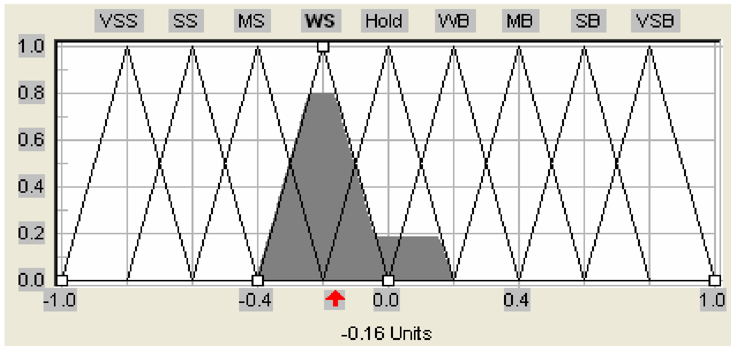


Figure 6. Defuzzification Trader's action using CoG method

In addition to the Trading strategy we can determine trading volume, assuming for simplicity that the trader's portfolio consists of one stock. As shown in Figure 6 the Trader's action is defined by 9 triangular membership functions in the interval [-1;1]. The decision-making is based on the shape of the membership functions for the defuzzified output variable z where negative value indicates to sell a fraction of endowment and positive value indicates to buy. The membership function of the term "Hold" is defined in the interval [-0.2;0.2]. There are 4 sub-ranges from 0.2 to the right: ([0.2;0.4],[0.4;0.6],[0.6;0.8]],[0.8;1.0]) and from -0.2 to the left: ([-0.2;-0.4],[-0.4;-0.6],[-0.6;-0.8],[-0.8;-1.0]) corresponding to the percentage of buying and selling volume ranges (See Table 3). For instance, in our example output z value of -0.16 would imply to hold but if the value of z was –0.35 this would imply to sell 35% of endowment.

Table 3. Decision intervals on traded volume

Trading Strategy	Output range	Range of Trading Volume (percent of endowment)
VSS	$-1 \le z \le -0.8$	Sell: 80%-100%
SS	$-0.8 \le z \le -0.6$	Sell: 60%-80%
MS	$-0.6 \le z \le -0.4$	Sell: 40%-60%
WS	$-0.4 \le z \le -0.2$	Sell: 20%-40%
Hold	$-0.2 \le z \le 0.2$	Hold
WB	$0.2 \le z \le 0.4$	Buy: 20%-40%
MB	$0.4 \le z \le 0.6$	Buy: 40%-60%
SB	$0.6 \le z \le 0.8$	Buy: 60%-80%
VSB	$0.8 \le z \le 1$	Buy: 80%-100%

5 Conclusion

In this paper two-layer feed forward back propagation ANN model based on Levenberg-Marquardt learning algorithm for stock market prediction has been considered. Developed ANN have 5-inputs, 50-neurons in hidden layer and a single output. The number of updated weights is 300. Experimental results show that relative error of prediction is less than 5 %. Results of prediction are used to establish a much finer trading strategy based on fuzzy logic approach that has advantages over a simple strategy of buy/hold/sell.

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The Application of Non-linear Model and Artificial Neural Networks to Exchange Rate Forecasting

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Abstract.

The hypothesis that foreign exchange rate behaviour is non-linear has been examined by several authors; others have proposed a linear framework. Here, evidence for a non-linear generating process is evaluated by an analysis of the comparative accuracy of short term forecasts of FX rates. Forecasts were generated by a linear model, and three non-linear models, including interaction model and artificial neural network (ANN). Nine data frequencies were used: five minutes, ten minutes, twenty minutes, thirty minutes, one hour, two hours, three hours, six hours and one day. Variables from the field of microstructure: order flow and market activity intensity are used to explain deutsche mark/dollar (DEM/USD) exchange rate movements. Two criteria are applied to evaluate model performance: root-mean squared error (RMSE) and the ability to predict the direction of exchange rate moves (PERC). The ANN is consistently better in terms of RMSE and PERC than random walk, linear model, and Interaction model for the various out-of-sample experiments.
Keywords: financial market forecasting, order flow, exchange rate, market microstructure, artificial neural networks, non linearity.

1. Introduction

Ever since the seminal work of Meese and Rogoff [1], the predictability of exchange rates has been an ongoing puzzle in international finance. They compare the predictive abilities of a variety of exchange rate models. They find that no existing structural exchange rate model could reliably out predict the random walk at short and medium run horizons. Baillie and McMahon [2] pointed out that exchange rates are not linearly predictable. These approaches try to explain exchange rate movements with macroeconomic variables. Lyons and Evans [3] incorporated a variable reflecting the microeconomics of asset pricing into a model of the exchange rate.

They introduced a new variable, order flow, as the proximate determinant of the exchange rate.
This paper examines whether the ANN technique can explain deutsche mark/dollar exchange rate movements better than random walk, linear and Interaction models. Two statistics are used to compare models: root-mean squared error (RMSE) and the percentage of correctly predicted exchange rate changes (PERC). Section 2 presents our analysis of the explanatory power of order flow for exchange rate. Section 3 studies the informative ness of order flow under trading intensity in the foreign exchange market. Section 4 introduces the ANN method and its application to the (FX) market. Section 5 describes the method used to assess the predictive performance of the models and the empirical results. Section 6 concludes the paper.

2. Order flow and foreign exchange rate determination

In the empirical finance literature, there is a long tradition of studying the higher frequency relationship between the price of financial assets and total volume of trade. Such analysis can not help resolve the Meese–Rogoff conclusion, not least because volume is directionless. Recently, researchers have investigated the impact of signed volume. The difference between seller and buyer initiated volume is termed order flow. Our objective is to investigate the relationship between order flow and exchange rate across frequencies, ranging from five minutes to one day using a four month span of data for DEM/USD (May 1 to August 31, 1996). We track how the explanatory power of order flow for price changes varies across sampling frequencies, [3, 4], by running a set of regressions of the following form:

$$\Delta P(k)_t = \alpha(k) + \beta(k) * \Delta X(k)_t + \varepsilon_t \tag{1}$$

Where $\Delta P(k)_t$ is the transaction price change at sampling frequency k and $\Delta X(k)_t$ is the order flow in the interval [t-1, t] at sampling frequency k.

At the highest frequencies (less than one hour) we observe non significant effects from order flow where R^2 ranges from 0.5% to 4.5%. From the point of view of exchange rate determination, lower frequency results are more relevant (for the daily frequency R^2 is about 22.3%). We demonstrate that using a linear model, order flow analysis has weak power to explain exchange rate changes.

Table 1. Explaining Exchange rate with order flow

Freq	$\hat{\beta}$	$t-stat^{(1)}$	R^2	$Serial^{(2)}$	$Hetero^{(3)}$
5mn	0.082	6.238	0.005	993	1096
10mn	0.080	5.810	0.010	186	430
20mn	0.098	6.195	0.021	63	206
30mn	0.128	7.406	0.045	56	89
1hr	0.181	9.281	0.127	25	59
2hrs	0.166	7.415	0.160	0.69	2.45
3hrs	0.143	6.304	0.172	2.81	2.87
6hrs	0.172	5.423	0.240	4.45	0.61
1day	0.152	3.553	0.223	6.06	0.67

(1) All t-stats are constructed using the Newey-West estimator of the coefficient variance-covariance matrix

(2) The serial column presents the p-value of Breusch-Godfey LM tests for residual serial correlation

(3) The Hetero column presents the p-values of Engle (1982) LM of tests for ARCH in the residuals

3. Market condition, order flow and exchange rates determination

The aim of this section is to characterize the information transmission mechanism of order flow under a market condition, namely the informativeness of order flow under one of the most important market statistic: trading intensity. Here the first basic question we would like to ask is that ‘‘Does Order flow is equally informative regardless of trading intensity?”

The asymmetric information pricing model of microstructure is based on learning process faced by market intermediaries. If a trader has superior information about the underlying value of the asset, his trades will reveal, at least partially, this private information about the value of the asset and will affect the behaviour of market prices. The central idea of the above information extracting process is that the market maker adjusts the quoting prices by observing order flow which in turn is driven by information. In this sense we say that information, through order flow, drives price movements. This idea can be put in the previous eq.(1), where β is a function of trading intensity. The question whether the informativeness of order flow is constant under trading intensity can be addressed by testing whether $\beta(Z_t) = \beta_0$, where β_0 is constant. Z_t is the trading intensity.

3.1 Nonlinearity test: Quintile model

A very straightforward way to test model stability is to split the sample into different sub-samples and see whether the model is stable across sub-samples [5]. In this paper, the sample is split into 5 sub-samples according to the variable Z_t. Dummy variable S_{jt} is used to distinguish each sub-sample. With this constraint, the eq. (1) can be expressed as

$$\Delta P_t = \alpha + \sum_{j=1}^{J} \beta_j * S_{jt} * \Delta X_{jt} + \varepsilon_t, \text{ j=1,..., 5} \tag{2}$$

Where $S_{jt} = 1$ for the corresponding sub-sample j and 0 otherwise and J is equal to 5. Under the null, all regression coefficients will be equal.

3.2 Nonlinearity modelling: Interaction model

The Interaction model can be written as

$$\Delta P_t = \alpha + \beta_1 * \Delta X_t + \beta_2 * Z_t * \Delta X_t + \varepsilon_t \tag{3}$$

In the interaction model, the regression coefficient β_2 captures the nonlinearity in the relationship between order flow and price change. A positive β_2 indicates that order flow is more informative under condition where Z_t is larger.

3.3 Estimation and analysis

The results of estimates of the quintile model are presented in table 2.

Table 2. Trading intensity effect on price impact of order flow.

Freq	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_4$	R^2	$F-stat^{(1)}$
5mn	0.008	0.083	0.060	0.075	0.181	0.006	4.504
10mn	0.023	0.097	0.044	0.050	0.213	0.010	8.711
20mn	0.041	0.061	0.086	0.110	0.245	0.021	8.218
30mn	0.005	0.001	0.032	0.111	0.323	0.045	11.346
1hr	0.048	0.073	0.119	0.152	0.395	0.127	9.389
2hrs	0.070	0.110	0.197	0.141	0.266	0.160	1.906
3hrs	0.026	0.112	0.026	0.191	0.342	0.172	4.353
6hrs	0.141	0.108	0.133	0.180	0.260	0.240	1.213
1day	0.116	0.089	0.102	0.046	0.326	0.223	0.992

([1]) the critical Fisher value for 5% significance level is 2.93

We note that F-tests are significant for sampling frequencies from 5 minutes to at least 3 hours. The results from the quintile model suggest that the null hypothesis (order flow is equally informative under trading intensity), is rejected in our samples.

The estimates of quintile model coefficients, eq. (3), are presented in table 3. The most notable feature is that β_2 is significantly away from zero for most of the sampling frequencies. The rejection of the null is equivalent to the rejection of linearity between order flow and price change. We note that β_2 is constantly positive for almost all time aggregation levels.

The significant positive β_2 suggests that the order flow tends to be more informative when trading intensity is higher and less informative when trading intensity is lower.

Table 3. Linear relationship between the informativeness of order flow and trading intensity.

Freq	$\hat{\beta}_1$	$t(\hat{\beta}_1)$	$\hat{\beta}_2$	$t(\hat{\beta}_2)$	R^2
5mn	0.081	6.196	0.305	9.521	0.0181
10mn	0.089	6.618	0.248	11.589	0.0181
20mn	0.096	6.254	0.153	9.815	0.0722
30mn	0.113	6.786	0.128	10.221	0.1230
1hr	0.179	9.655	0.044	8.275	0.2200
2hrs	0.164	7.579	0.022	4.551	0.2160
3hrs	0.150	7.157	0.020	5.605	0.2900
6hrs	0.182	5.724	0.005	1.861	0.2680
1day	0.154	3.606	0.006	1.190	0.2480

4. Application of artificial neural networks

This section examines whether introducing a market microstructure variables (that are , order flow and trading intensity) together with an artificial neural network (ANN) technique can explain deutsche mark/dollar exchange rate movements better than random walk, linear and Interaction models. The ANN model was developed based on data sets of two variables (order flow and trading intensity). The model was used to forecast the DEM/USD exchange rate movements. The networks trained and tested were the three-layer backpropagation ANNs with the non-linear sigmoid neuron activation function in hidden layer. The number of input neurons was two, while the number of hidden neurons was three. The last layer had one linear output neuron. The term backpropagation refers to the process by which derivatives of network error, with respect to network weights and biases, can be computed. This process can be used with a number of different optimization strategies. There are several different backpropagation training algorithms. One problem that can occur when training neural networks is that the network can overfit on the training set and not generalize well to new data outside the training set. This can be prevented by training with trainbr (Bayesian regularization).

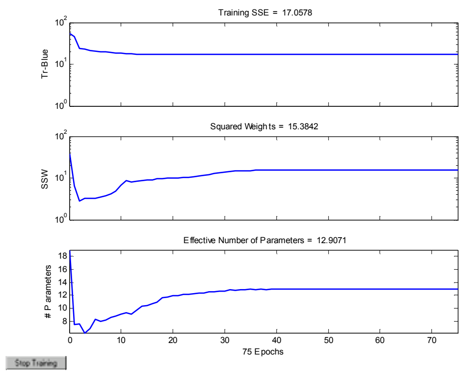

Fig. 1. Training, squared weights and effective nbr of parameters.

When using trainbr, it is important to let the algorithm run until the effective number of parameters has converged. The sum squared error (SSE) and sum squared weights (SSW) are relatively constant (SSE=17.05, SSW=15.38) over twenty iterations, fig.1.

5. Assessment of forecast performance and empirical results

In line with the Meese and Rogoff evaluation criterion, recursive estimation will be used to evaluate the models' predictive performance. The initial estimation starts with the first 85 per cent of the sample N. That comprises training and validation sets for the ANN. The remaining 15 per cent is a testing (forecasting). This paper considers whether the ANN model can outperform linear, random walk and Interaction models in terms of RMSE and PERC. In addition to RMSE, the PERC of the forecasted variable Δp_t is considered; this is the total number of correctly forecasted positive and negative movements [6]. We choose hourly frequency data as representative to estimate the model. After the initial estimation of the models in the first 500 observations, a set of out-of sample forecasts was used to generate RMSEs. Each recursive re-estimation added 4 observations, so that 10 RMSEs were calculated on out-of-sample data sets ranging in size from 88 to 52 observations. This led to the selection of an ANN model for one-hour-ahead forecasts of exchange rate changes, which were compared to linear, Interaction and random walk models. The experiments show that the ANN model forecasts one-hour-ahead exchange rate changes better than the linear, Interaction and random walk models, fig. 2.

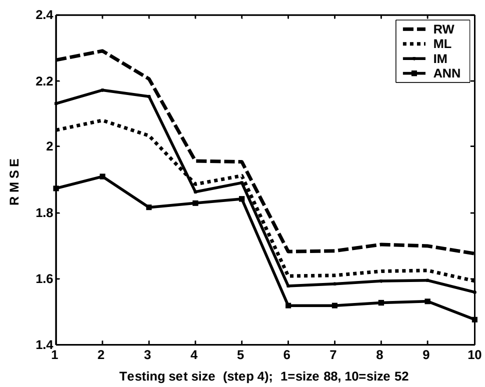

Fig. 2. : RMSE for ANN, linear, Interaction and RW models

Nevertheless, the primary indicator of good forecasting power is not necessarily RMSE, but the percentage of correctly forecasted directions of real exchange rate fluctuations. In turn, the presence of small RMSEs is not a guarantee that the prediction is accurate, and caution is required when interpreting the estimation results [6]. As noted above, PERC is also considered. Recursive regression for horizons between 52 and 88 observations (Step 4) reveals the superiority of the ANN model, fig. 3.

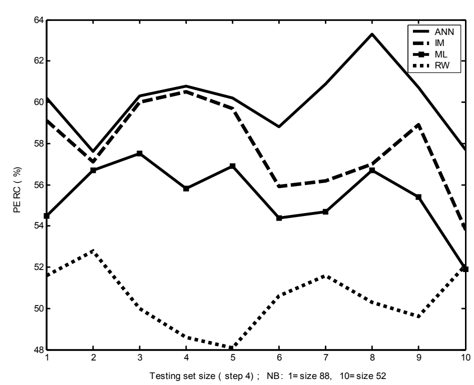

Fig. 3. Recursive estimation PERC comparison for ANN, Interaction, linear and random walk models.

Table 4 compares all the models used in terms of the second comparison criterion. ANN model correctly predicted, on average, 60.05 per cent of the direction of hourly exchange rate movements, Interaction model predicted 57.82 per cent, while linear model correctly predicted 55.45 per cent of such changes, and the random walk model predicted 50.53 per cent.

Table 4. The average percentages of correctly predicted signs for linear, Interaction, ANN and the random walk models

.

	RW	ML	IM	ANN
PERC (%)	50.53	55.45	57.82	60.05

6. Conclusion

An artificial neural network is employed for high-frequency DEM/US dollar exchange rate forecasting. Two criteria are applied to evaluate model performance: RMSE and the ability to correctly predict the direction of the exchange rate movements. The ANN is consistently better in terms of RMSE and percentage of correctly predicted signs than random walk, linear model and Interaction model for the various out-of-sample experiments.

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Financial Implications of Artificial Neural Networks in Automobile Insurance Underwriting

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Abstract

In the insurance industry, two primary things are considered when analyzing losses, the frequency and the severity. Previous research has investigated the use of artificial neural networks to develop models for use as an underwriter's aid when determining whether to accept an insurance policy, and what premium to set for it. This study furthers this line of research as a preliminary investigation into the financial impact of using a neural network model as an aid to underwriters.

Keywords: Automobile Insurance, Modeling, Neural Networks, Financial Analysis

Introduction

The processes by which the insurance market operates are based on the commonly held belief that an insurance underwriter is able to scrutinize a set of applications for insurance and select a set of applicants whose collective premium will be greater than the collective losses (Golding and King-Page 1952) (Gibb 1972).

Historically, an insurance underwriter evaluated each case based on variables he believed would contribute to subsequent losses. As the field developed; insurance companies hired actuaries. The actuary's function is to analyze past policy characteristics and loss experiences to find predictive relationships that the underwriter may use in the selection and pricing of individual process (Webb, Harrison et al. 1992) (Malecki and Underwriters 1986).

Historically, actuaries performed their duties using pencil and paper before the advent of computers. Today, more advanced computing tools are available (Hecht 1995) (Tauhert 1998) (Trencher 1998) (Daniels 1997).

One such technology-based tool that has potential as an aid to the underwriting process is the artificial neural network (Kitchens 2000).

The utility of artificial neural networks has been demonstrated in other fields such as predicting bankruptcy (Hawley, Johnson et al. 1990) (Wilson and Sharda 1994), insurance company insolvency (Huang, Dorsey et al. 1995), analyzing bank failures (Tam and Kiang 1992) and farm debt failures (Barney, Graves et al. 1999).

This paper describes the development and application of a preliminary study, focusing on the financial implications of the application of neural networks to the underwriting process in private passenger automobile insurance.

Literature Review

Private passenger automobile insurance underwriting is well suited to artificial intelligence applications. From a fixed set of finite data, underwriting decisions are made, policies are highly standardized, and deviations are rare.

Several studies have considered the use of computers in underwriting. They have attempted to predict “acceptability” (Gaunt 1972) (Rose 1986), incident of a loss (Retzlaff-Roberts and Puelz 1966) (Lemaire 1985), the relationship between premium and customer retention, and clustering methods (Smith 2000).

Artificial neural networks may be the most likely application of computers in underwriting. Originally, artificial neural networks were used to study the thought process of the human brain (Cowan and Sharp 1988). Early research proved that all processes that can be described with a finite number of symbolic expressions could be represented with a finite number of interconnected neurons (Wilson, Starkweather et al. 1990). Thus, artificial neural networks also provide a means of economic problem solving. The authors believe they also can be applied to insurance underwriting to reduce the ratio of insurance losses to insurance premiums.

Underwriting

The insurance business has certain characteristics that produce less than optimal financial results. Through the unique abilities of artificial neural networks, the underwriting process can be improved.

First, the inequity of the current rate classification system used in the United States will allow neural networks to more accurately asses the risk level of each policy holder, rather than a class of policy holders (Wood, Lilly et al. 1984).

Second, the current actuarial methods might benefit from recent developments in artificial intelligence. In spite of its importance, the actuarial approach used for insurance forecasting has been criticized as "rudimentary" (Cummins and Derrig 1993). The actuarial literature has been criticized for not devoting enough attention to the evaluation of alternative methods.

Third, the current actuarial work does not take an underwriting approach to the process. The primary method of research has been to predict the pure premium (AKA: relative rates).

Fourth, traditional statistical models have shown only incremental improvements. Perhaps technology can provide speed and ease to allow an artificial neural network to solve what is clearly a complex problem.

Fifth, even if the actuarial models were perfect, the neural network would at least be capable of matching the current results. Neural networks comprise a class of nonlinear statistical models whose information processing methods are generally cast in terms of the functioning of the human brain (Hawley, Johnson et al. 1990). Provided that the architecture is sufficient, a neural network output function can accurately approximate any function of x (White 1989). Further, any degree of desired accuracy can be achieved if the neural network is properly designed (Funahashi 1989).

Sixth, the potential financial savings is a strong incentive to make a reasonable attempt to apply neural networks to the underwriting process. Predicting the actual value of a paid claim, based on the information available to an underwriter, is exceedingly difficult (Kitchens, Johnson et al. 2001). As a rule-of-thumb, the average loss-to-gross-premium ratio is approximately 60 percent. The rest goes toward operating expenses and a small profit of approximately 3 percent. Thus, a 1 percent reduction in paid losses could equate to a 33 percent increase in operating profit (Kitchens Jr. 1999). While not a justification for using neural networks, it is incentive to try this and other nontraditional techniques.

Neural Networks.

For purposes of insurance modeling, the Genetic Adaptive Neural Network Training (GANNT) algorithm is a likely choice because it overcomes difficulties associated with the popular gradient and backpropagation techniques (Dorsey and Mayer 1994).

The application of a genetic algorithm was first proposed by John Holland in 1975 (Nygard, Ficek et al. 1992). It uses a system based on the reproduction of DNA molecules (Crane 1950). In summary, it uses replication, reproduction, crossover, and mutation to produce successive generations of possible solution arrays. The “fittest” solutions are retained and allowed to reproduce, creating new generations (Dorsey, Johnson et al. 1991).

Objectives

The loss frequency and severity each contribute to determining whether or not the insurance company will produce a profit. Loss frequency is an indication as to the likelihood that a loss will occur. Loss severity refers to the value of an insurance loss, regardless of probability.

In previous studies, the Genetic Adaptive Neural Network Training algorithm (GANNT) proved to be successful in predicting loss frequency in private passenger automobile insurance (Kitchens, Booker et al. 2002). Another study showed that loss severity could not be predicted using the same variables that are used to predict loss frequency. More research is needed in that area (Kitchens, Johnson et al. 2001).

This paper describes the development and application of a preliminary study, focusing on the financial implications of the application of neural networks to the underwriting process in private passenger automobile insurance. Three loss-frequency models will be used to predict policy acceptability; three sets of 500 actual policies will be tested. The losses and premiums will be compared.

Model Development

Three models were developed to predict the occurrence of a loss; a linear model, a logistic model, and a neural network model.

A Genetic Adaptive Neural Network Training Algorithm (GANNT) was used to train the neural network model. The GANNT algorithm selected was previously used successfully (Barney 1993) (Huang 1993) (Lin 1994).

The data used in this study came from a large international insurance company. It consisted of over 174,000 records from private passenger automobile policies in the United States.

The variables used in the model included those found on the insurance application, such as vehicle age, vehicle mileage, and driver age; as well as data concerning the driver’s previous behaviors such the number of at-fault and not-at-fault accidents found in the driver’s state-maintained Motor Vehicle Records. The dependant variable used was dichotomous representing a loss or no-loss on each policy.

A 16 hidden-node model was trained on 500 randomly drawn, balanced, in-sample policies using the GANNT algorithm. The resulting model was then tested using 500 random, balanced, out-of-sample policies.

The same data sets were used to train and test the linear regression, logistic regression, and neural network models. Using a threshold value of 0.5, the results were categorized into loss or no-loss predictions.

A confusion matrix was used to compare the predicted results to the actual results. The significance of these results was tested using the Wilcoxon Signed Ranks test. Only the policies that were "accepted" (no predicted losses) by the model were retained.

Results

The samples used were balanced samples - with an equal number of policies containing losses and no-losses. Thus, the loss-ratios presented here are highly exaggerated. The results are shown in Table 1.

Sample 1. Financial Results of Out-Sample Predictions

	Actual	Linear	Logistic	Neural Network
Predicted Losses/500	250	222	222	259
Sum of Losses	$1,255,846.39	$516,312.60	$516,312.60	$539,722.72
Sum of Premiums	$741,341.34	$340,980.52	$340,980.52	$318,407.85
Loss-Ratio	***1.694***	***1.514***	***1.514***	***1.695***
Average Loss	$2,516.73	$1,032.63	$1,032.63	$1,079.45
Average Premium	$1,482.68	$681.96	$681.96	$636.82

Sample 2, Financial Results of Out-Sample Predictions

	Actual	Linear	Logistic	Neural Network
Predicted Losses/500	250	184	192	155
Sum of Losses	$851,772.56	$533,713.59	$525,680.54	$609,664.76
Sum of Premiums	$952,807.31	$590,130.31	$607,901.04	$720,052.44
Loss-Ratio	***0.894***	***0.904***	***0.865***	***0.847***
Average Loss	$1,706.96	$1,067.43	$1,051.36	$1,219.33
Average Premium	$1,905.61	$1,180.26	$1,215.80	$1,440.10

Sample 3, Financial Results of Out-Sample Predictions

	Actual	Linear	Logistic	Neural Network
Predicted Losses/500	250	265	264	346
Sum of Losses	$1,189,997.26	$286,737.25	$269,633.56	$186,262.35
Sum of Premiums	$748,299.42	$237,885.38	$239,769.32	$195,655.47
Loss-Ratio	***1.590***	***1.205***	***1.125***	***0.952***
Average Loss	$2,384.76	$573.47	$539.27	$372.52
Average Premium	$1,496.60	$475.77	$479.54	$391.31

Table 1. Financial Results of Loss Predictions for Three Samples

The results for these three samples are mixed, and there is wide variation in the average, sum, and loss ratio values. In two out of three samples, the neural network model out-performed (resulted in a lower loss ratio) the linear model, the logistic model, and the actual results. In one case, the neural network model had the highest loss ratio. But, in this case, the difference as compared to actual results was negligible.

The sum-total of all three loss ratios for each model is depicted in Table 2. The results show that overall the Neural Network model out-performed the linear and logistic models, and improved upon the actual underwriting results.

Losses				
	Actual	Linear	Logistic	Neural Network
Sample 1	$1,255,846.39	$516,312.60	$516,312.60	$539,722.72
Sample 2	$851,772.56	$533,713.59	$525,680.54	$609,644.76
Sample 3	$1,189,997.26	$286,737.25	$269,633.56	$186,262.35
Total Losses	$3,297,616.21	$1,336,763.44	$1,311,626.70	$1,335,629.83

Premium				
	Actual	Linear	Logistic	Neural Network
Sample 1	$741,341.34	$340,980.52	$340,980.52	$318,407.85
Sample 2	$952,807.31	$590,130.31	$607,901.04	$720,052.44
Sample 3	$748,299.42	$237,885.38	$239,769.32	$195,655.47
Total Premium	$2,442,448.07	$1,168,996.21	$1,188,650.88	$1,234,115.76

Total Loss Ratio				
	Actual	Linear	Logistic	Neural Network
Total Loss Ratio	**1.35**	**1.14**	**1.10**	**1.08**

Table 2. Sum-Total of Loss Rations from All Three Samples

Conclusions

The results, while somewhat unstable, having a high degree of variation, indicate that further investigation is warranted. If a model can be developed and used as an underwriting tool to improve the loss ratio, the financial impact to the company's bottom-line could be tremendous.

It is also possible that rather than use a strict accept/reject rule for the model, a three-tiered recommendation system could be developed. The resulting model might recommend: “accept”, “accept with caution”, or “reject”. The “accept with caution” category could require precautionary action such as: additional endorsements, lower insurance limits, exclusion of certain drivers, reinsurance, or periodic re-evaluation.

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A Neural – Fuzzy Approach to Economic Data Classification

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Abstract. This paper offers a new approach to the development of the system based on neural-fuzzy analysis of economic data. The system is software and is used for solving management issues dealing with complex economic objects and fuzzy classification.

Introduction

Recent years have witnessed a large number of neural-fuzzy methods and soft computing [1,2,7,13] and their applications in economy and finance [3,8,11,13,14,17]. This article concerns the approach to construction of the system of data analysis within the framework of the neural network theory [4,5,6,9,12,16,] and fuzzy sets theory. There are many economic tasks requiring fast decision-making. The usage of hybrid approach may help perform some of these tasks.

The basis of our approach is Zuenkov's fuzzy classifier [10,15], and Kohonen's SOM, which could be vastly applied [8,11,14,17].

The Analogue of Kohonen's Map (AKM).

We begin the description of modified Kohonen's algorithm.

Let $\overline{x_j} = (x_{1j}, x_{2j}, ..., x_{nj})^T$ denote the $j-th$ input signal ($j = \overline{1,q}$) applied to the network, where $n-$ is the number of input terminals; $\overline{w_i} = (w_{1i}, w_{2i}, ..., w_{ni})^T$ is the corresponding synaptic weights of neuron i; $Net_i = \left\| \overline{x_j} - \overline{w_i} \right\|$ - the distance between $j-th$ input signal and $i-th$

prototype; $f(Net_i)$ is a competitive activation function. The basic steps of the algorithm are listed below.

1. Initialization. In this paper we used the following procedure of initial weights choosing. To choose the initial weight vectors $\overline{w_i} = (w_{1i}, w_{2i},..., w_{ni})^T$, we suggest the max-min classification of the input signals. The procedure is summarized as following:

1.1 Calculate the distance matrix between all input signals.

1.2 Calculate the proximity matrix of all input signals using the formula $\rho_{ij}^{/} = e^{-\frac{\ln 2}{r}\rho_{ij}}$

where ρ_{ij} is an element of distance matrix calculated in step 1, $\rho_{ij}^{/}$ is an element of the proximity matrix, r is the distance of uncertain proximity of input signals, i.e. the proximity of input signals equals 0.5.

1.3 Subtract the proximity matrix from the unitary matrix.

1.4 Define the value of difference between input signals.

1.5 Find the input signals satisfying the value of difference.

2. Sampling. Draw all samples $X = \{\overline{x_j}\}, j = \overline{1,q}$ from the input distribution.

3. Similarity matching. Find the winning neurons and the input signals which belong to each of the winning neurons at time t.

4. Updating. Adjust the synaptic weight vectors of all neurons, using the formula $\overline{w_i}(t+1) = \overline{w_i}(t) + \alpha(\overline{x_{av}^i} - \overline{w_i}(t)), i = \overline{0,m}$,

where $\overline{x}_{av}^{i}(t) = \dfrac{\sum_{j=1}^{q} \overline{x_j^i}(t)}{q}, i = \overline{1,m}$, q is the number of input signals which belong to $i-th$ winning neuron in $t-th$ epoch.

The task of visualization of prototypes is solved below. We considered two spaces: spatially input space R^n; spatially discrete output space R^k, which topology is endowed by the arrangement of neurons' set the computation nodes of a lattice. Nonlinear transformation, called AKM, maps the input space onto the output space and is shown by *Φ*: $R^n \to R^k$. The AKM feature map is represented by the set of synaptic weight vectors $\overline{w}_j$, which are topologically ordered in the sense that the spatial location of a neuron in the lattice corresponds to an input pattern.

Nonlinear AKM transformation is constructed as shown below.

The number of neurons in the network equals to the number of prototypes (m). The initial weight vectors $\overline{w_i}^{/} = (w_{i1}^{/}, w_{i2}^{/}, ..., w_{ik}^{/})^T$ are chosen randomly. The input signals of $i-th$ neuron are the weight vectors of other neurons. The outputs are the distances between the weight vector of $i-th$ neuron and the weight vectors of other neurons $\rho_{ij} = \left\| \overline{w_i}^{/} - \overline{w_j}^{/} \right\|, i \neq j$.

In Fig.1 we show the adaptive procedure of the weight updating. The adaptation of the weight vector $\overline{w}(n)$ is summed up in the form of an error-correction learning rule, as shown by

$$E(\rho) = \frac{1}{8}e^2 = \frac{1}{8}\sum_{i=1}^{m}\sum_{j=1}^{m}(t_{ij} - \rho_{ij})^2 \to \min,$$

where $i \neq j$, m is the number of neurons (prototypes), ρ_{ij} is the distance between $j-th$ input vector and $j-th$ weight vector, t_{ij} is the distance between prototypes
in R^n.

The neuron learning rule can be written as

$$w_{lm}(t+1) = w_{lm}(t) - \beta \cdot \Delta w_{lm},$$

where $\Delta w_{lm} = \dfrac{\partial E}{\partial w_{lm}} = -\sum_{j=1}^{m}(t_{mj} - \rho_{mj})(w_{lm} - w_{lj})$; $\beta \in (0;1)$ is the learning-rate parameter; $(t_{mj} - \rho_{mj})$ is the difference which can be interpreted as an error signal.

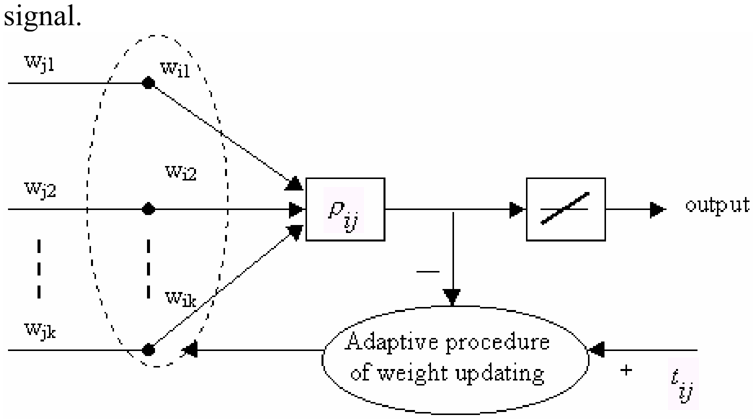

Fig. 1. Network for the realization of the AKM algorithm

The proposed algorithm was used to perform some application tasks.

Fuzzy classification

In objects' classification, the requirement is to classify the objects into a finite number of classes. The solution of the classification problem may be obtained by using the AKM map. To achieve the best results for the objects' classification, the use of the AKM map should be accompanied by methods of fuzzy sets theory. One option is the use of a hybrid approach shown in Fig.2.

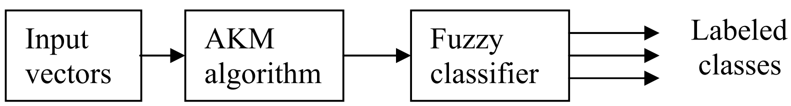

Fig.2. Two-stage classifier

Fuzzy classification is a classification of economic objects into possibly overlapping classes, where each of the objects s_j may belong to each of the classes ω_i to a certain degree.

The procedure aims at finding the number of the classes and the class membership functions which define the degree of each objects belonging to the each class. The number of the classes is chosen by the AKM clustering procedure. For fuzzy classification we used the method suggested by Zuenkov [4].

This method starts with the reflexive, symmetric fuzzy relation *R* on $S = \{s_1, s_2, ..., s_n\}$.

It is known that the transitive closure of *R* is $\hat{R} = R^1 \cup R^2 \cup \ldots \cup R^k$, $k \le n$; $\hat{R}$ is reflexive, symmetrical, max-min transitive, and more specifically, $\hat{R}$ is similarity relation on *S*.

The result of the classifying procedure can be represented as a membership function $\hat{\mu}^T(s, \omega_i)$, such as:

$$\hat{\mu}^T(s, \omega_i) = \underset{\xi \in S}{Sup} \min\left[\hat{v}^T(s, \xi), \mu(\xi, \omega_i)\right],$$

where $\mu(\xi, \omega_i)$ is membership degrees with the object ξ belonging to class ω_i.

$\hat{\nu}^T(s,\xi)$ is the membership function of the transitive closure in set $\{s, s_1, s_2, ..., s_n\} = \{s\} \cup S$ of fuzzy relation with membership function $\nu(s,\xi)$.

In this way, we assign the classifying object s to a class ω_m, if $\hat{\mu}^T(s,\omega_m) > \hat{\mu}^T(s,\omega_k)$, for $\forall k \neq m$.

The main problem is to choose a suitable relation *R*. In our case, fuzzy similarity relation is defined as a certain function f of the distance ρ between the economic objects that belong to *S*. This distance ρ is defined with AKM map and we get $\nu(s,\xi) = f(\rho(s,\xi))$.

Applications

Here we consider the three applications: banks clusterisation, evaluation of prices of apartments, bank planning.

1200 banks have been chosen to perform the first task. Each bank is characterized by 11 financial indices. To define the upper financial state, we used the information of the Central bank dealing with 30 biggest banks. To define the poor financial state, we chose the banks having the negative values of profit.

Using our software, we developed the AKM feature map, which comprised 50 clusters. They were identified as having upper, intermediate and poor financial states. Having analyzed the map, we chose the clusters that represented the similar financial states. These clusters were interpreted as three aggregated clusters and were associated with three fuzzy sets:
A= «Upper financial state», B= «Poor financial state»,
C= «Intermediate financial state».

Further analysis was channeled in two directions. Firstly, the construction of membership functions to each aggregated cluster A, B, C. Secondly, evaluation of proximity of a certain object to each cluster using AKM map. This enabled us to get the results represented in Fig.3.

1,2 – prototypes with upper financial state;
3 – prototype with intermediate financial state;
4,5 – prototypes with poor financial state;
6 – classifying bank.
From Fig.3 we can see that classifying bank is near to prototype with poor financial state.

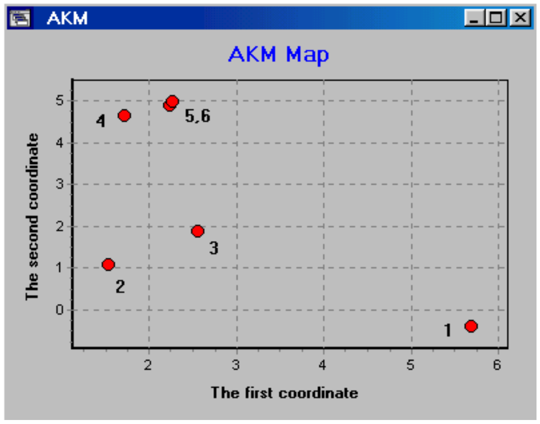

Fig.3. AKM Map

To perform the second task of evaluation of apartments' prices we used the database of "NeuroOK" company. To make clustering, we used the information about 1720 apartments in Moscow. The analysis of results showed that the characteristics of apartments in the database were insufficient. There were several instances in the database when apartments with similar characteristics had different prices.

In this paper we also used AKM for bank planning. Taking into consideration the bounds of all clusters, we could calculate the effective trajectory for the classified bank to reach the upper financial state.

Conclusion

Combinations of neural network and fuzzy sets theory have gained effectiveness during recent years. The hybrid approach is proposed as software that allows greatly decrease the time of network converging. The use of this approach enables us to perform the following tasks: bank planning, banks fuzzy classification and evaluation of apartments' prices.

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SOFT COMPUTING METHODS IN INVESTMENT AND RISK ANALYSIS AND IN PORTFOLIO OPTIMIZATION

Portfolio Optimization System (Siemens Business Services Russia)

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Software Solution Prehistory

The group of developers in the Solutions sub-unit of Siemens Business Services Russia (further – SBS Russia) was tasked to create a fundamentally new software solution that overcomes limitations of classical methods of portfolio optimization. It was particularly important since the customer of a new Portfolio Optimization system was the Pension Fund of the Russian Federation (further – PF) that was legislatively in charge of controlling the effectiveness of pension investments in the Russian stock market.

The scientific problem posed before the developers of new Portfolio Manager was triple:

- to find the adequate way of historical basic (initial) data interpretation;
- to develop the algorithms of perspective forecasting of stock indices;
- to develop valuation methods of real assets investment appeal.

All the three above-mentioned tasks were successfully settled.

Software Solution Scientific Characteristics

2.1. Fuzzy-plural approach to historical data modeling

Having declined the using of Wiener model, which consider the asset return as the current return we managed to make a model of index final return. The model parameters were fuzzy numbers of asset return and risk defined by specific rules. As a result we succeeded in raising reliability of estimation in those cases when the interpretation of stock market historical data is still possible (e.g., for the Russian conditions of 2001–2002 years). When the data interpretation in the way of fuzzy-plural models is not possible it is necessary to forecast stock indices without pre-processing of historical data but based on special theoretical prerequisites.

2.2. Stock indices forecasting

Based on the results of the rational investment choice theory SBS Russia employees managed to forecast the American stock market tendencies in 2001 and 2002 years thereby it confirms the precision of the mentioned theory and allows to work out a new theory of stock indices forecasting (see [1]). As a result of such a forecast stock indices are interpreted as triangular fuzzy functions. The parameters of indices return and risk are triangular fuzzy numbers correspondingly (the same as in the final return model).

The building of the new theory of stock indices forecasting saved from the necessity to interpret historical data for a forecast. It was shown that 2000–2001 years in the world history were the years of the economic paradigm changing and as a result the historical data for the preceding period decreased in value, it means that this data could not be used for the medium-term (all the more for the long-term) forecast any more.

2.3. Portfolio optimization in the fuzzy interpretation

If the parameters of the model have fuzzy description, regardless of the fact that they were got in the course of historical data processing or as forecasting results, then the portfolio optimization can be done in fuzzy-plural way. At that we get the efficient frontiers of portfolio set in the coordinates “risk-return” as a curved zone and the assets fractions in optimal portfolios as vectors of triangular numbers. The software solution allows scanning the efficient frontiers by risk level maximizing investment return.

The given optimization method is applied for the set of index assets. In cases when model classes are filled with real assets the other optimization methods are applied.

2.4. Valuation of assets investment appeal

When the optimal proportions of index (model) portfolio are defined it is necessary to fill the portfolio model components with real assets. At that it is rational to fix the fraction of a real asset in accordance with the investment appeal of this very security type, and to valuate this investment appeal on basis of unified complex criterion.

To build the method of assets appeal evaluation there were worked out specialized matrix methods of evaluation [1] successfully used before it to evaluate enterprise bankruptcy risk. For each type of real assets (debt instruments of the Russian Federation Subjects, corporate bonds, shares) there were worked out sets of criteria and their weights in the resulting criterion. All the separate criteria were normalized that allowed getting the resulting criterion of assets investment appeal to be normalized as well. Based on the criterion level evaluation the system automatically suggests trade advice (to buy, to sell or to keep).

2.5. Actuarial modeling

For the non-governmental pension funds it is necessary to make budgeting of money flows of fund earnings and payments that is to make actuarial estimation. The important part of estimation is the evaluation of the stock market expected return from the fixed investment portfolio.

The considered software solution gives the possibility to unite in the actuarial model estimation parameters of model portfolios (parameters set for the period of several years at the actuarial estimation level) as well as fund indices return forecasts for the corresponding estimation years. As a result the investment return is determined as a triangular fuzzy number for each estimation year.

Program Modules

3.1. Investment Profiles module

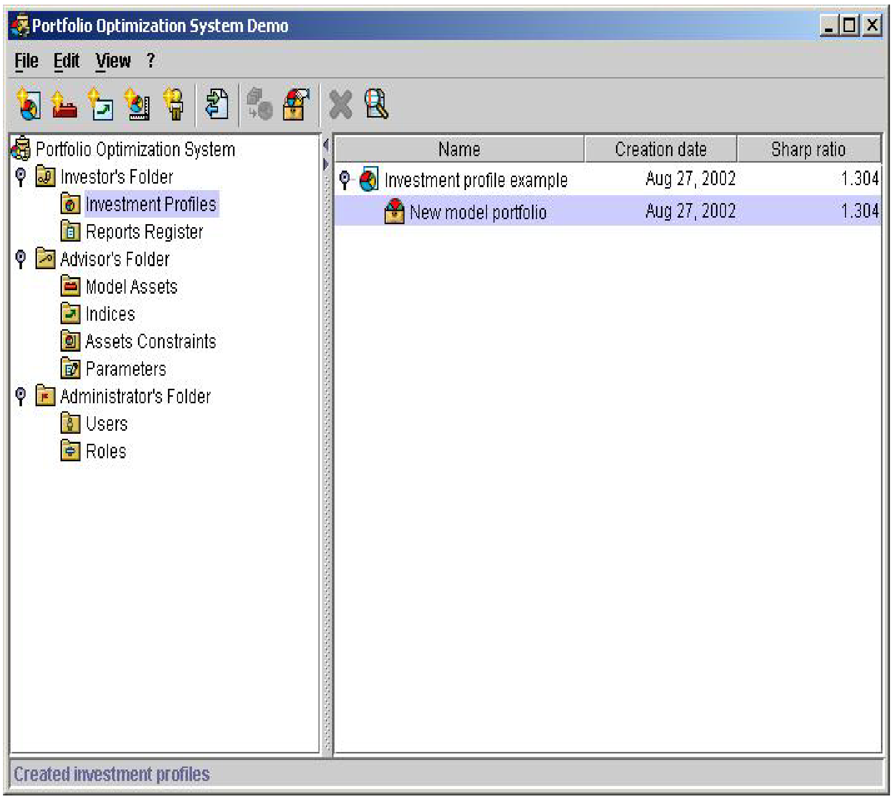

Fig. 1. The screenshot of the Investment Profiles module

The investment profile is a program information construction that contains the whole history of operations with investment portfolio. In the Pension Fund of the Russian Federation the investment profile means the managing company to which there is turned over the fixed amount of investments to be further managed. In the course of profile modifications PF employees can model the managing company operations with assets and valuate assets risk and performance.

The module functionality provides:

- the table view of all the created investment profiles with their properties such as an investment profile name, investment profile creation date and Sharp ratio;
- a new investment profile creation, rebalancing of the current model portfolio in the chosen profile, investment profiles consolidation, profile deleting, setting a model portfolio as the current portfolio in the chosen investment profile;
- the possibility to view and print end user's model portfolios reports (the results of end user's work with model portfolios) as well as to save reports as xml, html and pdf documents.

3.2. Investment Profile Creation module

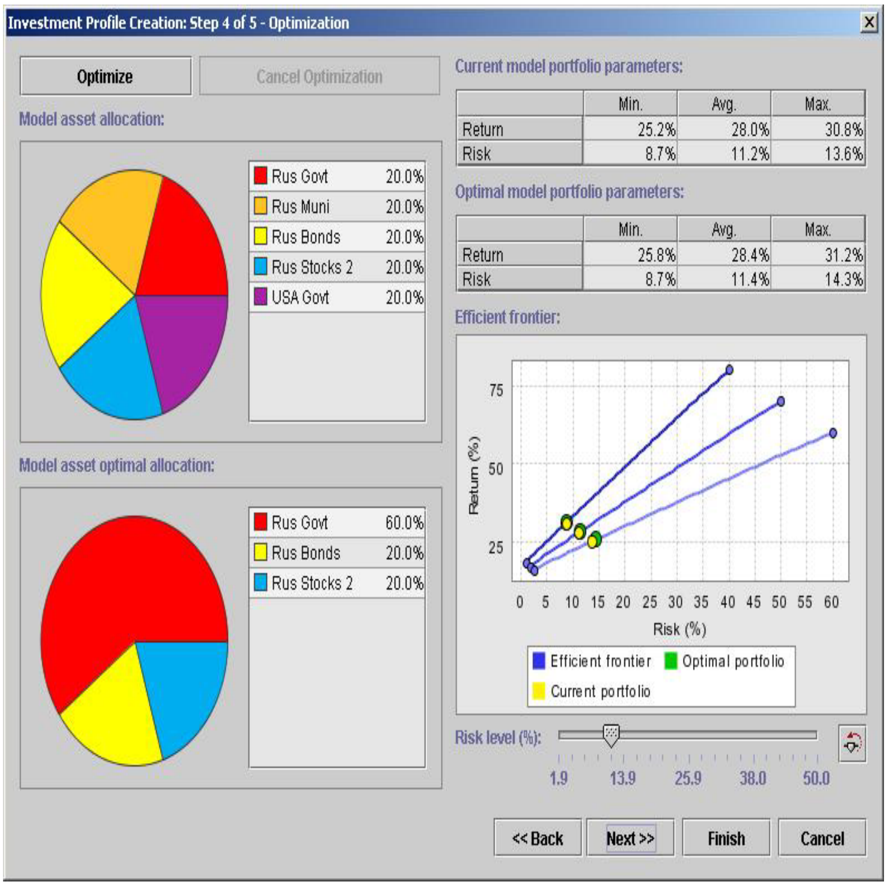

Fig. 2. The screenshot of the Investment Profile Creation module

The module functionality allows:

- to create an investment profile specifying the investment horizon and funds to be invested;
- to set benchmarks for an investment profile choosing planned dates to control return and the corresponding return values (no more than one benchmark per quarter);
- to choose model assets to be invested in specifying money volume for each asset; to mark assets that will take part in forming the efficient frontier; to represent asset allocation by way of a pie chart;

- to control the before fixed assets constraints on model assets volume providing the necessary notifications in case any constraints are broken;
- to rebalance model portfolios;
- to consolidate investment profiles;
- to view the properties of model assets stated in a profile such as return and risk values as triangular fuzzy numbers;
- to represent the model indices performance both in graphical and table forms, return values allocation by way of a bar chart and likelihood function slice;
- to represent the graphical result of model portfolio optimization in the way of blurred efficient frontier (the efficient frontier zone);
- to represent both the initial assets allocation in the way of three points and the desired allocation in the way of three points as well in the efficient frontier zone;
- to make model portfolio operative rebalancing specifying the assets optimal values;
- to change the portfolio risk level (along with the possibility to set the risk level back);
- to evaluate portfolio return retrospectively based on historical performance and prospectively (in the way of forecast) based on triangular fuzzy functions in three ways: in nominal price (RUB), in real price(RUB with taking inflation into account), in the before fixed currency (USD, GBP, EUR, JPY);
- to evaluate benchmark risk having the possibility to revalue it by changing the benchmark data;
- to compare the portfolio performance with the chosen model asset performance including the comparison with the Russian inflation level;
- to save the created investment profile (model portfolio);
- to create and represent reports by results of an investment profile creation or a model portfolio rebalancing.

3.3. Model Assets and Indices module

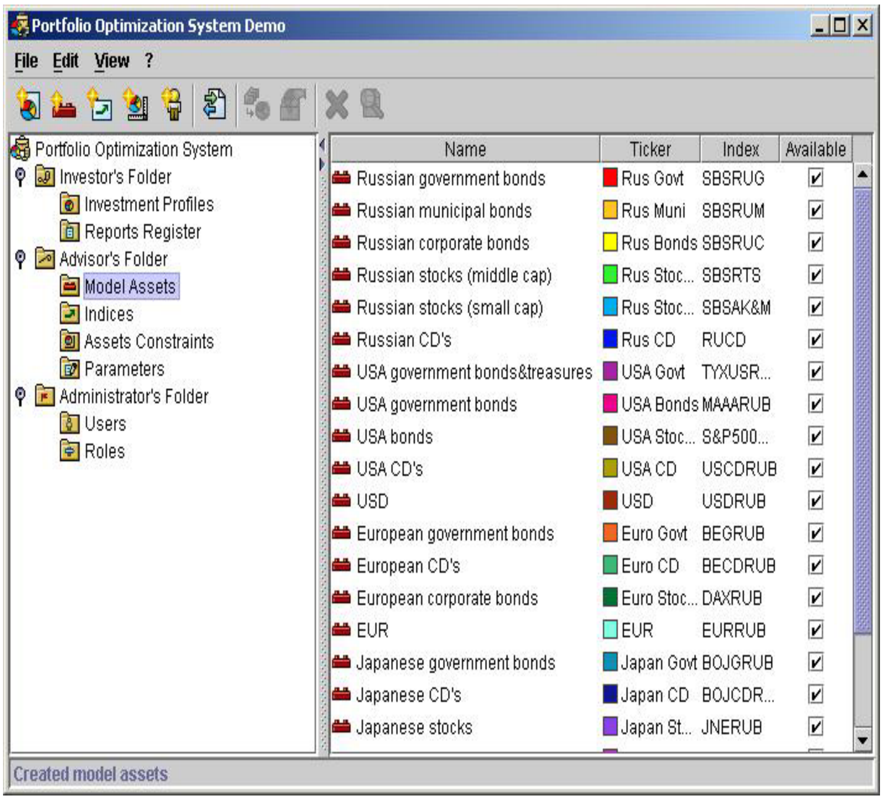

Fig. 3. The screenshot of the Model Assets and Indices module

The module functionality gives the possibility:

- to correct the number of model assets (classes) and to associate them with new indices;
- to add new indices as well as renew indices data;
- to set and correct assets constraints on model assets volume in a portfolio;
- to correct the program work parameters.

3.4. Economic Regions Profiles module

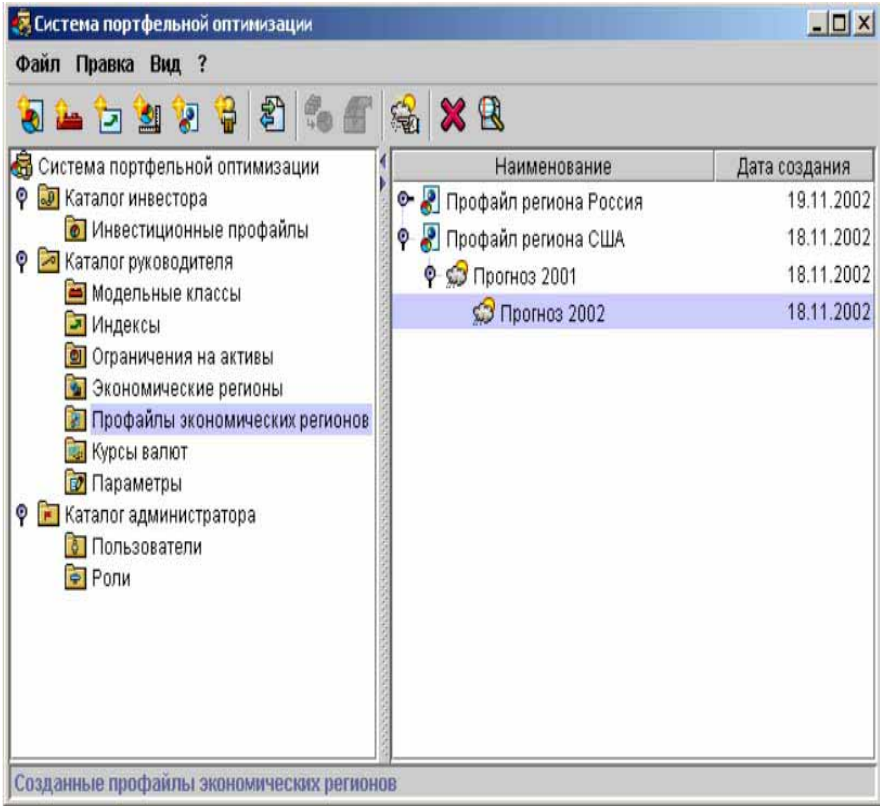

Fig. 4. The screenshot of the Economic Regions Profile module

The economic region profile is a program information construction that gives the possibility to consolidate the whole history of fund and macroeconomic indices for a country or a group of countries.

The module functionality provides:

- the table view of all the created economic region profiles with their properties such as an economic region profile name and economic region profile creation date;
- the possibility to correct any forecast in the chosen economic region profile;
- the possibility to view forecast results for all economic regions profiles;

- the possibility to view and print reports (the results of end user's work with forecasts and economic regions profiles) as well as to save reports as xml, html and pdf documents.
- the possibility to use the forecast values of indices risk and return as expert evaluations;
- the possibility to keep the economic regions reference book.

3.5. Economic Region Profile Creation module

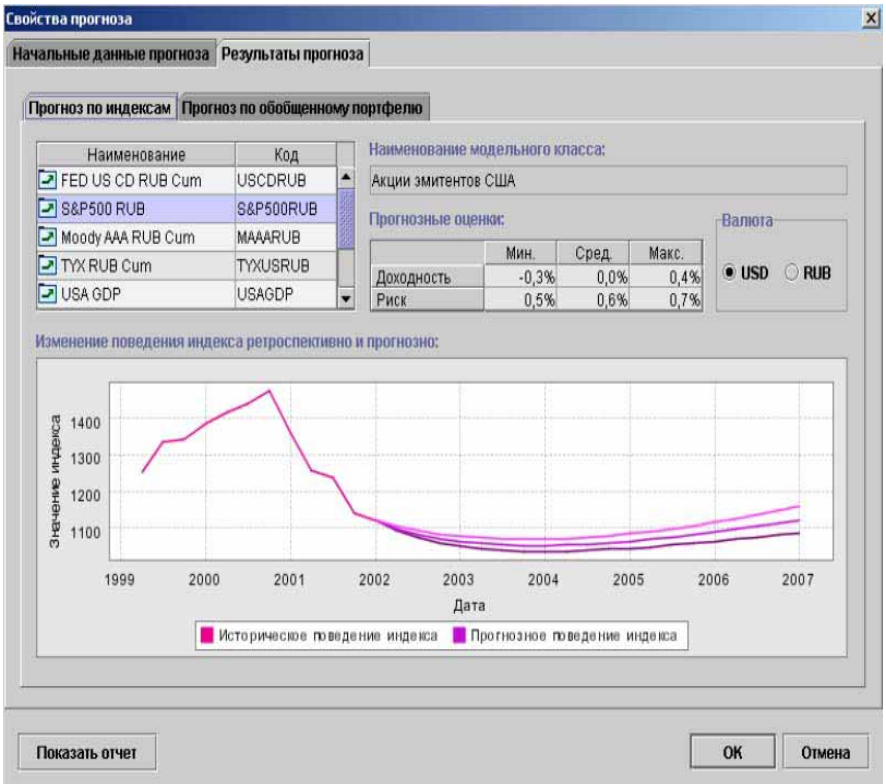

Fig. 5. The screenshot of the Economic Region Profile Creation module

The module functionality allows:

- to create economic regions profiles including the possibility to allocate indices by special groups and control the availability of national measures for the region specified;
- to set the required initial data for a forecast to be executed;

- to execute a forecast in accordance with the forecast algorithm [1];
- to get indices forecast results for generalized portfolio in a graphical form;
- to save the created economic region profile (forecast);
- to create reports by results of an economic region profile creation or a forecast changing.

3.6. Shares Scoring module

The module allows to choose and order the shares by the results of scoring (the method of scoring is stated in [1] and adapted both to the Russian and American stock markets).

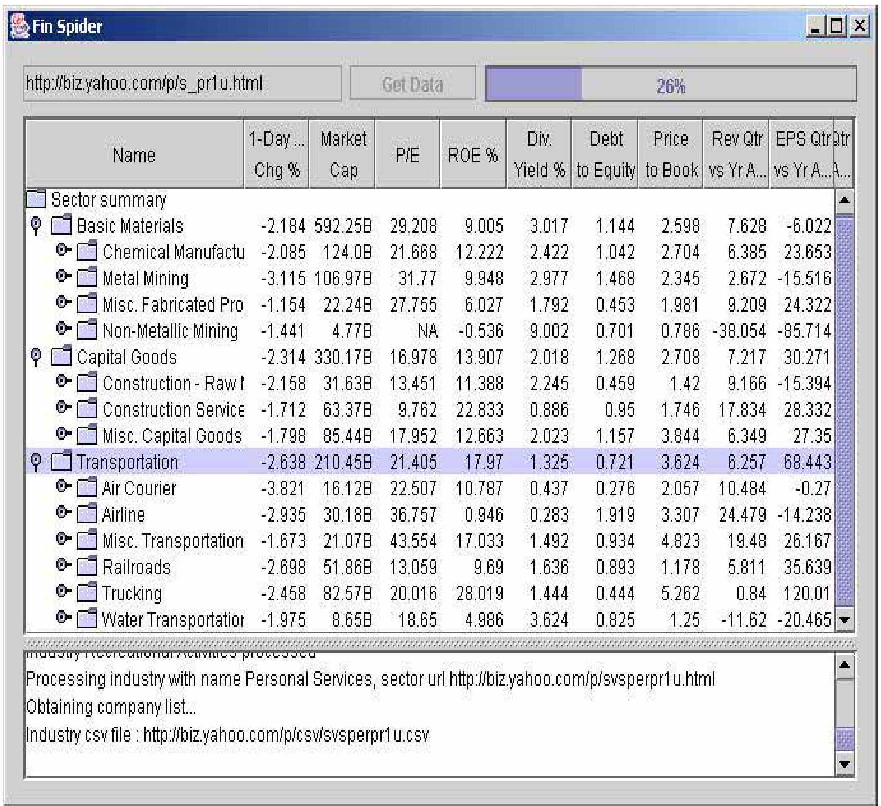

Fig. 6. The screenshot of the Shares Scoring module

As an illustrative example the system downloads the initial data about economic sectors, industries and some shares (USA) from the Yahoo.com portal and carries out data express-evaluation by results of which it gives trade advice concerning the shares of definite economic sectors, industries or separate shares (to buy, to sell or to keep). By that the system carries out the comparative analysis of shares concerning the industry, industries with the definite sector and sectors within the entire economy system suggesting comparative evaluations of the following type "much better, better, standard, worse, much worse". The comparison is based on the investment appeal evaluation of shares, industries and sectors.

3.7. Actuarial Estimations module

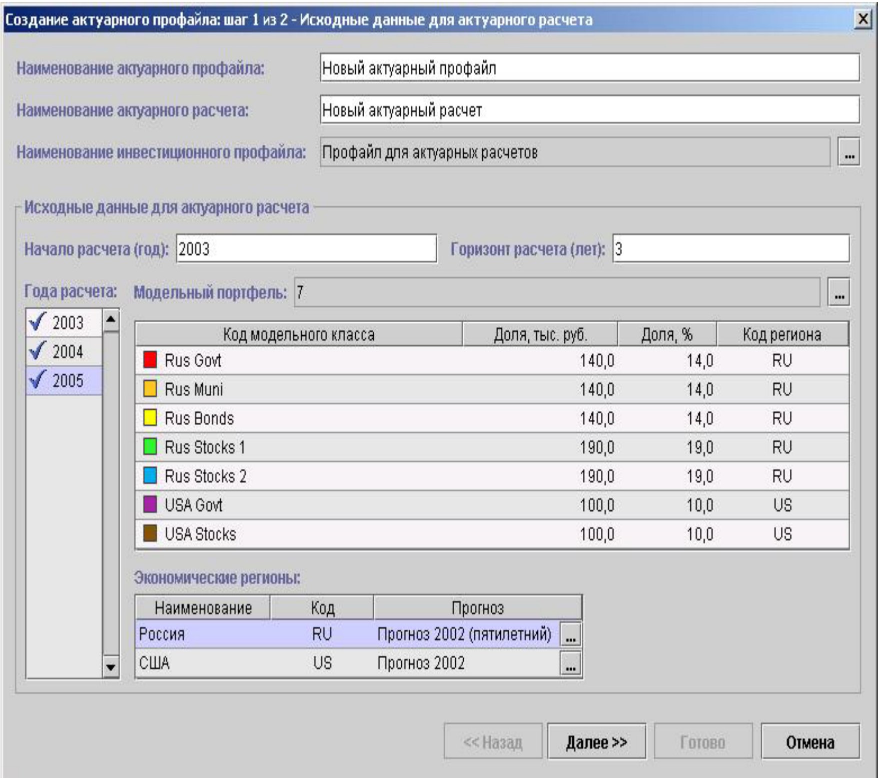

Fig. 7. The screenshot of the Actuarial Profile Creation module (actuarial estimation)

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Optimal Investment Using Technical Analysis and Fuzzy Logic

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Abstract. In this paper, we present the investment expert system based on fuzzy logic representation of rules that are used by investors applying technical analysis in the decision making. The method relies on fuzzy logic to aid decision making when certain price movements or certain price formations occur. Two different methods of fuzzy logic systems creating are compared. The first one is, in essence, a special adaptation of the classical Mamdani approach. The second method is based on so called "Logic-Motivated Fuzzy Logic Operators" i.e. on another mathematical representation of t-norm and Yager's implication rule. The preliminary results obtained on the basis of the real data of Warsaw Stock Exchange allow us to say that optimized expert system based on "Logic-Motivated Fuzzy Logic Operators" framework brings substantially grater benefits and is more reliable. Moreover, elaborated optimal investment strategy applying the technical analysis and the fuzzy logic makes it possible to gain profit even in the conditions of dropping stock prices.
Keywords. Fuzzy Logic, Investment expert system, Optimization.

1. Introduction

The departure point for the present work was the article [1] where the authors presented an expert system based on fuzzy logic representation of rules that are used by investors applying technical analysis in decision making. Since technical analysis theory consists of indicators used by experts to evaluate stock prices, the new method [1] maps these indicators into new inputs that can be fed into a fuzzy logic system. The only required inputs to these indicators are past sequence (history) of stock prices. This method relies on fuzzy logic to formulate a decision making when certain price movements or certain price formations occur. The main idea is to use technical indicators with fuzzy logic to create a fuzzy indicator that recommends sell, buy or hold position. To build the investment expert system the authors of [1] used directly the classical Mamdani's general form of the fuzzy rules in case of multi-input–single-output systems (MISO) [2]. In the paper [3], the new system of so called "Logic-Motivated Fuzzy Logic Operators" is proposed to better reflect the specifics of human reasoning in decision making processes. This system is based on other mathematical representation of t-norm and Yager's implication rule [4]. The rest of paper is organized as follows: Section 2 describes the set of technical analysis indices used in expert system and based on them membership functions representing expert's verbal estimations. In Section 3, the fuzzy logic investment expert system based on Mamdani's approach is presented; Section 4 is devoted to "Logic-Motivated Fuzzy Logic Operators" representation of expert system. Finally, in Section 5, we examine optimized investment strategy search using the elaborated expert systems.

2. Technical analysis indices and their fuzzy estimations

To show the merits of our optimization-based approach more transparently, we used nearly the same set of technical analysis indices as in the work [1]. Firstly, the historical data, $R(nT)$, representing the closing price of certain stock at sample (nT) were used to obtain the set of technical indicator inputs.
Factually, following inputs were included in the analysis:

- Closing price $R(nT)$
- Rate of change momentum indicator $ROC(nT) = R(nT) - R((n-r)T)$, where r is the depth of analysis $(n > r)$.
- The stochastic indicators

$$\%K(nT) = \left(\frac{R(nT) - R_{min}(nT)}{R_{max}(nT) - R_{min}(nT)} \right) \cdot 100,$$

$$\%D(nT) = \sum_{(n-3)}^{n} \frac{\%K(nT)}{3}, \qquad n \geq 3.$$

- Support level= $Avg(nT) - 2\sigma(nT)$,
- Resistance level= $Avg(nT) + 2\sigma(nT)$,

where $$\sigma(nT) = \sqrt{\frac{\sum_{n-g}^{n} (R(nT) - Avg(nT))^2}{g}},$$

$Avg(nT) = \dfrac{\sum_{n-g}^{n} R(nT)}{g}$ is g day price average.

The next step is the convergence of initial technical indicators into set of seven indexes more suitable for using in fuzzy logic system [1]:

$$Y_1 = Y_{ROC} = \frac{R(nT) - R((n-r)T)}{R((n-r)T)},$$
$$Y_2 = Y_1((n-2)T) - Y_1(nT) = Y_{ROC}((n-2)T) - Y_{ROC}(nT), \quad n \ge 2,$$
$$Y_3 = \%D(nT),\ Y_4 = Y_3(nT) - \%K(nT),\ Y_5 = Avg(nT) + 2\sigma R(nT),$$
$$Y_6 = R(nT) - (Avg(nT) - 2), Y_7 = R(nT) - Avg(nT).$$

If Y_1 is large, in another word if the *ROC* indicator is large, then the price is likely to move higher. If Y_6 is large (the price is close to the resistance level), then the price is likely to move lower and so on.

Fuzzy logic provides a mechanism to quantify such fuzzy concept. This is achieved by using membership function. Nevertheless, we have used only triangular membership functions in fuzzy logic system since there are no reasons and additional information for introducing more complicated forms in technical analysis setting. All the results presented in the paper were obtained using the membership function of type shown in Fig.1.

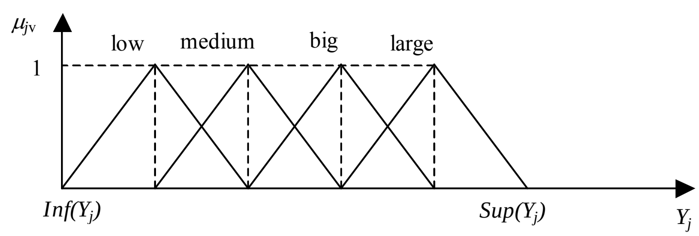

Fig. 1. Triangular membership functions

3. Fuzzy logic system: Mamdani's approach

We use the rule system proposed in [1].
Combined rules for the classes **large, big, medium, low** are**:**

IF {(Y_1 is **big)** *and* (Y_2 is **big**)} *or* {(Y_2 is **large**) *and* (Y_3 is **large**)} *or* (Y_7 is **large**) ***THEN*** C is **large,** (1)

IF (Y_2 is **large**) *or* (Y_6 is **large**) *or* (Y_3 is **large**) *or* (Y_5 is **low**) ***THEN*** C is **big,** (2)

IF (Y_4 is **big**) *or* (Y_7 is **medium**) *or* (Y_2 is **low**) ***THEN*** C is **medium,** (3)

IF {(Y_1 is **low**) and (Y_2 is **medium**)} or {(Y_6 is **low**) and (Y_7 is **low**)} ***THEN*** C is **low**. (4)

Let μ_{jv} denote the fuzzy membership grade of input Y_j to the class v. For example "Y_2 is **big"** denotes the membership of the fuzzy input 2 to the class **big** for the fuzzy input 2. Its value is computed using the membership grade μ_{23}.

Then the values of the antecedent of rules (1)-(4) may be calculated as

$$a_0 = \max\{\min(\mu_{22}, \mu_{11}), \min(\mu_{61}, \mu_{71})\},$$

$$a_1 = \max\{\mu_{21}, \min(\mu_{43}, \mu_{72})\}, a_2 = \max\{\mu_{24}, \mu_{64}, \mu_{34}, \mu_{51}\},$$

$$a_3 = \max\{\min(\mu_{23}, \mu_{13}), \min(\mu_{23}, \mu_{34}), \mu_{74}\}.$$

Mamdani's fuzzy implication method is used to combine the rules and calculate the outputs for classes **low, medium, big** and **large** respectively:

$$\mu_{C-0}(\bullet) = a_0 \wedge \mu_{c-0}(\bullet), \mu_{C-1}(\bullet) = a_1 \wedge \mu_{c-1}(\bullet),$$

$$\mu_{C-2}(\bullet) = a_2 \wedge \mu_{c-2}(\bullet), \mu_{C-3}(\bullet) = a_3 \wedge \mu_{c-3}(\bullet),$$

where $\mu_{c-0}(\bullet)$, $\mu_{c-0}(\bullet)$, $\mu_{c-0}(\bullet)$ and $\mu_{c-0}(\bullet)$ are membership functions corresponding to the classes **low, medium, big** and **large** of output variable, *C*, and symbol Λ denotes so-called clipping operation [5] that produces an output membership function $\mu_{C-i}(\bullet)$ clipped off at a height equal to the a_i.

Finally, Mamdani's process produces

$$\mu_C(C) = \mu_{C-0}(C) \vee \mu_{C-1}(C) \vee \mu_{C-2}(C) \vee \mu_{C-3}(C) = \\ = (a_0 \wedge \mu_{c-0}(C)) \vee (a_1 \wedge \mu_{c-1}(C)) \vee (a_2 \wedge \mu_{c-2}(C)) \vee (a_3 \wedge \mu_{c-3}(C)), \tag{5}$$

where $\vee$ is the max operator.

To get the real value estimation of C_r the standard center of area (COA) defuzzification method [6] was used.

Obviously, when C_r for the analyzed session is close to the 100 (high end), the stock is a strong buy.

4. Expert system on the base of Logic-Motivated Fuzzy Logic Operators

This system is based on other mathematical representation of t-norm and Yager's implication rule. Let us start with an implication operation $f_{\rightarrow}(a,b)$. According to [3], $f_{\rightarrow}(a,b)=b^a$ .. This operation was first introduced by R.Yager [4] and is called *Yager's* implication. As it is shown in [3], logic motivates the use of an algebraic product $f_{\&}(a,b)=a\cdot b$ as an "and"- operation (t-norm) and algebraic sum $f_{\vee}(a,b)=a+b-a\cdot b$ as an "or" –operation (t-conorm).

So, when using "Logic-Motivated Fuzzy Logic Operators" all the "and"–operators in the expressions (1)-(5) were presented by algebraic products and similarly all the "or"- operators in the preconditions of rules (1)-(4) and in the expression (5) were presented by algebraic sums.
Since instead of Mamdani's outputs we get implications of type

$$\mu_{C-i}(C)=\left(\mu_{c-i}(C)\right)^{a_1}, \tag{6}$$

some comments are necessary.

Firstly, only treatment of (6) in a fuzzy arithmetic spirit is reasonable. Therefore the α-cut representations of analyzed fuzzy values were used. As a result, the fuzzy values $\tilde{C}-i$ represented by membership functions $\mu_{C-i}(C)$ were presented as $\tilde{C}-i=\bigcup_{\alpha}[\underline{C},\overline{C}]_{\alpha}^{a_i}=\bigcup_{\alpha}[\underline{C}^{a_1},\overline{C}^{a_1}]_{\alpha}$,

where $[\underline{C},\overline{C}]_{\alpha}$ are crisp intervals such that $\mu_{c-i}(C)\geq\alpha$, $C\in[\underline{C},\overline{C}]_{\alpha}$.

To get a real value estimation, C_r, of fuzzy number $\tilde{C}-i$, two natural approaches may be used. The first of them is based on the results of [3] where the next logic-motivated expression proposed: $C_r=arg\,max\,\mu_{C-i}(C)$.For short, we will refer to this approach as *Yager$_{max}$* one.

The second approach is an application of interval arithmetic rules to the α-cut representations of analyzed fuzzy values:

$C_r = \sum_{\alpha} 0.5\alpha(\underline{C}^{a_1} + \overline{C}^{a_1}) / \sum_{\alpha} \alpha$. Further we will refer to this approach as $Yager_{ave}$. Finally, we create the expert system based on the Logic-Motivated Fuzzy Logic Operators which derives the real valued index C_r to determine buying and selling conditions.

Since Mamdani's type expert system described in Section 3 works the same way, it is possible to compare the results obtained with use of Mamdani's, $Yager_{max}$ and $Yager_{ave}$ approaches.

5. Case study: optimized investment strategy search using fuzzy expert systems

Elaborated expert systems create fuzzy logic outputs and output ranges, which determine how actions will be combined to form the executive action (decision stage). The success of the system is measured by comparing system output versus stock price movement. As we want to optimize the decisions, the control variables and target function must be chosen. In considered situation the natural target function is the total return (profit). We can obtain it using our optimized strategy during some control time period.

Since the value C_r may serve as an indicator of the good opportunity to buy or sell the stock, it seems natural to introduce the top C_{max} and bottom C_{min}, such that if $C_r > C_{max}$ the system buys and when $C_r < C_{min}$ the system sells. In our optimization task, parameters C_{max}, C_{min} were considered as the control variables. The third control variable was the number, *n*, of stock exchange sessions used in calculation of initial technical indicators inputs.

To illustrate the practical applicability of the proposed approach the teaching time periods (from one month to half of a year) were used to get optimal values of C_{max}, C_{min} and *n.* Optimization task was formulated as the maximization of the total return, R, obtained during the control time period:

$$(C_{min}, C_{max}, n)_{opt} = arg\,max\, R(C_{min}, C_{max}, n).$$

The simple random direct search method was used for the optimization and transaction costs were taking into account since they can significantly affect the total return in real-world situations.

Further, we obtained optimal C_{max} , C_{min} , n in simulation of decision making during the next time period (testing period) to get predicted total returns.

It is shown that the optimal approach give us much more benefits than method proposed in [1]. Moreover, elaborated optimal investment strategy utilizing the technical analysis and the fuzzy logic makes it possible to obtain a profit even under the conditions of dropping stock prices in opposite to results of [1] where only some reduction of total loss is indicated.

The results we got with use of Mamdani's, $Yager_{max}$ and $Yager_{ave}$ approaches were compared with those obtained using simplest "trend dependent" strategy, i.e. when we buy some stocks in the beginning and sell them only at the end of the analyzed time period.

The typical results we got for stocks of Polish company COMARCH are presented in Table 1, Fig.2. In all cases the initial investments of 10000PLN were simulated at the beginning of the control time period and testing period.

Table 1. Total returns obtained using different approaches (company COMARCH)

	Control time period (one month)	Testing time period (one month)
Mamdani's approach	590 PLN	514 PLN
$Yager_{max}$ approach	590 PLN	146 PLN
$Yager_{ave}$ approach	1100 PLN	200 PLN
Trend dependent strategy	80 PLN	429 PLN

Analyzing the Table 1 we can say that $Yager_{ave}$ approach seems to be more beneficial and reliable. The passive trend dependent strategy can bring less losses than Mamdani's approach when dropping trend takes place, but passive strategy is absolutely not paying for rising trends in comparison with any considered optimized strategy.

We can see that the optimization using fuzzy expert systems makes it possible to generate effective investment strategy, which brings the positive final profits even in a case of dropping prices.

The main advantage of our approach is the optimization, which significantly enhances the abilities of such expert system. Nevertheless, there are a lot of reserves to upgrade the fuzzy logic investment expert system.

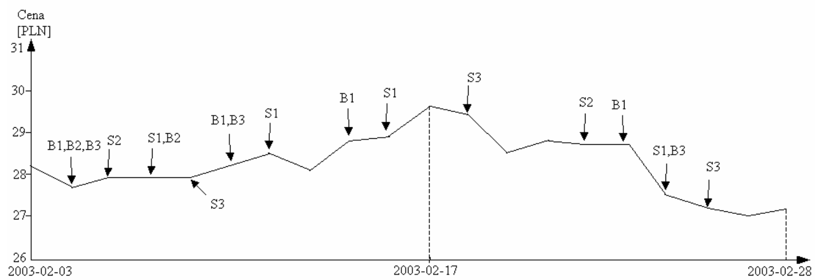

Fig. 2. Testing time period: B1, B2,B3 are buying signals according to Mamdani's , $Yager_{max}$ and $Yager_{ave}$ approaches respectively; S1,S2,S2 are corresponding selling signals.

In this paper we used only a few of hundreds parameters, which now examined in technical analysis practice. Moreover, the fuzzy logic expert system may be extended by introducing some qualitative verbally expressed characteristics (technical analysis charts and so on).

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Investment risk estimation for arbitrary fuzzy factors of investments project

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Introduction

Before setting a problem of investments risk estimation for general case, where project model parameters have fuzzy nature, we would like to point to the fact that the problem has already been solved in the next particular cases:

- for triangular numbers of general form [3], including solution in analytical form [2];
- when project effect is an arbitrary fuzzy factor and bottom constraint on this effect size is a point scalar number [4].

Now we came to the necessity and opportunity to solve the problem when effect and project constraint are fuzzy numbers of arbitrary form. Effect uncertainty is caused by project tasks unpredictability, but project constraint uncertainty is result of lack of clear understanding of project effectiveness norms by project owner. In this situation we have particular cases, when one or both numbers (effect and constrain) are:

- intervals;
- triangular numbers.

1. Interval case

We use NPV (Net Product Value) as project effectiveness factor. But all following discussions could be made for other factors of investment project.

We consider interval case. Let NPV = [NPV_1, NPV_2] is investment effectiveness, G = [G_1, G_2] is effectiveness constraint. We need to estimate possibility of event NPV < G. This event defines project inefficiency risk.

We take a phase space (NPV, G) and define there a rectangle limited with right and left points NPV and G. The rectangle is a field of equally possible events, which characterize investment process result. A shaded area (Figure 1 [3]**)** limited with straight lines $G = G_1$, $G = G_2$, $NPV = NPV_1$, $NPV = NPV_2$ and bisector of quadrantal angle $G = NPV$ represents ineffective investment. Correlation of $G_{1,2}$ and $NPV_{1,2}$ parameters gives us the following formula of shaded area [4] (unfortunately this formula is given with a mistake in [3], because there is not described one possible case of NPV and G correlation):

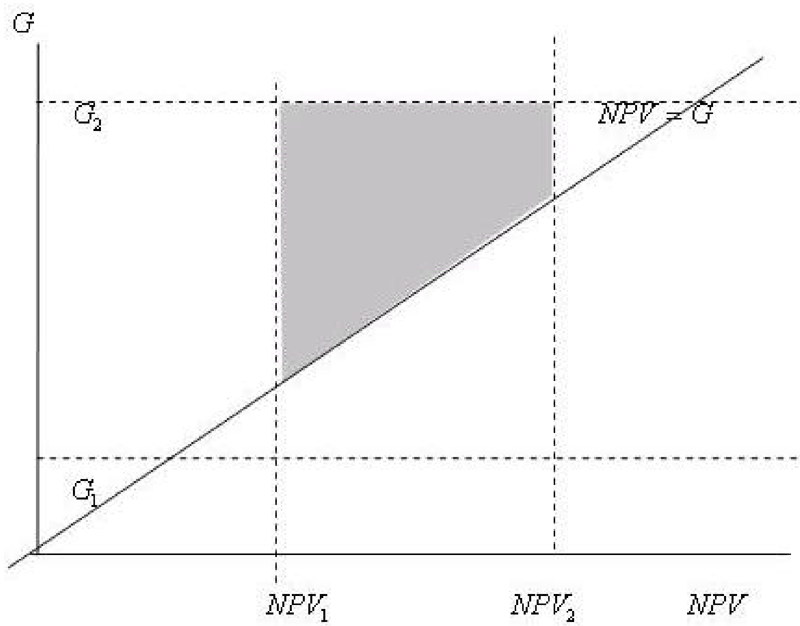

Fig. 1. Area of ineffective investment

$$S = \begin{cases} 0, G_2 \le NPV_1 \\ \dfrac{(G_2 - NPV_1)^2}{2}, \quad G_1 < NPV_1 < G_2 \le NPV_2 \\ \dfrac{(G_1 - NPV_1) + (G_2 - NPV_1)}{2} \cdot (G_2 - G_1), NPV_1 \le G_1 < G_2 \le NPV_2 \\ \dfrac{(G_2 - NPV_2) + (G_2 - NPV_1)}{2} \cdot (NPV_2 - NPV_1), G_1 \le NPV_1 < NPV_2 \le G_2 \\ (G_2 - G_1)(NPV_2 - NPV_1) - \dfrac{(NPV_2 - G_1)^2}{2}, NPV_1 \le G_1 \le NPV_2 \le G_2 \\ (G_2 - G_1)(NPV_2 - NPV_1), NPV_2 \le G_1 \end{cases} \tag{1}$$

It is clear that area depends on mutual position of [NPV_1, NPV_2] and [G_1, G_2] intervals. We explain this formula. Figure 1 shows situation when $G_1 < NPV_1 < NPV_2 < G_2$ and for which shaded area is equal to area of trapezium with bases (G_2 – NPV_2) and (G_2 – NPV_1) and altitude (NPV_2 – NPV_1). For other cases there are other graphs and shaded area could also be measured.

Since all variants (NPV, G) with given level of belonging α are equally possible, we can define project inefficiency risk level as geometric probability of hitting of point (NPV, G) into shaded area – ineffective investment area:

$$j = \frac{S}{(NPV_2 - NPV_1)\cdot(G_2 - G_1)} \qquad (2)$$

where S is defined according to the formula (1)

If factor $G = G_1 = G_2$ has point estimate, then (1) and (2)can be transformed to (using limit transition) [4]:

$$j = \begin{cases} 0 & , \text{ ïðè } \quad G < NPV_1 \\ \dfrac{G - NPV_1}{NPV_2 - NPV_1} & , \text{ ïðè } \quad NPV_1 \le G \le NPV_2 \\ 1 & , \text{ ïðè } \quad G > NPV_2 \end{cases} \qquad (3)$$

Vice versa, if $NPV = NPV_1 = NPV_2$ then

$$j = \begin{cases} 0 & , \text{ ïðè } \quad G_2 < NPV \\ \dfrac{G_2 - NPV}{G_2 - G_1} & , \text{ ïðè } \quad G_1 \le NPV \le G_2 \\ 1 & , \text{ ïðè } \quad G_1 > NPV \end{cases} \qquad (4)$$

In degenerate case (when we have $G = G_1 = G_2$ and $NPV = NPV_1 = NPV_2$ simultaneously) everything is absolutely simple:

$$j = \begin{cases} 0, & \text{ ïðè } \quad G \le NPV \\ 1, & \text{ ïðè } \quad G > NPV \end{cases} \qquad (5)$$

2. Transition to general case

Let data confidence level α is given, and

$$NPV = NPV_\alpha = [NPV_{1\alpha}, NPV_{2\alpha}], \qquad (6)$$

$$G = G_\alpha = [G_{1\alpha}, G_{2\alpha}]$$

are corresponding intervals of belonging, forming fuzzy sets. Practically (6) specifies two fuzzy numbers NPV and G in a segment way. If analytical form (6) exists, these fuzzy numbers have corresponding name (LR-numbers, triangular numbers, etc.).

In case of setting in form (6), risk level is defined according to formulas (1) and (2) for each level of α. Integral measure of possibility could be defined in two ways:

- **exactly** (through integral if possible)

$$Risk = \int_0^1 j(a)\,da \qquad (7)$$

where

$$j(a) = \frac{S_a}{(NPV_{2a} - NPV_{1e}) \cdot (G_{2e} - G_{1a})} \qquad (8)$$

- **approximately** (as finite sum)

$$\text{Risk} = \sum_{(i)} j(a_i) \times \text{Äá} \qquad (9)$$

where $\Delta\alpha$ - discrete value of segmentation (e.g. 0.1), i – segmentation index,

$$\alpha_i = (i\text{-}1) * \Delta\alpha. \qquad (10)$$

In terms of fuzzy sets intervals are rectangular fuzzy numbers, where the same interval corresponds to any level of belonging. Therefore in case of two interval factors $\varphi(\alpha)$ = const, and if we take this constant out of the integral (7), we get estimation (2).

In fact, things covered in this part are enough to implement automated risk estimation procedure (by analogy with investment calculator [1]). However, sometimes it is useful to have analytical statements for risk to perform test calculations. We demonstrate it basing on two triangular numbers.

3. Case of two triangular numbers of general form

Let G = (G_{min}, G_{av}, G_{max}) and NPV = (G_{min}, G_{av}, G_{max}) are triangular numbers of general form. We find level of ineffectiveness risk in general form [2] calculating integral (7). We consider function $j(a)$:

$$j(a)=\begin{cases} 0, & G_2 \le NPV_1; \\ j_1=\dfrac{(G_2-NPV_1)^2}{2(G_2-G_1)(NPV_2-NPV_1)}, & G_1 < NPV_1 < G_2 \le NPV_2; \\ j_2=\dfrac{(G_1-NPV_1)+(G_2-NPV_1)}{2(NPV_2-NPV_1)}, & NPV_1 \le G_1 < G_2 \le NPV_2; \\ j_3=\dfrac{(G_2-NPV_2)+(G_2-NPV_1)}{2(G_2-G_1)}, & G_1 \le NPV_1 < NPV_2 \le G_2; \\ j_4=1-\dfrac{(NPV_2-G_1)^2}{2(G_2-G_1)(NPV_2-NPV_1)}, & NPV_1 \le G_1 \le NPV_2 \le G_2; \\ j_5=1, & NPV_2 \le G_1. \end{cases} \tag{11}$$

It is necessary to note, that in the case of triangular numbers G and NPV, function $j(a)$ can not exist on all the intervals simultaneously. Integral (7) will look like

$$\int_0^1 j(a)da = \int_{a_0}^{a=a_1} j_1(a)da + \int_{a_1}^{a=a_2} j_2(a)da + \int_{a_2}^{a=a_3} j_3(a)da + \int_{a_3}^{a=a_4} j_4(a)da + \int_{a_4}^{a=a_5} j_5(a)da \tag{12}$$

however, some parts of the integral will be equal to 0, depending on type of numbers G and NPV.

We find integrals $\int_{a_{i-1}}^{a_i} j_i da$, $i=\overline{1,5}$. For this purpose we define

$$\begin{aligned} NPV_{\max} &= N; \\ NPV_{\min} &= M; \\ NPV_{av} &= S; \\ G_{\max} &= n; \\ G_{\min} &= m; \\ G_{av} &= s. \end{aligned} \tag{13}$$

It is also necessary to express formally function $j(a)$ through variable a, that is express values G_1, G_2, NPV_1 and NPV_2 through a. It is easy to do by means of taking general straight-line equation: $a(NPV_1) = a \cdot NPV_1 + b$ and finding factors a and b with aid of points of straight line: $(NPV_{\min}, 0)$ and $(NPV_{av}, 1)$. So

$$a(NPV_1) = \frac{NPV_1 - NPV_{\min}}{NPV_{av} - NPV_{\min}} \tag{14.1}$$

or

$$NPV_1 = a\,(NPV_{av} - NPV_{\min}) + NPV_{\min} \tag{14.2}$$

Similarly we get expressions for NPV_2, G_1 and G_2:

$$NPV_2 = NPV_{\max} - a\,(NPV_{\max} - NPV_{av}) \tag{15}$$

$$G_2 = G_{\max} - a\,(G_{\max} - G_{av}) \tag{16}$$

$$G_1 = a\,(G_{av} - G_{\min}) + G_{\min} \tag{17}$$

Using (13-17) we make substitution in function (11) and write resulting expressions for $\int\limits_{a_{i-1}}^{a_i} j_i da$, $i = \overline{1,5}$, which we got from integration and some transformations:

1. For $G_1 < NPV_1 < G_2 < NPV_2$

$$\int\limits_{a_0}^{a_1} j_1 da = \tag{18}$$

$$\int\limits_{a_0}^{a_1} \frac{[n - a(n-s) - a(S-M) - M]^2}{2[N - a(N-S) - a(S-M) - M]\cdot[n - a(n-s) - a(s-m}$$

$$=$$

$$= \frac{1}{2} * [\, a \cdot \frac{-(S - M + n - s)^2}{(n-m)(N-M)(-1+a)}$$

$$+ a^2 \cdot \frac{(S - M + n - s)^2}{(n-m)(N-M)(-1+a)} +$$

$$\frac{-(S-s)^2}{(n-m)(N-M)(-1+a)}+$$
$$\left.+\frac{2\cdot(S-s)\cdot(S-M+n-s)\ln(-1+a)}{(n-m)(N-M)}]\ \right|_{a_0}^{a_1}$$

If we substitute

$$\frac{1}{2(n-m)(N-M)}=k, \tag{19}$$
$$S-s=q,$$
$$S-M+n-s=p,$$

we can rewrite the expression in more simple way:

$$\int_{a_0}^{a_1} j_1 da = \tag{20}$$
$$k\cdot\left[-\frac{p^2a^2}{(1-a)}+\frac{p^2a}{(1-a)}+\frac{q^2}{(1-a)}+2qp\ln(-1+a)\right]\Bigg|_{a_0}^{a_1}$$

2. For $NPV_1 < G_1 < G_2 < NPV_2$

$$\int_{a_1}^{a_2} j_2 da = \tag{21}$$
$$\int_{a_1}^{a_2}\frac{[a(s-m)+m-a(S-M)-M]+[n-a(n-s)-a(S-M}{2\cdot[N-a(N-S)-a(S-M)-M]}$$

$$=\left(\frac{1}{2}a\cdot\frac{m-2\cdot M+n}{(N-M)}+a\cdot\frac{S-s}{(N-M)}+\frac{\ln(-1+a)(S-s)}{(N-M)}\right)\Bigg|_{a}^{a}$$

3. For $G_1 < NPV_1 < NPV_2 < G_2$

$$\int_{a_2}^{a_3} j_3 da = \tag{22}$$

$$\int_{a_2}^{a_3} \frac{[n-a(n-s)-N+a(N-S)]+[n-a(n-s)-a(S-M)\cdot}{2[n-a(n-s)-a(s-m)-m]}$$

$$=\left(\frac{1}{2}\cdot a\cdot\frac{2n-N-M}{n-m}+a\cdot\frac{S-s}{n-m}+\frac{\ln(-1+a)(S-s)}{n-m}\right)\Bigg|_{a_2}^{a_3}$$

4. For $NPV_1 < G_1 < NPV_2 < G_2$

$$\int_{a_3}^{a_4} j_4 da = \tag{23}$$

$$\int_{a_0}^{a_1}\left(1-\frac{[N-a(N-S)-a(s-m)-m]^2}{2[N-a(N-S)-a(S-M)-M]\cdot[n-a(n-s)-a(s}\right.$$

$$=a-\frac{1}{2}*[a\cdot\frac{-(s-m+N-S)^2}{(n-m)(N-M)(-1+a)}$$
$$+a^2\cdot\frac{(s-m+N-S)^2}{(n-m)(N-M)(-1+a)}+$$
$$+\frac{-(S-s)^2}{(n-m)(N-M)(-1+a)}$$
$$-\frac{2\cdot(S-s)\cdot(s-m+N-S)\ln(-1+a)}{(n-m)(N-M)}]\ \Bigg|_{a_3}^{a_4}$$

Or if we substitute

$$\frac{1}{2(n-m)(N-M)}=k, \tag{24}$$
$$S-s=q,$$
$$s-m+N-S=g,$$

We can rewrite it like this:

$$\int_{a_3}^{a_4} j_4 da = \tag{25}$$

$$\left(a - k \cdot \left[-\frac{g^2 a^2}{(1-a)} + \frac{g^2 a}{(1-a)} + \frac{q^2}{(1-a)} - 2qg \ln(-1+a) \right] \right) \Bigg|_{a_3}^{a_4}$$

5. For $NPV_2 < G_1$

$$\int_{a_4}^{a_5} j_5 da = \int_{a_4}^{a_5} 1 da = a \Big|_{a_4}^{a_5} \qquad (26)$$

If function $j(a)$ exists only on this interval, degree of risk is equal to 1, and investments will be definitely ineffective:

$$Risk = \int_0^1 1 da = 1 - 0 = 1 .$$

Expressions (18-26) could be used directly for evaluation of ineffective investment risk. In this case they should be placed into (12), with found value a_i before.

The following example clears up all the explained things.

Example. We consider an investment project "Equipment acquiring in the course of feed processing building modernization" [2]. Let project effectiveness constraint is a fuzzy number G = (-200, 0, 300) and NPV = (-817, 700, 1332). We need to evaluate inefficient investment risk degree for the project.

Solution. Membership function for NPV is:

$$m_{NPV}(x) = \begin{cases} 0, & x \le -817; \\ \dfrac{x+817}{1517}, & -817 < x \le 700; \\ \dfrac{1332 - x}{632}, & 700 < x \le 1332; \\ 0 & 1322 < x. \end{cases} \qquad (27)$$

Membership function for G is:

$$\mu_G(x) = \begin{cases} 0, & x \le -200; \\ \dfrac{x+200}{200}, & -200 < x \le 0; \\ \dfrac{300-x}{300}, & 0 < x \le 300; \\ 0 & 300 < x. \end{cases} \tag{28}$$

Graphical representation of these fuzzy numbers is given on Figure 2. For these correlation of fuzzy numbers NPV and G function $j(\alpha)$ exists only on the next three intervals: interval $NPV_1 < G_1 < G_2 < NPV_2$ for $\alpha \in [0; \alpha_0]$, interval $G_1 < NPV_1 < G_2 < NPV_2$ for $\alpha \in [\alpha_0; \alpha_1]$ and interval $G_2 < NPV_1$ for $\alpha \in [\alpha_1; 1]$. We need to find values for α_0 and α_1 before using formulas to define project risk degree. After equating μ_G to μ_{NPV} on the corresponding intervals we get:

$$\alpha_0 = 0{,}47 \text{ for } NPV = G = -106 \tag{29}$$

$$\alpha_1 = 0{,}615 \text{ for } NPV = G = 116$$

Basing on these data and with aid of formulas (12), (20), and (21) we find project inefficiency risk degree

$$Risk = \int_0^{0.615} j(\alpha)d\alpha = \int_0^{0.47} j_2(\alpha)d\alpha + \int_{0.47}^{0.615} j_1(\alpha)d\alpha \tag{30}$$

$$\int_0^{0.47} j_2(\alpha)d\alpha = 0{,}136\,,$$

$$\int_{0.47}^{0.615} j_1(\alpha)d\alpha = 0{,}0064$$

Risk degree for the project is

$$\text{Risk} = 0.1424. \tag{31}$$

If we want to define project risk by means of approximate method (according to (9)), we need to arrange table of segment intervals, evaluate risk for each level of belonging, according to (11), and sum all the risks with weight $\Delta\alpha$=0.1. Calculation results are given in Table 1.

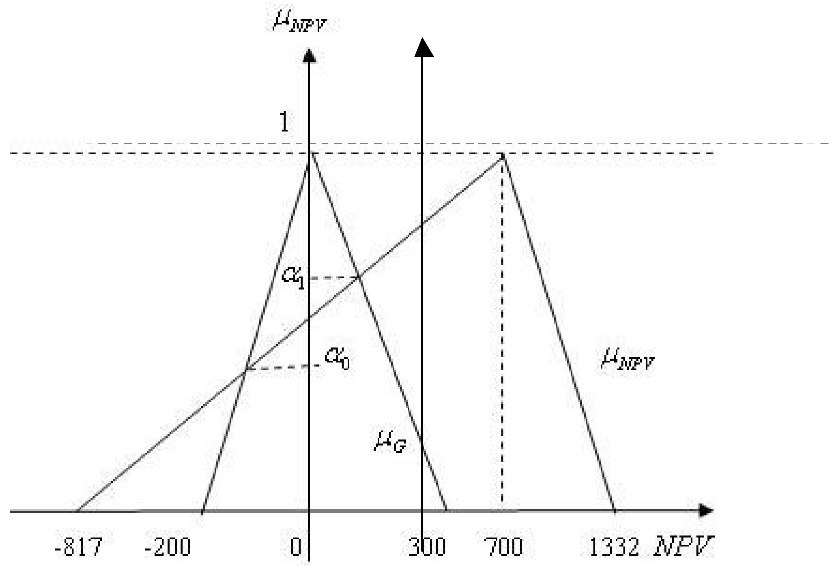

Fig. 2. Data of the example

Table 1. Risk evaluation using an approximate method

Alpha	NPV1	NPV2	G1	G2	Fi
0	-817	1332	-200	300	0.403
0.1	-665	1269	-180	270	0.367
0.2	-514	1206	-160	240	0.322
0.3	-362	1142	-140	210	0.264
0.4	-210	1079	-120	180	0.186
0.5	- 58	1016	-100	150	0.081
0.6	93	953	-80	120	0.002
0.7	245	890	-60	90	0.000
0.8	397	826	-40	60	0.000
0.9	548	763	-20	30	0.000
1	700	700	0	0	

As a result, according to (9)

$$\text{Risk} = 0.163. \tag{32}$$

This result differs from previous result (31) on 14%, which is within feasible accuracy. If we consider 20, 50, and 100 segment intervals, the results are

$$\text{Risk} = 0.152, \quad \text{Risk} = 0.146, \quad \text{Risk} = 0.144 \tag{33}$$

correspondingly, that is, we see convergence of approximate estimate to accurate estimate (31).

4. "Triangle + number" model

If one of two fuzzy numbers degenerates to a real number, all the formulas become much easier. In [3] there are analytical forms for the case when G is degenerate:

$$Risk = \begin{cases} 0, & G < NPV_{\min} \\ R \times \left(1 + \dfrac{1-\alpha_1}{\alpha_1} \ln(1-\alpha_1)\right), & NPV_{\min} \le G < NPV_{av} \\ 1-(1-R) \times \left(1 + \dfrac{1-\alpha_1}{\alpha_1} \ln(1-\alpha_1)\right) & NPV_{av} \le G < NPV_{\max} \\ 1, & NPV_{\max} \le G \end{cases} \tag{34}$$

where

$$R = \begin{cases} \dfrac{G - NPV_{\min}}{NPV_{\max} - NPV_{\min}}, & G < NPV_{\max} \\ 1, & NPV_{\max} \le G \end{cases} \tag{35}$$

$$\alpha_1 = \begin{cases} 0, & G < NPV_{\min} \\ \dfrac{G - NPV_{\min}}{NPV_{av} - NPV_{\min}}, & NPV_{\min} \le G < NPV_{av} \\ \dfrac{NPV_{\max} - G}{NPV_{\max} - NPV_{av}}, & NPV_{av} \le G < NPV_{\max} \\ 0, & NPV_{\max} \le G \end{cases} \tag{36}$$

Vice versa, if NPV is degenerate, we get mirror formulas:

$$Risk = \begin{cases} 0, & NPV > G_{\max} \\ R \times \left(1 + \frac{1-\alpha_1}{\alpha_1} \ln(1-\alpha_1)\right), & G_{av} \le NPV < G_{\max} \\ 1 - (1-R) \times \left(1 + \frac{1-\alpha_1}{\alpha_1} \ln(1-\alpha_1)\right), & G_{\min} \le NPV < G_{av} \\ 1, & G_{\min} > NPV \end{cases} \tag{37}$$

where

$$R = \begin{cases} \frac{G_{\max} - NPV}{G_{\max} - G_{\min}}, & NPV > G_{\min} \\ 1, & G_{\min} \ge NPV \end{cases} \tag{38}$$

$$\alpha_1 = \begin{cases} 0, & NPV > G_{\max} \\ \frac{G_{\max} - NPV}{G_{\max} - G_{av}}, & G_{av} \le NPV < G_{\max} \\ \frac{NPV - G_{\min}}{G_{av} - G_{\min}}, & G_{\min} \le NPV < G_{av} \\ 0, & NPV \le G_{\min} \end{cases} \tag{39}$$

5. "Triangle + interval" model

Figure 3 shows the case when NPV is a triangular number, and G is an interval. This case is typical for the situation, when project budget has a spread, and project owner can not define project efficiency norms exactly. On the one hand, he can afford project being unprofitable for some period of time (e.g., when project goal is to get temporary monopoly and increase market share by means of dumping, removing weak competitors). On the other hand, project owner does not understand how long such an unprofitableness could continue. As a result we get interval evaluation of extremely low NPV.

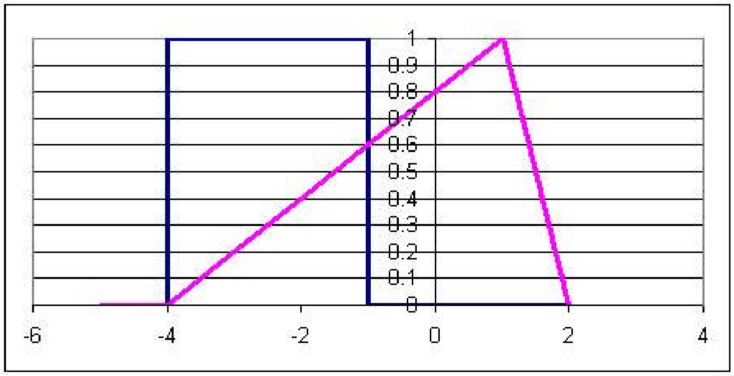

Fig. 3. "Triangle + interval" model

We can get compact analytical formulas for an important particular case, when the next condition is valid for all levels of belonging:

$$G_1 < NPV_1 < G_2 < NPV_2. \tag{40}$$

In this case we get

$$j\,(a) = \frac{\left(G_{\max} - NPV_1(a)\right)^2}{2\left(G_{\max} - G_{\min}\right)\left(NPV_2(a) - NPV_1(a)\right)}, \tag{41}$$

where

$$NPV_2(a) = NPV_{\max} - a\left(NPV_{\max} - NPV_{av}\right) \tag{42.1}$$

$$NPV_1(a) = NPV_{\min} + a\left(NPV_{av} - NPV_{\min}\right) \tag{42.2}$$

After integrating of (41) by α, by analogy with formulas for two triangular numbers, we get an expression for integral measure of risk. The whole development of formulas could be done by readers on their own.

Conclusion

Table 2 contains all the results found for today and connected with evaluation of investment project risk degree with arbitrary fuzzy parameters.

Table 2. Formulas for project risk estimation

Form of NPV	Numbers of formulas in this paper, according to the form of G			
	Point	Interval	Triangular number	General form
Point	(5)	(4)	(37) – (39)	(7) – (9)
Interval	(3)	(1) – (2)	In the future	
Triangular number	(34) – (36)	In the future	(11) – (26)	
General form	(7) – (9)			

When implementing risk calculators (by analogy with [1]) it is advised to use formulas of the most general form and increase number of intervals from 10 to 100 (when reconstructing segment intervals basing on analytical functions of known form). The task of risk estimation has arithmetic complexity in all the cases, therefore there should be no problems with unacceptable laborious of operations. Particular correlations could be used as examples for testing.

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Robust Selection of Investment Projects

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Abstract: The problem of choosing the best investment project in the situation of uncertainty with respect to cash flows is considered. Two approaches (together with the corresponding algorithms) are proposed to solve it. Both are based on approaches known from the literature, called robust. For each project they find the worst circumstances and finally choose the project which will be optimal if the circumstances are bad. The two approaches differ in the definition of possible scenarios and in the evaluation methods of the projects, although in both approaches NPV (Net Present Value) is the basis for the evaluation. A computational example illustrates the proposal.

1 Introduction

The problem of making the best investment decision is very difficult from the practical point of view. This is mainly because such decisions are based on predicted data (cash flows, discount rates, life time etc), which are bound to be known only approximately and their true realisations may be a surprise. That is why it is of utmost importance to develop methods which would help the decision maker to deal with uncertainty of investment decisions. There are several techniques that can be used, like fuzzy modelling ([3,4]). In this paper we would like to propose to apply to investment decision problems another approach known from the literature - the robust approach. As this notion comprises in the literature different things, our proposal comprises in fact two different approaches. The result each time would be a “robust” choice - i.e. one that would be good in the worst circumstances. Of course, it will be clarified (as it can of course have several meanings) what "the worst circumstances" and "good" means. The proposed approaches will be illustrated with an example. Calculations have been carried out by means of EXCEL.

2 Statement of the problem

We consider the following problem: there are given n investment projects $P_j \ (j = 1,...,n)$ with the same useful life of l years. A discount rate r is fixed. For each project $P_j \ (j = 1,...,n)$ there are given the predicted cash flows for the end of each year $k \ (k = 0,...,l)$, in the form of two numbers: $\overline{f}_j^k$ - the most possible cash flow, and θ_j^k - the maximal possible reduction of the cash flow $\overline{f}_j^k$, due to purchase prices increase, selling price decrease, underestimated operating expenses or other problems. Thus, the true cash flow of project $P_j \ (j = 1,...,n)$ at the end of the k-th year can take on values $F_j^k(\lambda) = \overline{f}_j^k - \lambda\theta_j^k \ \ \lambda \in [0,1]$.

The problem consists in choosing exactly one project which would be considered the best one. The basic criterion used is Net Present Value, abbrev. NPV (the basic criterion used in the evaluation of investment projects in practice), which for project $P_j \ (j = 1,...,n)$ and for a fixed $\lambda \in [0,1]$ takes on the following value:

$$NPV(P_j, \lambda) = \sum_{k=0}^{l} \frac{F_j^k(\lambda)}{(1+r)^k} = \sum_{k=0}^{l} \frac{\overline{f}_j^k}{(1+r)^k} - \lambda \sum_{k=0}^{l} \frac{\theta_j^k}{(1+r)^k} \qquad (1)$$

Let us consider the following example which will accompany us through the paper:

Example: There are 6 investment projects, we want to choose one to be carried out. The two tables below contain values $\overline{f}_j^k$ and θ_j^k

Table 1: Values $\overline{f}_j^k$ for the example

k	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6
0	-4000	-1000	-10000	-10000	-2000	-1000
1	0	0	14300	14080	3000	0
2	10000	0	0	0	300	0
3	0	0	0	0	2000	200
4	0	0	0	0	200	7000
5-9	0	0	0	0	0	0
10	0	10000	0	0	0	0

In the years 5-9 the predicted flows are each year, for each project the same - equal to 0, that is why those 4 years are put in one row. We assume that the predicted flows equal to 0 are sure, i.e. for those years no reduction (to a negative flow) is possible, but this assumption is of no other importance than simplicity.

Table 2: Values θ_j^k for the example

k	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6
0	200	100	1000	1000	10	100
1	0	0	1000	1400	300	0
2	1000	0	0	0	30	0
3	0	0	0	0	200	20
4	0	0	0	0	20	700
5-9	0	0	0	0	0	0
10	0	100	0	0	0	0

We assume $r = 10\%$

The choice of one single project is not unequivocal here. If no disturbance occurs and all the flows take on values $\overline{f}_j^k$, project 1 has the highest NPV. On the other extreme, if everything goes wrong and all the flows take on values $\overline{f}_j^k - \theta_j^k$, project 6 is the best one. In an extreme case, we might have 6 disjoint subsets for λ of the interval [0,1], for each of whom another project has the best NPV if the flows take on values $F_j^k(\lambda) = \overline{f}_j^k - \lambda\theta_j^k$.

But we are obliged to select exactly one project. To solve this problem, we propose to use two robust approaches from the literature.

3 First robust approach to the selection of investment projects

The first approach we want to propose as a method of selecting the best investment project has already been applied to an economic problem - to the economic order quantity problem ([4]). Here we want to use it to select the best investment project. Its idea can be summarized as follows:

a) we assume that we are trying to choose the best object from a set of objects
b) the goodness of each object is measured by a function, whose value changes according to which scenario it is calculated
c) we have no influence on the scenario which will actually occur, but we know the whole set of possible scenarios *S*.
d) first for each scenario we find the best object from among those being evaluated
e) then, for each object and for each scenario we find out how worse this object is from the optimal one for the scenario in consideration
f) then, for each object we determine the worst case: the scenario when it is most worse from the optimal object for this scenario
g) finally we choose the object for which the value calculated in f) is minimal (the object whose worst case is the best).

In order to apply this approach, we have to define objects, scenarios, the function which measures the goodness of the objects and the distance function, which measures the distance between a given object and the optimal one for a given scenario.

The obvious choice for the objects are projects P_j $(j = 1,...,n)$, for the function which measures the goodness of the objects we choose NPV. Scenarios can be defined in various ways, let us here assume the following set of scenarios: $\{\lambda \in [0,1]\}$. Each scenario defines exactly the cash flow of each project, which will be, for a given scenario λ, equal to $F_j^k(\lambda) = \overline{f}_j^k - \lambda\theta_j^k$. Each scenario corresponds to a certain market situation: λ=0 corresponds to the best market situation, the maximal cash flows will occur, λ=1 corresponds to the worst market situations - all the projects will have the minimal cash flows, the interior points from [0,1] are related to in-between market situations.

In the footsteps of [4], we propose to measure, for a given scenario, the distance of a given project from the optimal one for this scenario by means of the following function:

$$S(P_j,\lambda) = \frac{\left|\min\limits_{j=1,...n}\left[NPV(P_j,\lambda)\right] - NPV(P_j,\lambda)\right|}{\min\limits_{j=1,...n}\left[NPV(P_j,\lambda)\right]} \tag{2}$$

The final choice will be the project P_j for which $\max\limits_{\lambda\in[0,1]} S(P_j,\lambda)$ attains its minimal value.

In our example, project P_1 is optimal for scenarios $[0,0.77]$ and project P_6 for scenarios $(0.77,1]$. The following table gives us the values of $S(P_j,\lambda)$ for selected scenarios:

Table 3: Values $S(P_j, \lambda)$ for the example ($\max\limits_{\lambda \in [0,1]} S(P_j, \lambda)$ are in bold).

Project	λ=0	λ=0.3	λ=0.6	λ=0.9	λ=1
1	0	0	0	0,02	**0,03**
2	**0,33**	0,32	0,3	0,29	0,29
3	0,3	0,39	0,49	0,62	**0,67**
4	0,34	0,46	0,61	0,78	**0,84**
5	0,34	0,37	0,36	0,36	0,36
6	**0,08**	0,05	0,02	0	0

It is easy to see that the final choice in our example, i.e. the robustly optimal project, would be P_1. The values calculated in Table 3 also help us to evaluate other projects. For example, we can see that P_6 is clearly the second best choice and that the other projects differ a lot from the two best with respect to the optimality criterion $\max\limits_{\lambda \in [0,1]} S(P_j, \lambda)$.

4 Second robust approach to the selection of investment projects

Here we would like to make an attempt to use in the investment projects selection problem another approach called "robust" in the literature. To our knowledge, this the first attempt to apply this approach to economic problems. The approach is proposed in [1], where the authors apply it to linear and discrete optimization problems. This approach can summarized as follows:

h) we assume that we are trying to choose the best object from a set of objects
i) the goodness of each object is measured by a function, whose value changes according to which scenario it is calculated
j) we have no influence on the scenario which will actually occur, but we know the whole set of possible scenarios *S*.
k) set *S* can be divided into *m* disjoint subsets S_t $(t = 0,\dots,m)$ such that $S = \bigcup\limits_{t=1}^{m} S_t$, where set S_t $(t = 0,\dots,m)$ can be called "set of scenarios pessimistic to degree *t*".

l) first, we set $t = t^*$ - the degree of pessimism (the greater t, the greater the pessimism).
m) then for each object we find the worst value of the evaluation function that can occur for scenarios from $\bigcup_{t=1}^{t*} S_t$.
n) finally we choose the object for which the worst value from m) is the best.

The authors of [1] justify their approach saying that in the nature rarely everything goes wrong, it is thus unnecessary to consider the most pessimistic scenarios. Thus, the decision maker can choose the degree of pessimism. The solution determined according to the approach will be the best in the worst case - where the "worst case" means "worst within a framework of a certain pessimism level".

In our case objects are again projects $P_j \ (j = 1,...,n)$, for the function which measures the goodness of the objects we choose again NPV. But scenarios will be defined as follows:

- $S = \left\{(\lambda_k)_{k=0}^{l}, \lambda_k = 0 \text{ lub } 1\right\}$
- $S_t = \left\{(\lambda_k)_{k=0}^{l} : \left|\{k : \lambda_k = 1\}\right| = t\right\}, t = 0,...,l$, where $|\ |$ denotes the power of a set.

λ_k =0 in a scenario means that the k-th year will be a good one, all the flows in this year will be maximal, i.e. equal to $\overline{f}_j^k \ (j = 1,...,n)$. λ_k =1 stands for the other extreme: the k-th year will be bad, all the flows will be equal to $F_j^k(1) = \overline{f}_j^k - \theta_j^k \ \ (j = 1,...,n)$. Set $S_t \ (t = 0,...,l)$ comprises all the scenarios in which exactly t years are bad and l-t years good.

If for a fixed t^* and for a selected project we want to find the worst value of NPV for this project for scenarios from $\bigcup_{t=0}^{t^*} S_t$, it is enough to consider just S_{t^*} - all the other scenarios give NPV which are not worse than the one from S_{t^*}. Thus, it is enough to find for each project the worst (the minimal) value of NPV for scenarios from S_{t^*}. The final choice will be then the project whose worst value of NPV found in this way is maximal.

For our example, using this approach and assuming the degree of pessimism 10 we would finally choose project P_6.

4 Conclusions

In this paper we propose to apply to the choice of investment projects in the situation of uncertainty the robust approaches from the literature. The robust solution is one that is reasonably good even if the circumstances turn out to be the worst. The decision maker can define himself notions like “the worst circumstances”, “set of possible scenarios”, “reasonably good”. Further research is still needed to explore all the possibilities of robust approaches in capital budgeting.

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Analyzing the Solvency Status of Brazilian Enterprises with Fuzzy Models

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Abstract

The goal of this work is to present the basic concepts that guide fuzzy logic systems to be used in credit risk analysis. Here we used fuzzy quantification theory to make a kind of multivariate analysis. Moreover, since Brazil has becoming an interesting emergent economy this work uses and shows to the international community some interesting accounting indicators that can efficiently point the health of a brazilian company.

1. Introduction

Credit risk analysis has been growing in the financial literature. In general, models begin with subjective data and try to give a scientific shape to the classifications they pose. Those classifications are criticized either for being tendentious or by not taking in account important aspects, although subjective, of people, enterprises and governments realities. It would be interesting that we had systems that would be completely independent from psychological aspects and that at the same time would take in account peculiarities of those who are looking for credit. Credit risk has become in Brazil the great concern of risk managers in financial institutions and of regulators. There are some reasons for that. The first one we can point is the Real Plan in 1994 that brought inflation rates to tolerable levels since then. Now brazilian market risk is much better understood and tracked. Another reason is that the larger part of banks' economic capital is generally used for credit. The sophistication of traditional standard methods of measurement, analysis and management of credit risk might, therefore, not be in line with its significance. A third reason points to the fact that a great number of insolvencies and restructuring of banks all over the world were influenced by prior bankruptcies of creditors.

The credit risk of a single client is the basis of all subsequent risk analysis and of credit portfolio risk modeling. A concept of default can be client oriented, i. e. the status of default is a state of a counterpart such as insolvency or bankruptcy. This work analyses the short to mid-term probability of insolvency for a brazilian company. It has evolved from a previous work [1] that used the traditional Z-score technique. To determine if a company has the ability to generate sufficient cash flow to service its debt obligations the focus of traditional credit analysis has been on the company's fundamental accounting information. Evaluation of the industrial environment, investment plans, balance sheet data and management skills serve as primary input for the assessment of the companies likelihood of survival over a certain time horizon. In this work we suppose a multivariate normality to warranty our results are adequate.

2. Fuzzy Quantification Theory

Since sample sets are commonly called groups in multivariate analysis, we call the fuzzy sets that form the samples here simply fuzzy groups. Now we will write the probability *P(A)* of a fuzzy event determined by a fuzzy set *A* over a *n*th dimensional interval R^n, which is defined by degree of probability *P* as [2]

$$P(A) = \int_{R^n} \mu_A(x)dP = E(\mu_A) \tag{1}$$

where $E(\mu_A)$ is the expected value of membership function μ_A. From this, we can calculate the fuzzy mean and fuzzy variance for variable *x* as [2]

$$m_A = \frac{1}{P(A)}\int_{R^n} x\mu_A(x)dP \tag{2}$$

$$\sigma_A^2 = \frac{1}{P(A)}\int_{R^n} (x - m_A)^2 \mu_A(x)dP \tag{3}$$

When we consider fuzzy event *A* is occurring, the probability of fuzzy event *B* to occur, $P_A(B)$ is

$$P_A(B) = \frac{1}{P(A)}\int_{R^n} \mu_B(x)\mu_A(x)dP = E_A(\mu_B) \tag{4}$$

and so the following relationship concerning the fuzzy event arises:

$$\sigma_A^2 = E_A\left[\left(x - m_A\right)^2\right] = E_A(x^2) - E_A^2(x) \tag{5}$$

For a given sample (x_1, …, x_n), when we are concerned with the fuzzy event A, can we now define the sample mean and variance. The size of the fuzzy set can be expressed, using the elements of the set, as

$$N(A) = \sum_{\omega=1}^{n} \mu_A(x_\omega) \tag{6}$$

Applying this idea of the size of a fuzzy set $N(A)$ to the sample, one can define the sample mean m_A and variance σ_A^2 as [2]

$$m_A = \frac{1}{N(A)} \sum_{\omega=1}^{n} x_\omega \mu_A(x_\omega) \tag{7}$$

$$\sigma_A^2 = \frac{1}{N(A)} \sum_{\omega=1}^{n} (x_\omega - m_A)^2 \mu_A(x_\omega) \tag{8}$$

Using the definitions above we can now explain variation between groups, variation within groups and total variation for fuzzy groups. Let sample $x_\omega (\omega = 1, \ldots, n)$ be given and the membership function of fuzzy group A_i (i = 1, …, K) be defined by $\mu_{A_i}(x_\omega)$. In this instance, the total mean m and mean m_{A_i} using fuzzy group A_i are expressed by

$$m = \frac{1}{N} \sum_{i-1}^{K} \sum_{\omega=1}^{n} x_\omega \mu_{Ai}(x_\omega) \tag{9}$$

$$m_{Ai} = \frac{1}{N(A_i)} \sum_{\omega=1}^{n} x_\omega \mu_{Ai}(x_\omega) \tag{10}$$

where we have

$$N = \sum_{i=1}^{K} N(A_i) \tag{11}$$

The total variation T, variation between fuzzy groups B and variation within a fuzzy group E are then respectively defined as

$$T = \sum_{\omega=1}^{n} \sum_{i-1}^{K} (x_\omega - m)^2 \mu_{Ai}(x_\omega) \tag{12}$$

$$B = \sum_{\omega=1}^{n} \sum_{i=1}^{K} (m_{Ai} - m)^2 \mu_{Ai}(x_\omega) \tag{13}$$

$$E = \sum_{\omega=1}^{n} \sum_{i=1}^{K} (x_\omega - m_{Ai})^2 \mu_{Ai}(x_\omega) \tag{14}$$

and

$$T = B + E \tag{15}$$

The relationship above shows that ideas like multivariate analysis derived from relationships such as maximum variance ratio can easily be extended to fuzzy events [2]. The object of fuzzy quantification theory as it is used in this work is to express several fuzzy groups in terms of qualitative descriptive variables that take the form of values on [0,1][2][3]. Table 1 shows the data handled. We try to express, as well as possible using a linear equation of category weight a_i of category A_1, the structure of the external standard fuzzy groups on the real number axis:

$$y(\omega) = \sum_{i=1}^{K} a_i \mu_i(\omega) \quad ; \qquad \omega = 1, \ldots, n \tag{16}$$

So it is determining the a_i that gives the best separation of the external standard fuzzy groups on the real number axis. The degree of separation of the fuzzy groups is defined as the fuzzy variance ratio η^2, which is the ratio of the total variation T and variation between fuzzy groups B from equations (12) and (13):

$$\eta^2 = B/T \tag{17}$$

	Fuzzy External Standard	Category
ω	$B_1 \cdots B_M$	$A_1 \cdots A_i \cdots A_K$
1	$\mu_{B1}(1) \cdots \mu_{BM}(1)$	$\mu_1(1) \cdots \mu_i(1) \cdots \mu_K(1)$
2	$\mu_{B1}(2) \cdots \mu_{BM}(2)$	$\mu_1(2) \cdots \mu_i(2) \cdots \mu_K(2)$
$\vdots$	$\vdots \quad \vdots$	$\vdots \quad \vdots$
ω	$\mu_{B1}(\omega) \cdots \mu_{BM}(\omega)$	$\mu_1(\omega) \cdots \mu_i(\omega) \cdots \mu_K(\omega)$
$\vdots$	$\vdots \quad \vdots$	$\vdots \quad \vdots$
n	$\mu_{B1}(n) \cdots \mu_{BM}(n)$	$\mu_1(n) \cdots \mu_i(n) \cdots \mu_K(n)$

0Table 1 – Data used in Fuzzy Quantification Theory

Now we determine a_i for linear equation (16) which maximizes fuzzy variance ratio η^2. The fuzzy mean $\bar{y}_{Br}$ within fuzzy group B_r for value $y(\omega)$ for the linear equation and total fuzzy mean $\bar{y}$ come out as

$$\bar{y}_{Br} = \frac{1}{N(B_r)} \sum_{\omega=1}^{n} y(\omega)\mu_{Br}(\omega) \quad ; \quad r = 1, ..., M \tag{18}$$

$$\bar{y} = \frac{1}{N} \sum_{r=1}^{M} \bar{y}_{Br} N(B_r) \tag{19}$$

Fuzzy mean $\overline{\mu}_i^r$ within each fuzzy group B_r for the membership value of category A_i and total fuzzy mean $\overline{\mu}_i$ are expressed as

$$\overline{\mu}_i^r = \frac{1}{N(B_r)} \sum_{\omega=1}^{n} \mu_i(\omega)\mu_{Br}(\omega) \quad ; \quad i = 1, ..., K \ ; \ r = 1, ..., M \quad (20)$$

$$\overline{\mu}_i = \frac{1}{N} \sum_{r=1}^{M} \overline{\mu}_i^r N(B_r) \quad ; \quad i = 1, ..., K \quad (21)$$

Now, to simplify the notation, we define the (Mn , K) matrices $\boldsymbol{A}$, $\overline{\boldsymbol{A}}_{\boldsymbol{G}}$ and $\overline{\boldsymbol{A}}$ for $\mu_i(\omega)$, $\overline{\mu}_i^r$ and μ_i as

$$\boldsymbol{A} = \begin{bmatrix} \mu_1(1) & \cdots & \mu_i(1) & \cdots & \mu_K(1) \\ \vdots & & \vdots & & \vdots \\ \mu_1(\omega) & \cdots & \mu_i(\omega) & \cdots & \mu_K(\omega) \\ \vdots & & \vdots & & \vdots \\ \mu_1(n) & \cdots & \mu_i(n) & \cdots & \mu_K(n) \\ \mu_1(1) & \cdots & \mu_i(1) & \cdots & \mu_K(1) \\ \vdots & & \vdots & & \vdots \\ \mu_1(n) & \cdots & \mu_i(n) & \cdots & \mu_K(n) \end{bmatrix} \qquad \overline{\boldsymbol{A}}_{\boldsymbol{G}} = \begin{bmatrix} \overline{\mu}_1^1 & \cdots & \overline{\mu}_i^1 & \cdots & \overline{\mu}_K^1 \\ \vdots & & \vdots & & \vdots \\ \overline{\mu}_1^1 & \cdots & \overline{\mu}_i^1 & \cdots & \overline{\mu}_K^1 \\ \vdots & & \vdots & & \vdots \\ \overline{\mu}_1^1 & \cdots & \overline{\mu}_i^1 & \cdots & \overline{\mu}_K^1 \\ \overline{\mu}_1^2 & \cdots & \overline{\mu}_i^2 & \cdots & \overline{\mu}_K^2 \\ \vdots & & \vdots & & \vdots \\ \overline{\mu}_1^M & \cdots & \overline{\mu}_i^M & \cdots & \overline{\mu}_K^M \end{bmatrix}$$

$$\overline{\boldsymbol{A}} = \begin{bmatrix} \overline{\mu}_1 & \cdots & \overline{\mu}_i & \cdots & \overline{\mu}_K \\ \vdots & & \vdots & & \vdots \\ \overline{\mu}_1 & \cdots & \overline{\mu}_i & \cdots & \overline{\mu}_K \\ \vdots & & \vdots & & \vdots \\ \overline{\mu}_1 & \cdots & \overline{\mu}_i & \cdots & \overline{\mu}_K \\ \overline{\mu}_1 & \cdots & \overline{\mu}_i & \cdots & \overline{\mu}_K \\ \vdots & & \vdots & & \vdots \\ \overline{\mu}_1 & \cdots & \overline{\mu}_i & \cdots & \overline{\mu}_K \end{bmatrix} \quad (22)$$

In addition, the K dimensional row vector $\boldsymbol{a}$ for category weight a_i and the (Mn , Mn) diagonal matrix $\boldsymbol{G}$ formed from membership value μ_{Br} are defined as

$$\boldsymbol{a}' = \begin{bmatrix} \boldsymbol{a}_1 & \cdots & \boldsymbol{a}_i & \cdots & \boldsymbol{a}_K \end{bmatrix} \quad (23a)$$

$$\boldsymbol{G} = \begin{bmatrix} \mu_{B1}(1) & & & & & \\ & \ddots & & & 0 & \\ & & \mu_{B1}(n) & & & \\ & & & \mu_{B2}(1) & & \\ & 0 & & & \ddots & \\ & & & & & \mu_{BM}(n) \end{bmatrix} \tag{23b}$$

This way, the total variation *T* and variation between fuzzy groups *B* from equation (12) and (13) can be written in matricial form as [2][3]

$$\boldsymbol{T} = \boldsymbol{a}'(\boldsymbol{A} - \overline{\boldsymbol{A}})'\boldsymbol{G}(\boldsymbol{A} - \overline{\boldsymbol{A}})\boldsymbol{a} \tag{24}$$

$$\boldsymbol{B} = \boldsymbol{a}'(\overline{\boldsymbol{A}}_{\boldsymbol{G}} - \overline{\boldsymbol{A}})'\boldsymbol{G}(\overline{\boldsymbol{A}}_{\boldsymbol{G}} - \overline{\boldsymbol{A}})\boldsymbol{a} \tag{25}$$

If we now substitute equation (24) and (25) in equation (17) and partially differentiate by *a* we obtain the relationship

$$\begin{aligned} &\left[\boldsymbol{G}^{1/2}(\overline{\boldsymbol{A}}_{\boldsymbol{G}} - \overline{\boldsymbol{A}})\right]'\left[\boldsymbol{G}^{1/2}(\overline{\boldsymbol{A}}_{\boldsymbol{G}} - \overline{\boldsymbol{A}})\right]\boldsymbol{a} = \\ &\eta^2\left[\boldsymbol{G}^{1/2}\left(\boldsymbol{A} - \overline{\boldsymbol{A}}\right)\right]'\left[\boldsymbol{G}^{1/2}\left(\boldsymbol{A} - \overline{\boldsymbol{A}}\right)\right]\boldsymbol{a} \end{aligned} \tag{26}$$

Now, defining $\boldsymbol{S_G}$ and $\boldsymbol{S}$ by the (K, K) matrix as

$$\boldsymbol{S}_{\boldsymbol{G}} = \left[\boldsymbol{G}^{1/2}\left(\overline{\boldsymbol{A}}_{\boldsymbol{G}} - \overline{\boldsymbol{A}}\right)\right]'\left[\boldsymbol{G}^{1/2}\left(\overline{\boldsymbol{A}}_{\boldsymbol{G}} - \overline{\boldsymbol{A}}\right)\right] \tag{27a}$$

$$\boldsymbol{S} = \left[\boldsymbol{G}^{1/2}\left(\boldsymbol{A} - \overline{\boldsymbol{A}}\right)\right]'\left[\boldsymbol{G}^{1/2}\left(\boldsymbol{A} - \overline{\boldsymbol{A}}\right)\right] \tag{27b}$$

it is possible to decompose $\boldsymbol{S}$ using triangular matrix Δ as $\boldsymbol{S} = \Delta'\Delta$ getting

$$\left[(\Delta')^{-1}\boldsymbol{S}_{\boldsymbol{G}}\Delta^{-1}\right]\Delta\boldsymbol{a} = \eta^2\Delta\boldsymbol{a} \tag{28}$$

Because of that, category $\boldsymbol{a}$ for equation (16), which maximizes fuzzy variance ratio η^2, can be obtained from eigenvector $\Delta\boldsymbol{a}$, which maximizes eigenvalue η^2 of the matrix $\left[(\Delta')^{-1}\mathbf{S}_{\mathbf{G}}\Delta^{-1}\right]$.

3. Data analysis

The theory shown in previous section was used to develop a fuzzy system that was then applied to simulation data based on a work that analyzed the probability of insolvency for brazilian enterprises using traditional multivariate analysis [1]. That work selected some key accounting variables that can efficiently pinpoint an enterprise financial health. These variables are here named as X1-X5 and their significance are as follows. Table 2 below explain the meaning of the abbreviators in the variables.

LS	Liquid Sales
CP	Circulating Passive
T	Treasury Balance
LA	Liquid Asset
TSI	Tributary and Social Insurance obligations
SGC	Sold Goods Costs
OI	Operational Investment
MMS	Monthly Medium sales

Table 2 – Abbreviations used for accounting data

(CP / LA) / Sector Median – X1 - This is an indicator of the financial structure that represents the short term indebtedness. The interpretation of the results reached in this index confirm that the great use of third-party money by an enterprise, carries it towards insolvency. It is very common in Brazil that enterprises finance their projects with credit lines that have terms and characteristics incompatible with their cashflows, carrying, this way, to the intensive use of short term resources. So, they fall in major financial costs and, in consequence, not only their liquidity but also their return are affected. One should note that this variable is not being used here in its absolute form; instead it is computed in relation to the index of the sector the enterprise is located in.

OI / LS – X2 - When positive, OI refers to the short term liquid investment. In a static situation, it represents those resources necessary to keep the enterprise's actual level of operational activity. Most of the time, those resources are obtained by means of financing from onerous sources with equivalent or even longer terms. One can consider that this element keeps proportionality to the financial cycle and to the volume of sales. The sample's result can show great difficulties for insolvent enterprises to finance their operations by means of natural sources linked to their activities.

T / LS – X3 - This indicator is the essence of a brazilian solvency model [4] and describes, from a more dynamical point of view than the traditional analysis of balance budget, the financial situation of an enterprise. The treasury balance signalizes the enterprise's financial policy. If it is positive, this represents availability of resources that warrants the enterprise short term liquidity. However, when it is negative, it is important to set up a relationship with the sales level because the index expressivity can show imminent financial difficulties, specially when negative treasury balances are kept in successive periods of time and/or they are crescent. Sample data clearly reflect major difficulties for insolvent enterprises in get operational financing, this way using erratic sources.

Stores / SGC – X4 **-** Industrial sector have been being object of major changes regarding the productive chain, gathering various logistic methods that invariably conduct to the reduction or elimination of stores. It should nowadays be seen as resources application. From high financial costs in Brazil, this rotating coefficient, when high, influences the rentability and manly the liquidity of an enterprise. A question that requires great analyst's attention is when those enterprises start having very high stores. That can mean either an strategic policy guided by predictive premises regarding the market although with remarkable risk degree or formation of stores that can allow the enterprise to product for a given amount of time, in case it faces an imminent insolvency. When such a situation is identified by the market the regular credit offer is, normally, affected. From the reasons pointed, one can see the coherence of the selection of this index as a discriminant element.

TSI / MMS – X5 - This coefficient attests a liquidity measure. Brazilian confuse and onerous fiscal policy allied to conjectural economic problems is determining the growing of the tributes in relation to the enterprise's invoices, more and more biding their operational margins. This situation have been being an element for a non-book acknowledgment of these obligations by a very expressive number of enterprises. It is also true that the number of fiscal draw ups have been growing, this resulting in a non-spontaneous acknowledgment of these engagements, driving enterprises to parcel these obligations up and, by consequence, making the accounted values in this title to grow even more. So, this indicator gained importance to differentiate healthy enterprises from those with social insurance and/or tributary problems in Brazil.

Each variable was assigned three membership functions namely, *insolvent, undetermined* and *solvent*. Table 3 shows, for each variable, the means that have been chosen for centers of the membership functions and figure 1 draw them.

Variable	Insolvent	Undetermined	Solvent
X1	5.2	3	0.8
X2	-3.5	-1.35	0.8
X3	3.2	1.85	0.5

X4	-2.9	-1.45	0
X5	0.4	0.3	0.2

Table 3 – Center of the membership functions used for the variables

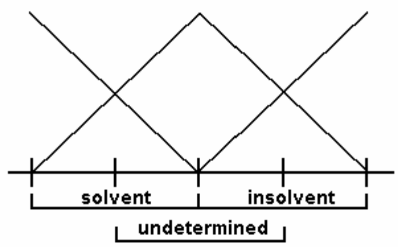

Figure 1 – Partition of unit between membership functions

From a previous analysis regarding the use of some curves as membership functions [8], triangles were chosen for the membership functions because of their facility of use and the partition of unit between neighbor functions was followed. These means come from simulation data of the status of enterprises and follows real data analyzed on a previous work [1]. The model was then tuned by finding the weights a_i that give the desired responses to training data according to equations (16) and (17).

4. Results and conclusion

From a group of 1500 companies, 500 were randomly chosen to tune the model and the remaining 1000 were used to test it in a cross-validation style. From these, 150 were undetermined. Once the model was tuned, it were applied, for each company of the validation group, the values for variables X1-X5 so that crisp classification of companies is found as being the result of equation (16). Table 4 shows the results reached. As we can see from table 4 the results achieved show the model's efficiency is very high. For comparison, the model that used a Z-Score [1] got a whole precision of 98.45%.

Despite the results reached be excellent by themselves, the fuzzy model here developed is much more understandable since it shows how the state of solvency of an enterprise can be seen either from the individual accounting variables or from their combined result. Moreover, linguistic variables show exactly in which group a company belongs to whereas a Z-Score model attributes to variables weights that cannot be easily understood.

Also one can see that the variables chosen for the model are very representative of the health of a brazilian enterprise. None of them, if alone, can be said to be highly representative of the state of solvency of a company but, as we have seen, taken together they can be of great help in analyzing a company's financial health. We then conclude that the model is very usable for everyone that wants to know if a brazilian company is facing future difficulties or not.

	Tuning		**Validation**	
Solvent	300	100%	450	100%
Insolvent	200	100%	400	100%
Total	500	100%	850	100%

Table 4 – Results reached

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Opportunities within The New Basel Capital Accord for assessing banking risk by means of Soft Computing

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Summary. The New Basel Capital Accord (Basel II) is defining regulation for risk assessment and management in the financial industry of next years. In last years, soft computing provided a significant effort into developing innovative risk assessment models. However, validation of these models is still at beginning, and they have been adopted only partially by industry. The New Accord means to foster financial institution to develop internal risk assessment models. Although, industry is inclined to adopt econometric models as VaR, soft computing can play a relevant role. To be effective, research effort should work in the direction of upcoming new regulation. This paper means to provide a reading of the third Basel II Consultive Paper in order to highlight constraints and possibilities for developing bank risk models based on soft computing approaches.

1 Introduction

In April 2003, the Basel Committee on Banking Supervision released its third Consultative Paper (CP3) on the New Basel Capital Accord (commonly known as Basel II) [1]. Basel II means to foster financial and banking institutions to better assess and manage risk. Differently by the current Capital Basel Accord (Basel I) the risk is differentiated according to its different nature. The New Accord considers three sources of risk: *credit risk*, *market risk* and *operational risk*. Moreover the New Accord allows an estimation of risk by means of different levels of sophistication: several approaches can be adopted by banks according to their ability in measuring risk, so that internal risk assessment models can be adopted in place of standard approaches. Indeed the New Accord wishes to promote a higher sensibility of financial institutions for risk management and mitigation based on their own organization and background.

Risk analysis is usually addressed by means of statistical tools. Although the New Accord provides strict guidelines in risk assessment, often it does not prescribe any particular risk model: it is open to the experimentation of innovative techniques. This is particularly relevant in the area of operational risk assessment, due to the fact that reliable models do not exist at the moment. Soft computing techniques can be applied in the assessment of financial risk, in condition that they respect Basel II requirements. This paper is a commented review of the current proposal for the New Accord, looking for which opportunities of soft computing application are reasonable in the perspective of the New Accord.

The New Accord is structured in three sections (pillars): Capital Requirements, Regulatory Review Process, Disclosure & Market Discipline. In particular Capital Requirements is devoted to calculate capital charge based on credit, market and operational risk. The New Accord does not prescribe one way to determine capital charges, but it provides many options on approach to calculation.

2 Risk nature

Pillar 1 considers capital charges according to different sources of risks: credit risk, market risk and operational risk. The minimum capital requirements (MCR) are computed as (par.22)

$$MCR = \frac{TotalCapital}{CreditRisk + 12.5\,(OpRisk + MarketRisk)} \tag{1}$$

The MCR ratio cannot be less than 8%. For each 100 euros of capital exposure, a bank should reserve between 38 euro-cents and 42 euros as capital charge for risk coverage. Therefore banks and financial institutions are motivated to develop better risk assessment models to reduce capital charge. In the next section, the papers intends to review CP3 in order to highlights opportunities for the development of industry soft computing models within the upcoming regulation.

2.1 Credit Risk

The credit risk is due to uncertainty in a counterparty's ability to meet its obligations. The New Accord considers two possible options for measuring the credit risk. The first option is known as Standard Approach (STD). The STD Approach requires banks classify their credit exposures. Rating will be applied to claims on sovereigns, non-central government public sector entities (PSEs), multilateral development banks (MDBs), banks, securities firms, corporates, retail. Rating is provided by External Credit Assessment Institutions (ECAI) such as Standard & Poor's. Each class has an associated risk weight. Risk weights lead to capital charge. For instance if a bank has an exposure of 1Ml euros with 50% risk weight, this lead to a *risk weighted asset* (RWA) for 500.000 Euros. The New Accord requires a 8% capital charge on RWAs. In our example, the capital charge for that exposure would be 40.000 Euros. An example of Corporate rating is reported in Tab.1.

Table 1. Standard's & Poor's Corporate Rating

Rating	Risk Weight
AAA to AA-	20%
A+ to A-	50%
BBB+ to BB-	100%
Below BB-	150%
Unrated	100%

ECAI methodology must be rigorous, systematic, and subject to validation. Moreover an ECAI should disclose methodologies, time horizon, meaning or rating, default rates and transitions matrix.

We could think to adopt fuzzy classification in order to provide a smoother rating, according to which criteria are satisfied, and which not. Although the idea of adopting soft computing techniques, fuzzy classification in particular, appears appealing by a scientific perspective, the idea that ECAI can follow this way does not appear realistic within mid-term and at the present consultive document.

The other option for assessing the credit risk is based on Internal Rating-Based (IRB) Approach. In particular Basel II considers two IRB variants: Foundation and Advanced. This approach allows banks to use their own internal risk assessment models in order to derive RWAs, although they must respect qualitative and disclosure requirements (parr.349-500). The Accord defines a categorization of assets in corporate, sovereign, bank, retail, and equity. The main discussion regarded corporate rating. In particular discussion interested SME based economies such as in Italy [27] and Germany [28]. For each assessment class, the New Accord requires to estimate four parameters: probability of default (PD), loss given default (LGD), exposure of default (EAD) and, in some cases, effective maturity (M) (par.180).

Each of this parameter must be related to the *borrower grade* and to transaction specific factors (par.358-362). A borrower grade is a classification based on the assessment of borrower specific factors (par.367). The New Accord requires a minimum of seven grades in order to avoid an over population in some classes. The New Accord requires to assign all exposures related to one borrower to the same grade except special cases (par.359). Therefore there is no space for fuzzy classification techniques. However the Accord does not prescribe any particular method or model in order to grading borrowers, and it recognizes the central role of human judgment (par.379). In particular the Accord states that models should be only used as a supportive source of information to human decision making processes. Banks must prove that adopted models are logical and good predictors of risk and business procedures should be in place in order to correct model's errors and improve its performance. Therefore, adaptive models such as Artificial Neural Networks are applicable when their application respects requirements prescribed by the Accord. However, since they are black box models, their fitness could be proven only with large data set on statistical basis, but the evidence of their logic assumptions would be harder. Differently, causal models could be adopted in order to facilitate their verification by Supervisors. Thus, bayesian belief networks and fuzzy rule based models are good soft computing candidates to play a role in credit scoring. These models could be outlined by means of ANN in early stage of rating system design. However, the final model should be refined and maintained by experts.

For each borrower grade, banks have to provide an internal estimate for the associated probability of default. PD estimation can be based on techniques that mainly stochastic but not necessary (par.423). LGD, is expressed as percentage of exposure and can be determined in two ways (par.255). If the bank adopted the foundation approach, standard LGD are given within the New Accord(parr.256-7). Banks that adopted the advanced approach, can decide to provide internal estimates of LGD (parr.430-5). In that case, LGD must be estimated as a long-run average LGD for each facility. Maturity (M) is a factor that reduce the credit risk (par.288). The bank adopting the advanced IRB approach must provide an internal maturity assessment for each facility (par.289).

Financial institutions adopted several stochastic approaches mainly based on Structural approach [8, 23, 7, 19] or on the Reduced Form approach [13, 15, 16, 20]. The role of soft computing in this area is still at beginning. Early experimentation regarded artificial neural networks (ANN) for bankruptcy prediction (see [5] for a review). Although results of ANN application are encouraging, current models are still not able to provide an estimation for PD, that is essential in the Basel II context. Another promising approach is application of Bayesian Belief Networks (BBN) to credit scoring [2], for their ability in modelling causal dependencies. Datamining techniques [21] are able to derive data-driven risk assessment models, but their reliability is still not proven in PD estimation. Results in fuzzy and approximate reasoning are still at early stages [6, 14].

Although to a lesser extent, other possibilities for soft computing techniques are in the area of Credit Risk Mitigation (CRM). Banks use different techniques to mitigate their credit risk exposures. The New Accord extends the set of of credit risk mitigants compared to the present accord. In particular a transaction can be collateralized. In this case, the New Accord includes two methods to calculate the the exposure amount after risk mitigation.

$$E^* = \max\left(0, [E \cdot (1 + H_e) - C \cdot (1 - H_c - H_{fx})]\right) \quad (2)$$

where

- E is the exposure value before risk mitigation;
- H_e is the haircut appropriate to the exposure;
- C is the current value of the collateral received;
- H_c is the haircut appropriate to the collateral;
- H_{fx} is the haircut appropriate for currency mismatch between the collateral and exposure.

Supervisors may permit banks to calculate H using their own internal estimates of market price volatility and foreign exchange volatility. Soft computing models could be adopted in haircut calculus.

2.2 Market Risk

Market risk is linked to possible losses in on and off-balance-sheet positions arising from undesirable market movements. The current Capital Accord requires to estimate the market risk by means of the 99%-quantile of the distribution of a portfolio's loss over a certain time horizon. The use of VaR models has became an industry standard. Probability distributions are usually computed by three approaches: variance-covariance approach; Monte-Carlo simulation; historical simulation. However VaR has some limitations [17] as:

- market prices do not have a properly normal distribution (fat tails)
- it is not able to capture extreme events
- might discourage diversification
- it is not coherent (sub-additive).

A risk measure ρ on a random payoff X is *coherent* if it satisfies the following four axioms [3, 4]:

1. Translation invariance: $\rho(X + \alpha) = \rho(X) - \alpha$ for all $X \in \mathcal{G}$ and $\alpha \in \mathbb{R}$.
2. Subaddativity: $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$ for all $X_1, X_2 \in \mathcal{G}$.
3. Monotonicity: $\rho(X_1) \leq \rho(X_2)$ if $X_1 \geq X_2$ a.s.
4. Positive Homogeneity: $\rho(\lambda X) = \lambda\rho(X)$ for any small $\lambda \geq 0$

The debate if VaR approach should be adopted in the New Accord as the regulatory measure of market risk is going on [17].

The adoption of bayesian and neural networks have been investigated for calculating VaR by unconditional mixture densities [31, 29], in order to model the density conditionally [18]. This is in line with current Capital Accord and CP3 document (parr.642-676). In some cases they are adopted to estimate volatility [25] or to make directly asset portfolio allocation with optimal VaR

[11]. Other techniques involve the use of evolutionary computing [24]. At the moment, these methods are not allowed as regulatory or for capital charge determination, and they can only be used as internal market risk benchmarks.

2.3 Operational Risk

The New Accord defines the operational risk (OR) *as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events.* (par.607) In more details, the Accord includes three ways for OR assessment: the Basic Indicator Approach; the Standardised Approach; and Advanced Measurement Approaches (AMA). (par.608) The Basic Indicator Approach compute the OR capital charge as

$$K_{BIA} = GI \cdot \alpha \tag{3}$$

where K_{BIA} is the average annual gross income over the previous three years, and α is 15% at the moment. (par.612) The Standardised Approach makes a separation of banks' activities into eight business lines: corporate finance, trading & sales, retail banking, commercial banking, payment & settlement, agency services, asset management, and retail brokerage. (par.615) The capital charge is computed as

$$K_{TSA} = \sum_{i=1}^{8} GI_i \cdot \beta_i \tag{4}$$

where GI_i is the average of an exposure indicator over the past three years for each business line, and β_i is a fixed percentage varying between 12% and 18% depending on the severity of operational risk associated to each line. (par.619).

Both methods are expensive in terms of capital charge. This should lead banks to adopt Advanced Measurement Approaches in order to better assess their operational risk exposure and to reduce the required capital charge. The New Accord does not prescribe any particular analytical model. (par.627). However, it is reasonable to consider the OR capital charge as

$$K_{AMA} = \sum_i f_i(AV_i, PE_i, LGE_i) \tag{5}$$

due to the bank's different business lines. For each business line, f_i is a model suitable for that line, deriving from line assets values AV_i, probability of OR events PE_i, and losses given events LGE_i. A simple model could be

$$f_i(AV_i, PE_i, LGE_i) = \sum_j AV_{j|i} \cdot \left(\sum_k PE_{k|i} \cdot LGE_{k,j|i} \right) \tag{6}$$

Although, we can think to extend quantitative approaches adopted for market and credit risk as VaR or actuarial methods as Extreme Value Theory

(EVT) aiming to derive capital charges, there are some several limitations to take into account. The main limitation is about availability of data, on which to build reliable risk models. The main concern is about high impact low frequency (HILF) events, for which it is hard to build functional OR models. It appears necessary to build models that integrate data with expert knowledge, in building causal models. Causal models allow analysis in "what-if" fashion. There several approaches coming from the soft computing [22]

In a 2000 PinceWaterhouseCooper survey among 55 financial institutions, banks consider self assessment as the most significant tool to asses operational risk (45%), with 70% of them that already have already started to use it, and 22% that have planned it. In this context fuzzy set theory can play a relevant role providing a powerful way to model vagueness [12, 30]. In particular, as part of the AMA, financial institutions may use scorecards as a means to adjust qualitatively operational risk capital initially determined on a quantitative basis [26]. Scorecards combine both qualitative and quantitative measure: they can adopt soft computing models such as fuzzy approximate reasoning or belief networks for merging this information. This could be possible by the adoption of proper *judgment functions* [10]. Finally another interesting area of investigation regards the use of aggregation operators [9], in order to formulate coherent measures of risk.

3 Conclusions

The New Basel Accord is promising to better structure the nature of risk and its assessment than what the current Capital Accord do. In particular, the New Accord would foster financial institution to develop internal risk assessment models. The different nature of risk require ad-hoc assessment models. If credit risk and market risk assessment model can adopt econometric approaches, operational risk requires to be treated apart. Indeed, HILF events cannot be easily modeled by means of traditional stochastic methods. Soft Computing techniques can provide a remarkable contribute for assessing each source of risk, and in particular the operational risk. However, the development of soft computing approaches must consider the regulatory scenario that is coming from Basel Committee. Without a careful review of regulation, soft computing techniques take a risk of being regarded as exotic by the financial industry world dominated by econometric models.

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Risk Analysis in Granular-Information-Based Decision aid Models

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1 Introduction

We live in the world where uncertainty is inherent in the vast majority of decisions that we have to make. It is no surprise that uncertainty prevails decisions, which are made in almost every field of human activity, from deciding whether to take an umbrella or not when going outside to managerial and political decisions. This requires the development of more robust decision models. There are several analytical approaches to deal with uncertainty. This paper proposes to use granular bodies of evidence to model uncertainty of our world. One of the advantages of granular information is that it incorporates fuzzy information and probabilities. Moreover, these paradigms enable one to use natural language to describe the problem, which facilitates modelling of the decision. However, generally fuzzy logic and probability theory are not used together to deal with uncertainty.

We show some of the advantages of using two of these paradigms together and propose a methodology for risk analysis based on this representation.

Generally speaking, uncertain events can be considered as opportunities or risks, depending of whether they turn out to be favourable or not. The main tool for dealing with uncertainties is risk analysis. In the following sections we consider our approach in detail. We assume that the reader is well acquainted with the domain of fuzzy logic and the notion of entropy, so we skip the explanation of these topics, but you can refer to [3, 4 and 5]. In section 2 we present a short overview of fuzzy granular information. Section 3 considers the possibility of using fuzzy granules and entropy in risk assessment. Moreover, in this section we suggest a generalized definition of entropy and we present its analysis. In section 4 concluding remarks are given.

2 Fuzzy Granular Information

The idea of information granularity and its application in the context of fuzzy logic is presented in [6]. The idea of information granularity is very close to that of fuzzy information. Information is fuzzy granular in the sense that, (a) the perceived information is fuzzy and, (b) the perceived information is granular with a granule being a clump of values drawn together by similarity or indistinguishability.

In this paper conditioned π-granules are considered, which are characterised by propositions of the following form:

$$g_i \overset{\Delta}{=} \text{If } X = u_i \text{ then } Y \text{ is } G_i ,$$

where G_i and u_i are fuzzy values and X, Y are linguistic variables. Evidence E can be regarded as a collection of such granules:

$$E = \{g_1, \ldots, g_N\}.$$

Variable X assumes its value with a specified probability. Thus, evidence is probability distribution P_X of conditional π- granules. Moreover, each granule can be regarded as conditional possibility $\Pi_{(Y|X=u_i)} = G_i$. Hence, evidence can be regarded as conditional possibility distribution $\Pi_{(Y|X)}$. Thus, evidence can be considered as the following construction:

$$E = \{P_X, \Pi_{(Y|X)}\}.$$

Given a collection of bodies of evidence $E = \{E_1, \ldots, E_K\}$, one can ask questions about the information contained in these evidences. The main question is: "what is the probability of (Y is Q)?", where Q is a fuzzy subset. The probability of such an event is an *interval value* in which the expected possibility $\mathrm{E\Pi}(Q)$ is regarded as the upper boundary, and the expected certainty $\mathrm{EC}(Q)$ is regarded as the lower boundary of the probability sought. In [6] the following formulae are proposed:

$$\mathrm{E\Pi}(Q) = \sum_{i,j} p_{ij} \sup\left(Q \cap G_i \cap H_j\right),$$

$$\mathrm{EC}(Q) = 1 - \mathrm{E\Pi}(Q'),$$

where Q' is a complement of set Q. These formulae are for a special case in which there are two evidences in the system.

In paper [7] you can find description of an adaptive network, which can be used to calculate the upper and the lower boundaries of the probability.

3 Uncertainty and Granular-Information-Based Risk Assessment

As we have learnt from previous sections, granular information can be represented with the help of conditional granules of the "IF…THEN…" form. In order to adapt this methodology to risk assessment, we can describe our problem domain with the help of such granules putting in the consequence ("THEN") part evaluation of some criterion, which is important for us and which is in direct or indirect relation to the risk inherent in our problem domain.

For example, if we are considering a project of constructing a power plant, we ought to consider to what extent it will influence the environment and the locality. Thus, we can describe our problem with a set of bodies of evidence made up of the following granules:

Evidence I, for description of distance to the closest inhabited area:

IF (plant is pretty close to inhabited area) with probability $p_{1,1}$
THEN influence is medium
IF (plant is close to inhabited area) with probability $p_{1,2}$
THEN influence is moderately high
IF (plant is very close to inhabited area) with probability $p_{1,3}$
THEN influence is high

Evidence II, for description of forecasted air pollution:

IF (air pollution is medium) with probability $p_{2,1}$
THEN influence is moderately high
IF (air pollution is not high) with probability $p_{2,2}$
THEN influence is low

Evidence III, for description of forecasted water pollution:

IF (water pollution is very low) with probability $p_{3,1}$
THEN influence is low
IF (water pollution is low) with probability $p_{3,2}$
THEN influence is medium
IF (water pollution is medium) with probability $p_{3,3}$
THEN influence is high

Probabilities $p_{i,j}$ are subjective evaluations based on decision maker's experience or expert's opinion. In this setting these are crisp values, but in a general case they can be fuzzy. Formulae presented in Chapter 2 are used to aggregate these rules. As will be seen later, the result of aggregation is an interval-valued probability.

We can continue describing parameters that can influence the evaluation of the chosen criterion and, as can be seen, we can incorporate our confidence in the values of these parameters, e.g. with the help of subjective probabilities. Moreover, we can construct other sets of bodies of evidence, which describe evaluation of other criteria.

After we define bodies of evidence, we can calculate the probability that influence on the environment will be high or medium. Obtained probability will have an interval value $[p_{min}, p_{max}]$, which defines the lower and the upper bound of the probability.

We can calculate what are the probabilities that the criterion takes different values, e.g. "high", "low" etc. Moreover, we can consider different criteria.

In the following section we discuss how we can assess the uncertainty related to the probabilities obtained and how it can be related to risk.

3.1 Entropy and Interval-Valued Probabilities

In this section we show how a generalised definition of entropy can be obtained, which is suitable for interval-valued probabilities. Obviously, the entropy itself will be interval-valued. As before, we assume that system can be in n states, but the probability that the system is in i-th state is interval and is equal to $[p_i^{min}, p_i^{max}]$. In case when probabilities are single-valued rather than interval-valued, it is required that probabilities sum to 1:

$$\sum_i p_i = 1. \tag{3}$$

If probabilities are interval valued, then (3) can be rewritten as (4):

$$\sum_i p_i^{\min} \le 1 \le \sum_i p_i^{\max} . \tag{4}$$

It is easy to show that (3) is a special case for (4) when $p_i^{min} = p_i^{max}$ for each i. Moreover, if we define p_i^{avg} as (5) then it can be shown that *heuristics* (6) hold.

$$p_i^{avg} = \frac{p_i^{\min} + p_i^{\max}}{2}, \tag{5}$$

$$\forall i : p_i^{\min} \le p_i^{\max} \Rightarrow -p_i^{avg} \log p_i^{\min} \ge -p_i^{avg} \log p_i^{\max} . \tag{6}$$

It should be noted that states with lower probability values are more informative. Thus, we can expect that in order to calculate the upper boundary of entropy $H^{\max}$ we should use the lower probability bounds p_i^{min}. We can find the upper boundary of entropy for a system as follows:

$$H^{\max} = -\sum_{i=1}^{n} p_i^{avg} \log p_i^{\min} , \tag{7}$$

and the lower boundary of entropy:

$$H^{\min} = -\sum_{i=1}^{n} p_i^{avg} \log p_i^{\max} , \tag{8}$$

From (6) it follows that $H^{\min} \le H^{\max}$. Moreover, as was mentioned above, entropy for a system with interval-valued probability is interval-valued as well and is equal to (9).

$$H = [H^{\min}, H^{\max}] . \tag{9}$$

The obtained definition of interval-valued entropy is a heuristic generalisation of the 'traditional' entropy of a system with single-valued probabilities. However, it should be noted that it is a heuristic approach towards generalization of entropy, which gives an *approximate* result. An in-depth study of the entropy generalization will be described in upcoming papers, as it is the topic of our current research. Some of the advances are reported in [8]. In the following section we examine whether the additivity feature holds for the generalized heuristic version of entropy defined by (7), (8).

3.2 What About Additivity?

If additivity holds, it means that if we have two independent systems, say, X and Y, then entropy of a system that is obtained by joining systems X and Y is equal to sum of individual entropies for X and Y. In other words, if additivity holds, then

$$H(X, Y) = H(X) + X(Y). \tag{10}$$

If we define entropy as (7) and (8), then it can be shown that if we have two systems X and Y with states accordingly $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$, moreover, $P(X \sim x_i) = p_i$ and $P(Y \sim y_j) = r_j$, then

$$H^{\min}(X,Y) = -\left(\sum_{i=0}^{n} p_i^{q.avg} \log p_i^{\max} + \sum_{j=0}^{m} r_j^{q.avg} \log r_j^{\max}\right), \approx H^{\min}(X) + H^{\min}(Y) \tag{11}$$

$$H^{\max}(X,Y) = -\left(\sum_{i=0}^{n} p_i^{q.avg} \log p_i^{\min} + \sum_{j=0}^{m} r_j^{q.avg} \log r_j^{\min}\right), \approx H^{\max}(X) + H^{\max}(Y) \tag{12}$$

where $p_i^{q.avg} = \frac{\left(p_i^{\min} \sum_{j=0}^{m} r_j^{\min} + p_i^{\max} \sum_{j=0}^{m} r_j^{\max}\right)}{2}$, $r_j^{q.avg} = \frac{\left(r_j^{\min} \sum_{i=0}^{n} p_i^{\min} + r_j^{\max} \sum_{i=0}^{n} p_i^{\max}\right)}{2}$. (13)

As can be seen, (13) is very similar to (5), except for the summation factors (14) that appear in (13). If we are dealing with single-valued probabilities then it is obvious that these factors are equal to *one* and the additivity feature holds. If the probabilities are interval-valued then from (4) it follows that formulae (14) hold.

$$0 \le \sum_{j=0}^{m} r_j^{\min} \le 1,\ \sum_{j=0}^{m} r_j^{\max} \ge 1,\ 0 \le \sum_{i=0}^{n} p_i^{\min} \le 1,\ \sum_{i=0}^{n} p_i^{\max} \ge 1. \tag{14}$$

Summation factors (14) can be considered as a sort of scaling factors, where the first may have reducible influence and the second may have augmenting influence, so we may expect that two of these factors compensate each other to some extent. Hence, entropy for the joined system calculated according to (11) and (12) *should not* differ much from the sum of individual entropies of the systems considered. Thus, it can be stated that we have *quasi-additivity*, as the summation factors compensate each other to some extent.

3.3 Incorporating Entropy and Risk Assessments

In the previous sections we described how we could use granular information to describe problem domain. After we construct fuzzy bodies of evidence, we can use them to calculate probability that criteria will take different values. Entropy values for systems corresponding to different criteria should be calculated separately and then summed according to the additive property of entropy. Obtained entropy is interval-valued and we can use it to measure uncertainty of our problem domain and of evaluations of the criteria chosen.

In general terms this approach can be described as follows. First we have to define what criteria are of interest for us. After that for each alternative we construct fuzzy granules that describe each criterion and evaluate the probabilities that a particular criterion will take some value. The probabilities are interval-valued. A particular value of some criterion corresponds to a separate state in a system. After having evaluated probabilities of values for all criteria we can calculate entropy for each system. In order to get evaluation of overall entropy, we can sum entropies calculated for separate systems. Now it is clear why the additivity feature is so important. If it would not hold, we could not just sum up individual evaluations in order to get overall evaluation.

The overall entropy value obtained can be considered as evaluation of *uncertainty* for a particular alternative and, thus, as a measure of *risk*. The evaluations obtained are interval, so it may be hard to compare them, as the intervals may overlap or include each other. However, since the proposed risk value represents the amount of information shortage, a high risk value may indicate that there is too little information available about an alternative. In such a case it would be advisable for decision maker to revise the decision model and to supplement it with additional data, if it is possible.

You can find more detailed explanation of this approach and a simple case study in [2].

4 Conclusion

This paper shows how fuzzy granular information can be used in order to measure risk and uncertainty. The uncertainty assessment is based on the generalised definition of entropy.

One of the advantages of the approach proposed in this paper is that one can use elements of natural language to describe problem domain, upon which the uncertainty is assessed. This is due to fuzzy logic upon which the approach is based, which enables one to use fuzzy rather than crisp values.

Moreover, we show how entropy can be generalized to the case of interval-valued probabilities and we analyse the new definition to see whether the additivity holds. We conclude that quasi-additivity holds for the generalized entropy.

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A new approach to optimizing portfolio funding in an fuzzy environment

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Abstract

In this article, the authors continue to develop their approach to optimizing portfolio funding in fuzzy environment. Based on [6], they suggest a new approach to solving portfolio optimization problems.

The approach attempts to ascertain the amount of risk that the portfolio will yield less than was expected. Based on this value, the authors present a method of maximizing the minimum expected portfolio return. Expected portfolio return and respective asset risk levels, represented as fuzzy values, are the suggested input parameters for the model. Analysis is conducted through non-linear optimization methods, particularly the gradient method, which is used to solve sample problems.

The article provides an innovative approach to optimizing (mutual) fund portfolios despite uncertain input parameters (a sample case is discussed).

1. The essence of approach

We consider fund portfolio which consists of N components and its expected return on [0, T]. Each of the components is characterized by its final revenue r_i (estimated in T as relative increment of an asset during the period), when r_i is **fuzzy value**.

Let's consider the model from [8] where final revenue is treated as random process with linear trend, at cross section of which a normally distributed random value is situated. Parameters are triangle fuzzy values. Thus each asset can be estimated using two triangle values – average expected return and risk. Also there's a correlation triangle matrix which defines correlation between assets.

During assets behavior simulation with gauss distribution, we define asset by two moments of this distribution. Thus, we can solve Markovitz's problem in fuzzy target setting. Then effective frontier of portfolio set is curved line represented in coordinates of basis “risk – revenue”, and optimal portfolio in each point has fuzzy frontiers. This idea is at the basis of model portfolio optimization [3,7].

There are some problems which restrain use of approach [8]

1. We succeed in overcoming the Markovitz approach limitations, connected with non-Wiener character of price processes; however we couldn’t overcome symmetric portfolio risk measure. This is especially important for investor because the risk of losses is much more difficult to stand with that with the risk of gains. Such asymmetric requires us to use downside risk measure, which is impossible in Markovitz approach.
2. It was mentioned in [4] that correlation is a thing that doesn’t apply to market. We can prove it changing the time period of correlation matrix estimation in wide ranges and we would see that correlation ratios would change the modulus values also in wide ranges. At the same time in articles [1, 2] we have shown that influence of correlation ratios on final decision is one degree lower than influence of risk revenue parameters. In addition we know that all financial markets are changing simultaneously (because of increasing communications), thus correlation of leading indices is almost always positive. Finally we can conclude that correlation is a) strained interpretation b) non-informative elements of the model, so using the correlation value doesn’t make sense in this model.

Above described problems brings to life an approach as implemented in [4] where:

1. Portfolio risk is an opportunity that expected portfolio revenue would be lower than fixed value.
2. Assets correlation in portfolio isn’t considered.
3. The revenue of each asset is determined by (non-random) fuzzy value. Similarly, the limitation to minimum revenue can be estimated as scalar or fuzzy value. Thus we put two sources of information (average return and asset volatility) in one.

Optimizing portfolio in such conditions is the same to solve problem of **maximizing minimum of expected portfolio return** in T while risk level is fixed (similarly as it applied in [4] and [9]). Effective frontier of return in such case is curved line in basis of “unbearable low level of return – minimum level of expected return”. Optimal portfolio with non-fuzzy frontiers is corresponds to each point on the such effective frontier. So far the fuzziness of the model is in unbearable low level of return.

Let's consider solving the problem on the basis of described model, using assumptions of the widest tolerances to the fuzzy parameters of the model.

2. Possible optimization of the portfolio.

So far we decide to develop approach described in [4] but exclude all probability descriptions from the model.

Assume that there's fund portfolio which consists from N components on [0, T] interval. Forecast performance of each component i = 1..N in time point T is characterized by its **final revenue** r_i (estimated in T as relative increment of price asset for period) where r_i is **fuzzy value**. This fuzzy value we can determine either as an analytical function or as sequence of intervals like $[r_{imin}(\alpha), r_{imax}(\alpha)]$ for each attribute level. Further we consider the second way. Sometimes final revenue may be non-fuzzy value (for instance for bonds).

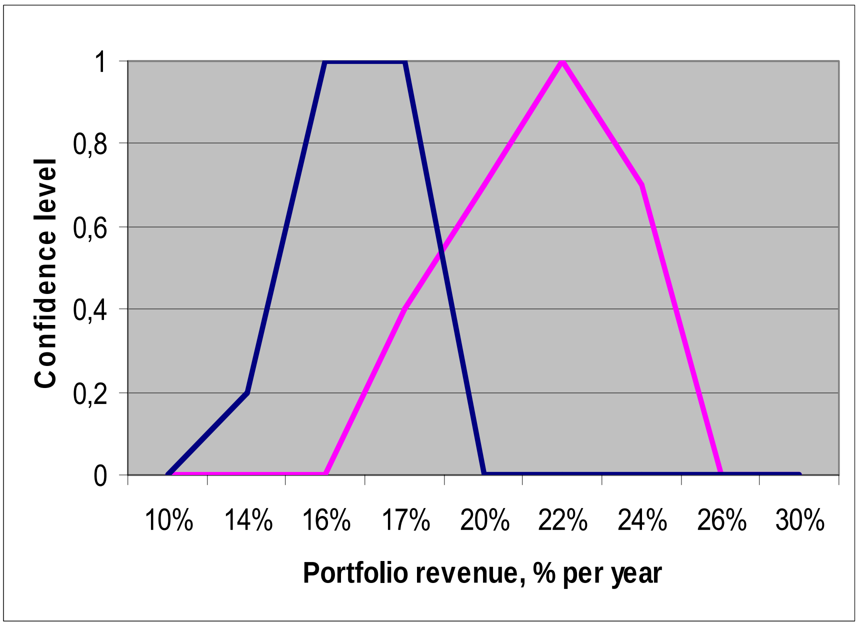
If we know estimated return of assets, we can conclude that:

$$[r_{min}(\alpha), r_{max}(\alpha)] = \sum_i x_i \otimes [r_{i\min}(\alpha), r_{i\max}(\alpha)] = [\sum_i x_i \otimes r_{i\min}(\alpha), \sum_i x_i \otimes r_{i\max}(\tag{1}$$

for $\forall\ \alpha \in [0,1]$, where x_i – is weight of i assets in portfolio. Also

$$\sum_{i=1}^{N} x_i = 1,\ 0 \le x_i \le 1. \tag{2}$$

Let's determine the lower boundary of portfolio on T as fuzzy value which determined on $[r_{Pmin}(\alpha), r_{Pmax}(\alpha)]$ interval. Let's consider an example with inflation. It's obviously that investors desire to have portfolio revenue above that inflation rate. The picture below describes the relationship between portfolio revenue and inflation level.



Corresponding risk is equal accordance [10] to:

$$\text{Risk} = \text{Poss}\ \{r < r_P\} \approx \sum_{(\alpha)} \frac{S_\alpha \Delta\alpha}{(R_{2\alpha} - R_{1\alpha})(G_{2\alpha} - G_{1\alpha})}, \tag{3}$$

where $\Delta\alpha$ - discretization level by belongings α (for instance, 0.01), $[R_{1\alpha}, R_{2\alpha}] = [r_{min}(\alpha), r_{max}(\alpha)]$, $[G_{1\alpha}, G_{2\alpha}] = [r_{Pmin}(\alpha), r_{Pmax}(\alpha)]$), and

$$S_\alpha = \begin{cases} 0, G_{2\alpha} \le R_{1\alpha} \\ \dfrac{(G_{2\alpha} - R_{1\alpha})^2}{2}, \qquad G_{1\alpha} < R_{1\alpha} < G_{2\alpha} \le R_{2\alpha} \\ \dfrac{(G_{1\alpha} - R_{1\alpha}) + (G_{2\alpha} - R_{1\alpha})}{2} \cdot (G_{2\alpha} - G_{1\alpha}), R_{1\alpha} \le G_{1\alpha} < G_{2\alpha} \le R_{2\alpha} \\ \dfrac{(G_{2\alpha} - R_{2\alpha}) + (G_{2\alpha} - R_{1\alpha})}{2} \cdot (R_{2\alpha} - R_{1\alpha}), G_{1\alpha} \le R_{1\alpha} < R_{2\alpha} \le G_{2\alpha} \\ (G_{2\alpha} - G_{1\alpha})(R_{2\alpha} - R_{1\alpha}) - \dfrac{(R_{2\alpha} - G_{1\alpha})^2}{2}, R_{1\alpha} \le G_{1\alpha} \le R_{2\alpha} \le G_{2\alpha} \\ (G_{2\alpha} - G_{1\alpha})(R_{2\alpha} - R_{1\alpha}), R_{2\alpha} \le G_{1\alpha} \end{cases} \tag{4}$$

Thus we can define an expected portfolio revenue for each correlation of portfolio portions $\{x_i\}$ as fuzzy value in (1) – portfolio risk and as a common scalar value in (3) – (4). Now we can proceed to setting and solving the portfolio optimization problem.

Let's assume that optimal portfolio which has under given unbearable risk level (3) the minimal value of revenue is maximal. Using formula it means

$$r_{min}\ (\alpha=0) \ \rightarrow \max \qquad (5)$$

under constraints on the portfolio portions size (2) and the type of equality on size Risk = $Risk_0$.

As there's no common analytical type for dependence Risk (3), there's no common solving of optimization problem (5). This problem can be solved using only approximate approaches, for example **gradient approach**, which is described by the following algorithm.

1. We determine an efficient portfolio quantity frontier in coordinates (risk of portfolio inefficiency – minimum portfolio revenue). Let mark max $\{r_{min}\ (\alpha=0)\}$ = Eff (от efficient frontier). Then equation for efficient frontier is

$$Eff = Eff\,(Risk_0) = Eff\,(Risk). \qquad (6)$$

2. The right point of frontier (6) is defined by asset with maximum $r_{imin}(\alpha=0)$. Let's give this asset the number 1. Then in appropriate optimal portfolio

$$x_I = 1,\ x_i = 0,\ i \neq I$$

Abscissa of the right point of frontier (6) is defined by (3-4), let's mark it Risk2. and mark the ordinate of right point of frontier as Eff2 = Eff(Risk2) = $r_{Imin}\ (\alpha=0)$

3. Let's control degeneracy of frontier. If the left edge of frontier coincides with right, it degrades to a point, and the algorithm stops. Control is realized in the following way. For each i asset, $i \neq I$, formed a relatively small increment of portion Δx_i,, followed with appropriate lowering of portion x_i, and appropriate changes of levels ΔEff and ΔRisk are measured. Let's name a ratio ΔEff/ΔRisk a **gradient** (it's approximation of first derivative of efficient frontier). Then, if the gradient is negative for each I-asset (i.e. decrease of value is followed with risk increase), construction of efficient frontier is finished and algorithm stops.

Generally speaking, each point of efficient frontier has a **minimum of gradient**, and it determines **concavity** of efficient frontier in each point. If it weren't so, a portfolio situated above efficient frontier would exist, which contradicts the definition of efficient frontier.

4. If there's no degradation of efficient frontier (it doesn't shrink in one point), algorithm of gradient optimization begins to work. A couple of assets (i, j) are chosen for which a positive gradient has **minimum** value when a small portion is taken from asset-i and transferred to asset-j. A new point with coordinates (Riss, Eff) is appeared on the efficient frontier, which corresponds to portfolio with optimally re-distributed portions. And then algorithm turns back to the step 3 with control of completing process of restoring the efficient frontier (until all possible gradients are negative).

Gradient method described here is at the basis of decision [3, 7]. The main advantage of this algorithm is its adaptability to each coordinate net (risk-value), including the classical net of Markovitz. That is, the type of risk and the way of its calculation for gradient algorithm doesn't matter.

Let's take up the simplest calculating example

3. Calculating example

The fund portfolio contains N = 4 assets, initial datum on which are presented in table 1. All yields are formulated as interval estimation and, accordingly, are rectangular fuzzy values (interval estimation doesn't depend on implement level). The restriction on the lowest limit of asset yield – r_P = 9.5%..11% annual is given in interval form too. In the last column of table 1 a finish counting of business risk level is made in relation to plan level of yield rp using simplified formula [10], which is apparent from (3-4) for interval case.

$$
\text{Risk} = \begin{cases} 0, G_{2\alpha} \le R_{1\alpha} \\ \dfrac{(G_{2\alpha} - R_{1\alpha})^2}{2(G_{2\alpha} - G_{1\alpha})(R_{2\alpha} - R_{1\alpha})}, \quad G_{1\alpha} < R_{1\alpha} < G_{2\alpha} \le R_{2\alpha} \\ \dfrac{(G_{1\alpha} - R_{1\alpha}) + (G_{2\alpha} - R_{1\alpha})}{2(R_{2\alpha} - R_{1\alpha})}, R_{1\alpha} \le G_{1\alpha} < G_{2\alpha} \le R_{2\alpha} \\ \dfrac{(G_{2\alpha} - R_{2\alpha}) + (G_{2\alpha} - R_{1\alpha})}{2(G_{2\alpha} - G_{1\alpha})}, G_{1\alpha} \le R_{1\alpha} < R_{2\alpha} \le G_{2\alpha} \\ 1 - \dfrac{(R_{2\alpha} - G_{1\alpha})^2}{2(G_{2\alpha} - G_{1\alpha})(R_{2\alpha} - R_{1\alpha})}, R_{1\alpha} \le G_{1\alpha} \le R_{2\alpha} \le G_{2\alpha} \\ 1, R_{2\alpha} \le G_{1\alpha} \end{cases} \tag{8}
$$

Table 1. Initial datum for businesses

Asset №	r_{min}, % year	r_{max}, % year	Risk of an asset by level r_P
1	9	14	0.250
2	8	18	0.225
3	6	25	0.224
4	5	30	0.210

From data of table 1 it can be said that all four assets (and, of course, their random portfolio combinations) have non-zero risk measure. It can be seen also that with increasing asset number a vagueness in part of its expected efficiency increases also. Right there we can accentuate convergence of our approach and Markovitz approach and its derivative. The monotonous grow of volatility and risk by Markovitz (along with asset number) doesn't correspond to risk tendency in our measuring, because in all cases grow of yield run (risk by Markovitz) entails average yield value moving away from frontier values, which gives the risk decreasing.

Iterative algorithm of gradient optimization enables to reconstruct the efficient frontier given on figure 2.

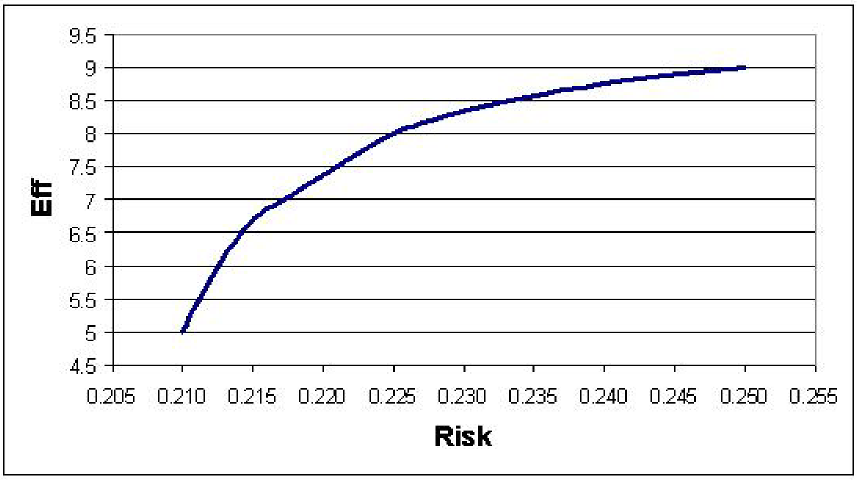


Fig. 2. Efficient frontier

It's clear from figure. 2 that minimal gradient grows when it passes round the efficient frontier from right to left. It can be seen also, that all four assets take part in constructing the efficient frontier (asset 3 is almost imperceptible on the frontier), minimal yields and risks decrease in one consequence, which characterizes this portfolio **as monotonous [5]** (in the work [5] it's shown that all assets of monotonous portfolio take part in constructing the efficient frontier of portfolio quantity).

Conclusion

Quite new results [9, 10] received in 2003, allowed us to compare fuzzy values of random type, estimating the risk that one of them would be more than another in some future moments). At the same time we found that estimated risk can be used as criterial parameter not only in investment projecting, but in optimization of business portfolios of random nature. All this in the sum with result [4] allowed us to come back to the problem of optimization of fund portfolio in determinist-possibilistic statement and to look at the available arsenal of means of optimization in other way.

It begins to seem that compromise achieved when constructing the modified Markovitz method (where yield of asset is fuzzy random value or fuzzy random process), starts to contradict with possible accuracy of forecasting estimations. It's known for a long time that real process of stock price growth isn't wiener. It's also clear that stock market is closely connected with dynamics of macroeconomic parameters and conditions of rational investment [8]. But forecasts made on this basis are very fuzzy and it contradicts with accuracy necessary for describing parameters of fuzzy random value. In addition to everything, a problem of symmetric risk remains which couldn't be solved without complete breach from Markovitz idea.

Thus we have to come away from Markovitz method in fuzzy statement, having changed the paradigm of our investigations. We exclude probabilistic descriptions from assets growth model without any regret, because we don't have any preconditions for constructing such probabilistic formalisms. But we have some experience in forecasting fund indices in which probabilistic problematic disappears by itself, giving the place for calculation range of accounting. Knowingly coarsening of model in that way, leads to increasing of reliability of modeling results, and transition in particularly possibilistic space of model gives us already well-developed set of methods for risk portfolio estimations and optimization in fuzzy set of problem. By the way, in system (2,3) this idea is already realized for benchmark-risk estimation, when the benchmark-risk itself is a simple scalar value, and calculation range of portfolio yield – triangle estimation. The risk we estimate is a benchmark risk in pure form.

In the work [5] we visually show that having only ratios of degrees of assets yields and risks we can make a portfolio optimization and get quite trustworthy results. It means that in the near time we should reconstruct the similar method, but not in fuzzy-probabilistic space, but in opportunities of space. A question of what happens as result of combined optimization of volatile assets and assets with fixed yield which appears instead of Sharp beta-factor deserves an individual place.

An approach developed by us can be successfully used as in fund management, as in optimization of business-portfolios of any nature (for example business-processes in trade network or portfolio of direct investments with scalable insertions). It is clear that the same problem can be solved taking into consideration an extra two-sided constraints on asset size (these considerations will be considered in the body of gradient optimization algorithm).

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Comparative Study of Aggregation Methods In Bicriterial Fuzzy Portfolio Selection

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Abstract. Portfolio selection problem is formulated as a nonlinear fuzzy bicriterial optimization task. The first local criterion is the fuzzy return of portfolio. The second criterion is a measure of financial risk considered as a degree of the portfolio uncertainty. Special numerical optimization method based on the probabilistic approach to crisp and fuzzy interval comparison has been elaborated. The key issue is a proper aggregation of usually competing profit maximization and risk minimization local criteria. Three most popular local criteria aggregation methods were compared using optimization problem of five-stock portfolio in a fuzzy setting as an example. It is shown that all the results obtained with common (single criterion) methods can be considered as special cases of bicriterial portfolio problem formulation.

Keywords: Portfolio selection; Fuzzy nonlinear programming; Bicriterial optimization task, Aggregation method.

1. Introduction

The classical portfolio selection problem was formulated by Markowitz as the quadratic programming task in which the risk variance is minimized and the investment diversification is treated in computational terms. Markowitz's portfolio optimization model, contrary to its theoretical reputation, has not been used extensively in its original form to construct a large-scale portfolio [1]. The first reason behind this is in the nature of input required for the portfolio analysis. If accurate expectations about future mean returns for each stock and the correlation between each pair of stocks could be obtained, the Markowitz model would produce optimal portfolios. But to get such an accurate data the basic assumption of symmetrical Gaussian distributions of all returns must be adopted. Unfortunately, in practice symmetrical Gaussian distribution is rather seldom case [2]. Indeed, Markowitz model is, in essence, the single criterion one, whereas in real–world problems we usually deal with the set of particular criteria reflecting our various portfolio requirements. As it stated in [3], portfolio selection is usually a multiobjective problem. Problems mentioned above can be alleviated using fuzzy approach [4-8] to the portfolio optimization. It is worth noting that in all cases the portfolio selection problem was expressed as the fuzzy linear programming task with a single criterion, whereas it was shown in [9] that in general problem is at least bicriterial one.

In this paper we consider local criteria of return rate maximization and risk minimization, which usually are taking into account when assessing the portfolio. Thus, the portfolio selection problem is formulated as the bicriterial fuzzy nonlinear optimization task. The nonlinearity is a consequence of the bicriterial task nature and used numerical approach to crisp and fuzzy interval comparison.

Proposed approach is based on α-level representation of fuzzy intervals and estimation of the probability of certain interval being greater/equal than another interval.

2. The basic mathematical tools

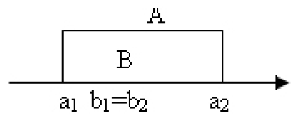
An approach based on the α-cuts presentation of fuzzy numbers is used. So, if $\widetilde{A}$ is a fuzzy number, then $\widetilde{A} = \bigcup \alpha A_\alpha$, where A_α is a crisp interval $\{x: \mu_A(x) \geq \alpha\}$, αA_α is a fuzzy interval $\{(x,\alpha): x \in A_\alpha\}$. It has been proved that if $\widetilde{A}$ and $\widetilde{B}$ are fuzzy numbers (intervals), then all the operations on them may be presented as operations on a set of crisp intervals corresponding to their α-cuts : $(A@B)_\alpha = A_\alpha @ B_\alpha$. There are some definitions of interval arithmetic [11], but in practical applications the so-called «naive» form is proved to be the best one. According to it, if $A = [a_1, a_2]$ and $B = [b_1, b_2]$ are crisp intervals, then

$$Z = A@B = \{ z = x@y, \quad \forall x \in A, \forall y \in B \}.$$

As a natural consequence of the basic concept assumed, the method for fuzzy interval comparison must be elaborated on a basis of the crisp interval comparison. The complete set of expressions for interval relations from [10] is shown in Table 1, obvious cases (without overlapping and inclusion) are omitted.

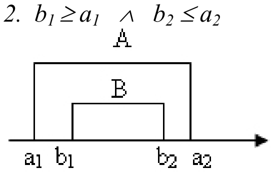
Table 1. The probabilistic interval relations

$P(B>A)$	$P(B<A)$	$P(B=A)$
1. $b_1>a_1 \;\wedge\; b_1<a_2 \;\wedge\; b_1=b_2$		



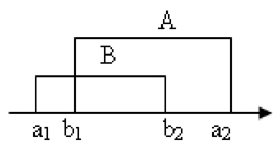
$\dfrac{b_1 - a_1}{a_2 - a_1}$	$\dfrac{a_2 - b_1}{a_2 - a_1}$	0

2. $b_1 \geq a_1 \quad \wedge \quad b_2 \leq a_2$



$\dfrac{b_1 - a_1}{a_2 - a_1}$	$\dfrac{a_2 - b_2}{a_2 - a_1}$	$\dfrac{b_2 - b_1}{a_2 - a_1}$

3. $a_1 \geq b_1 \quad \wedge \quad a_2 \geq b_2 \quad \wedge \quad a_1 \leq b_2$



0	$1 - \dfrac{(b_2 - a_1)^2}{(a_2 - a_1)(b_2 - b_1)}$	$\dfrac{(b_2 - a_1)^2}{(a_2 - a_1)(b_2 - b_1)}$

The set of fuzzy interval relations was created using α-cut representation of fuzzy values.

Let $\tilde{A}$ and $\tilde{B}$ be fuzzy intervals (numbers) on X with corresponding membership functions $\mu_A(x)$, $\mu_B(x)$: $X \rightarrow [0,1]$ and $\tilde{A}$ and $\tilde{B}$ be sets of α-levels $A_\alpha = \{x \in X: \mu_A(x) \geq \alpha\}$, $B_\alpha = \{x \in X: \mu_B(x) \geq \alpha\}$.

Then all the fuzzy interval relations $\tilde{A}$ rel $\tilde{B}$, $rel = \{<,=,>\}$ may be presented by a set of α-level relations $\tilde{A}\ rel\ \tilde{B} = \bigcup_\alpha A_\alpha\, rel\, B_\alpha$. Since A_α and B_α are crisp intervals, the probability $P_\alpha(B_\alpha > A_\alpha)$ for each pair A_α and B_α can be calculated in the way described above. The set of the probabilities $P_\alpha(\alpha \in (0,1])$ may be treated as the support of fuzzy subset

$P(\widetilde{B} > \widetilde{A}) = \{\alpha / P_\alpha(B_\alpha > A_\alpha)\}$, where the values of α denotes the grade of membership to fuzzy interval $P(\widetilde{B} > \widetilde{A})$.

In this way, the fuzzy subset $P(\widetilde{B} = \widetilde{A})$ may also be easily created.

It can be shown that in any case $P(\widetilde{A} > \widetilde{B}) + P(\widetilde{A} = \widetilde{B}) + P(\widetilde{A} < \widetilde{B})$ ="near 1" , where "near 1" is a symmetrical relative to 1 fuzzy number. Thus the main properties of probability are remained in the introduced operations, but in a fuzzy sense.

Nevertheless, in practice, the real number indices are needed for fuzzy interval ordering. For this purpose it seems natural to use the defuzzification, which for a discrete set of α-levels takes following form:

$$\overline{P}(\widetilde{B} > \widetilde{A}) = \sum_\alpha \alpha \cdot P_\alpha\left(B_\alpha > A_\alpha\right) \Big/ \sum_\alpha \alpha$$

The expression indicates that contribution of α-level to the overall probability estimation is rising along with the rise in its number.

2. Two-objective interval and fuzzy interval comparison

In our case the criterion of interval/fuzzy objective function minimization/maximization may be presented using probabilistic approach.. On the other hand, local criterion of uncertainty minimization may be presented in a natural way through the relation of widths of compared intervals or fuzzy intervals. Let's consider the local criteria of the interval comparison that can be introduced as the mathematical formalization of the above inexact reasoning. As the first criterion it is possible to accept directly the probability that one of the compared intervals is greater/less than another one $\mu_p(P(A>B))$, $\mu_p(P(A=B))$, $\mu_p(P(A<B)$.

To define the second criterion, the next parameters are introduced

$$x_A = \frac{W_A}{\max(W_A, W_B)},\; x_B = \frac{W_B}{\max(W_A, W_B)},$$

where W_A, W_B are the widths of intervals A and B, respectively.

Parameters x_A , x_B may be used to introduce the criteria $\mu_W(x_A) = 1 - x_A$, $\mu_w(x_B) = 1 - x_B$ that explicitly reflect our intention to decrease the uncertainty (width of interval objective function) on successive stages of the numerical optimization procedure.

Obviously, for the estimation of the possibility $A < B$ it is necessary to use the pair of criteria $\mu_p(P(B<A))$ and $\mu_A(x_A)$, otherwise for the appreciation of possibility $B < A$ the local criteria $\mu_p(P(A<B))$ and $\mu_B(x_B)$ must be considered. In our case, satisfaction the local criterion $\mu_A(x_A)$ in optimization tasks may be rather desirable; however, it is not necessary.

We apply three the most popular methods of the aggregation of local criteria. Additive form may be presented as:

$$D_A(A,B) = \frac{1}{2} \cdot (r_p \mu_p(P(A > B)) + r_w \mu_w(x_A)) \tag{1}$$

$$D_B(A,B) = \frac{1}{2} \cdot (r_p \mu_p(P(B > A)) + r_w \mu_w(x_B)) \tag{2}$$

- general criteria in multiplicative form:

$$D_A(A,B) = \mu_p(P(A > B))^{r_p} \cdot (\mu_w(x_A))^{r_w} \tag{3}$$

$$D_B(A,B) = \mu_p(P(B > A))^{r_p} \cdot (\mu_w(x_B))^{r_w} \tag{4}$$

- general criteria in maximum pessimism form:

$$D_A(A,B) = \min\{(\mu_p(P(A > B)))^{r_p}, (\mu_w(x_A))^{r_w}\} \tag{5}$$

$$D_B(A,B) = \min\{(\mu_p(P(B > A)))^{r_p}, (\mu_w(x_B))^{r_w}\} \tag{6}$$

where r_p, r_w are ranks or parameters of relative importance of considered local criteria. Of course, if $D_A > D_B$, then $B < A$ and otherwise $B > A$. Thus, on each step of the numerical optimization we have a small local two-criteria optimization task.

The fuzzy extension of two-objective comparison can be easily derived with use of α-cut representation of fuzzy intervals.

So, if $\widetilde{A}$ and $\widetilde{B}$ are fuzzy intervals (numbers) then

$$\overline{D}(\tilde{B} > \tilde{A}) = \sum_{\alpha} \alpha \cdot D_{\alpha}(B_{\alpha} > A_{\alpha}) / \sum_{\alpha} \alpha \tag{7}$$

To implement presented mathematical tools the specialized software based on the object-oriented approach using C++ has been elaborated.

3. Bicriterial fuzzy portfolio optimization

The problem of portfolio selection is formulated as follows: maximize the fuzzy total return rate $\tilde{F}$

$$\tilde{F} = \sum_{j=1}^{n} x_j \cdot \tilde{c}_j \quad \text{subject to} \quad \sum_{j=1}^{n} x_j = 1 \ ,$$

where: $\tilde{c}_j$ is the fuzzy return rate of the j-th asset, x_j is a real-valued decision variable which shows the investment rate of the j-th asset.

The well-known direct search method was modified and used as a numerical algorithm with use of additive, multiplicative and maximal pessimism forms of global criterion introduced in previous section (see expression (1) - (6)). To make it possible to compare the results obtained using elaborated bicriterial method with those derived from single criterion approaches, the example of five stocks portfolio optimization in fuzzy setting was adopted from [8]. Since the stock return rates were presented in [8] by the normal fuzzy numbers (see Fig.1), the special method for the transformation of probability distributions into fuzzy numbers was elaborated. As a result, all five fuzzy numbers representing stock returns were expressed in a form of α-cut sets, and then the described above algorithm was used.

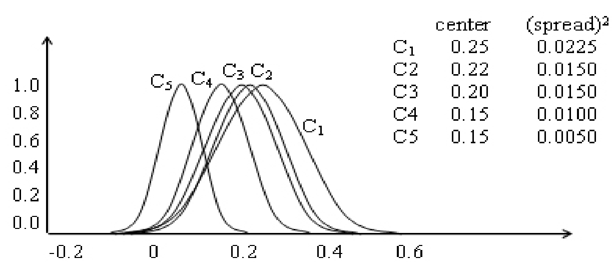


Figure. 1. Normally distributed fuzzy stock return rates from [8].

The results gained using different ranks of portfolio return maximization criterion r_p and risk minimization criterion r_w, and additive form of global criterion are presented in Table 2. In Fig. 2, the results obtained in [8] for the considered example of five-stock portfolio using the fuzzy versions of widely reputed and popular single criterion approaches are presented.

Table 2. The results of bicriterial portfolio optimization using additive form of global criterion.

r_p	r_w	x_1	x_2	x_3	x_4	x_5	Single criterion method [8] (see Fig. 2)
2	0	1	0	0	0	0	Modality model
0,5972	1,4028	1	0	0	0	0	Modality model
0,5971	1,4029	0,70	0,30	0	0	0	
0,5960	1,4040	0	1	0	0	0	Fractile model
0,2620	1,7380	0	0,92	0	0,08	0	Spread min model
0,2600	1,7400	0	0,35	0	0,64	0	Spread min model
0,2500	1,7500	0	0	0	0,99	0	
0,2400	1,7600	0	0	0	0,18	0,82	
0,2300	1,7700	0	0	0	0	0,99	

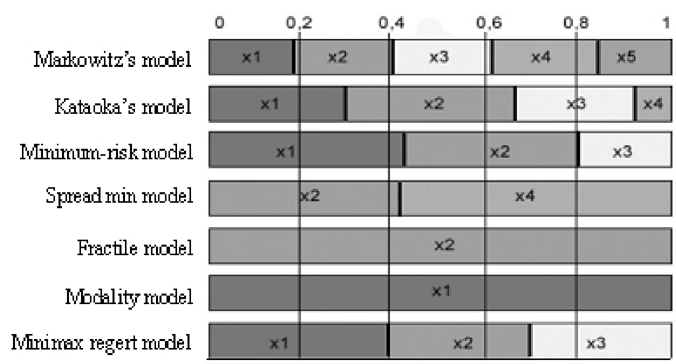


Figure 2. The results of five-stock portfolio optimization obtained using different single criterion models [8].

It is easy to see when comparing the results presented in Table 2 and Fig. 2 that varying the ranks of local criteria produced almost all the results that had been obtained earlier with using the fuzzy approach [8]. It is worth to note that such approaches are not, in essence, multicriterial ones, since they are based on the maximization or minimization of one local criterion (return or financial risk), whereas the other local criterion is considered to be only a restriction. It is interesting that stocks of type C_3 are not involved in the portfolio in any case when the bicriterial approach is used. This fact is easy to explain. The stocks C_3 and C_2 have the same variance (and as the consequence the risks) (see Fig. 2) but the mean of return of C_2 is greater than that of C_3, therefore in any case the stocks C_2 must be preferred. It seems quite natural that in a presence of the stocks C_2 the right portfolio policy is to reject the stocks C_3 from consideration. It is worth noting that the most optimal portfolio obtained using single criterion models (see Fig.2) includes the stocks C_3. Thus, the bicriterial approach is not only generalizing one but also better reflects the nature of the problem on the qualitative level.

The results of the portfolio optimization using multiplication and maximal pessimism types of local criteria aggregation differ substantially from those obtained using additive aggregation and single criteria methods. Nevertheless, the stocks of type C_3 rejected from optimal portfolio just as in the case of additive aggregation. So we can say that the proper choice of aggregation method in bicriterial nonlinear fuzzy portfolio optimization problem can play a key role.

4. Summary

The two-objective method for comparison of interval and fuzzy values taking into account the widths of compared uncertain values used for the building of the generalized criterion on the base of local criteria profit maximization and risk minimization is proposed.

The three most popular methods for the local criteria aggregation are compared using the example of portfolio consisting of five alternative stocks. The results compared with those obtained using single–criterion approach to the portfolio optimization in the fuzzy setting.

It is shown that proposed bicriterial fuzzy portfolio selection method based on the probability approach to crisp and fuzzy interval comparison produces all the results obtained using fuzzy versions of widespread single criterion approaches. Proposed method reflects better than traditional approach the qualitative nature of the considered portfolio optimization problem. The method makes it possible to take into account in a natural way the local criteria of portfolio return maximization and risk minimization with their ranks. The problem is formulated as the nonlinear optimization task, so the all possible forms of stock return membership function can be used without restrictions. Since the generalized criterion is formulated as an aggregation of local criteria, the method may be easily extended by inclusion additional criteria such as stock's liquidity, transaction costs and so on.

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On one method of portfolio optimization with fuzzy random data[1]

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Abstract

In the paper the method of portfolio optimization is offered for the case the expected values of securities are modeled by a fuzzy random variable. The way of the second order moments definition in nonfuzzy form is considered [1]. The proper calculation of the fuzzy random variables is elaborated. Within the developed approach to portfolio optimization with the use of fuzzy random data the optimization models are constructed. The solution methods of portfolio analysis problems by the models are developed.

Keywords:

fuzzy random variable, nonfuzzy moments of the second order, portfolio optimization

Introduction

The most useful information connected with the fuzzy random variables is revealed by the moment of the first and second order. There is a number of approaches to their definition. In all the approaches the expected value is a fuzzy variable and the moments of the second order can be either fuzzy[2] or nonfuzzy[1], that depends on their definition. In the paper the second approach is developed. The calculation methods of the fuzzy random variables numeric characteristics are offered. The application of

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the obtained results to the portfolio optimization problems is considered.

1. Basic definitions

Following [3-5], we introduce some necessary notions. Let $(\Gamma, P(\Gamma), \pi)$ be a possibilistic space, (Ω, B, P) be probability space. Here Γ is a set of elements denoted as $\gamma \in \Gamma$, $P(\Gamma)$ is a power set of Γ, π is a possibility measure [1].

Definition 1. Fussy random variable X is a real function $X(\cdot,\cdot): \Omega \times \Gamma \to E^1$, such that for any fixed $\gamma \in \Gamma$ the variable $X_\gamma = X(\omega, \gamma)$ is a random variable on (Ω, B, P).

Definition 2. *r*-level set of a fuzzy random variable is given by $X_\omega(r) = \{t \in E^1 : \mu_{X_\omega}(t) \geq r\}$.

In case when possibilistic distribution μ_{X_ω} is a quasiconcave and upper semicontinuous function, let $X_\omega^-(r)$ and $X_\omega^+(r)$ be lower and upper endpoints of *r*–level set.

Let's introduce the numeric characteristics of a fuzzy random variable according to the accepted notation [1].

Definition 3. The covariance of two fuzzy random variables X and Y is defined as

$$Cov(X,Y) = \frac{1}{2}\int_0^1 (Cov(X_\omega^-(r), Y_\omega^-(r)) + Cov(X_\omega^+(r), Y_\omega^+(r)))dr .$$

The consequence of Definition 3 is

Definition 4. The variance of a fuzzy random variable *X* is defined as $DX = Cov(X, X)$.

Definition 5. The expected value of a fuzzy random variable is a fuzzy variable, such that $[EX]^r = E[X]^r = [EX^-(r), EX^+(r)], \quad 0 < r \leq 1$.

For the further constructions we need the following elements of the fuzzy random variable calculations [6].

Let $X_1, \ldots, X_n$ be a collection of the fuzzy random variables, each of them have the following presentation $X_i(\omega, \gamma) = a_i(\omega) + \sigma_i(\omega) X_i(\gamma)$, where $a_i(\omega)$, $\sigma_i(\omega)$ – are random variables, which are shift and scale coefficients. If the fuzzy variables $X_i(\gamma)$ are min-related, the following results are correct:

1. The expected value of the weighted sum of the fuzzy random variables is defined as $E\{\sum_{i=1}^{n} \varpi_i X_i(\omega,\gamma)\} = \sum_{i=1}^{n} \varpi_i (a_i^0 + \sigma_i^0 X_i(\gamma))$, where a_i^0, σ_i^0 are the expected values of the shift and scale coefficients, ϖ_i are the scalars.
2. The variance of the weighted sum of the fuzzy random variables is defined as

$$D(\varpi_1 X_1 + \ldots + \varpi_n X_n) = \sum_{i=1}^{n} \varpi_i^2 DX_i + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \varpi_i \varpi_j Cov(X_i, X_j).$$

Let's turn to formulating a portfolio analysis problem with the fuzzy random data.

3. Setting the problems of portfolio analysis with fuzzy random data

Let's generalize the classical Markowitz model to the fuzzy random data. Let's introduce the following notations.

Let ϖ_i be a capital part spent on buying an *i*–type security, $\sum_{i=1}^{n} \varpi_i = 1$.

Let $R_1(\omega,\gamma),\ldots,R_n(\omega,\gamma)$ be fuzzy random variables, which are the expected values of the securities. Then, according to section 1, the expected value is a fuzzy random variable $\widetilde{R}(\varpi,\gamma) = \varpi_1 R_1(\gamma) + \ldots + \varpi_n R_n(\gamma)$, where $R_i(\gamma) = a_i^0 + \sigma_i^0 X_i(\gamma)$, and the portfolio risk, considering shift-scale presentation of $X_i(\omega,\gamma)$, is a function $V(\varpi)$.

$$V(\varpi) = \sum_{i=1}^{n} \varpi_i^2 DX_i + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \varpi_i \varpi_j Cov(X_i, X_j) =$$

$$= \sum_{i=1}^{n} \varpi_i^2 (D(a_i) + \frac{1}{12} D(\sigma_i)) + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (Cov(a_i, a_j) + \frac{1}{6} Cov(\sigma_i, \sigma_j)).$$

Considering the expected portfolio profitability is a fuzzy variable, Morkoviz models should be generalized. Let's introduce the following generalized portfolio optimization models, according to the results [7] and decision making principals with the fuzzy random variables [3,8].

Model of minimizing the possible risk under the preset level of possible income

$$V(\varpi) \to \min,$$
$$\begin{cases} M\{\widetilde{R}(\varpi,\gamma)\} = E_*, \\ \sum_{i=1}^{n} \varpi_i = 1, \end{cases}$$

where M denotes the transition to the modal value of the expected portfolio profitability, E_* is a preset profitability level.

Model of maximizing the expected income under the preset level of possible risk

$$k \to \max,$$
$$\pi\{\widetilde{R}(\varpi,\gamma) = k\} \ge \pi_0,$$
$$\begin{cases} V(\varpi) \le r, \\ \sum_{i=1}^{n} \varpi_i = 1. \end{cases}$$

In this model k is an additional level variable, π_0 is possibility level.

Let's turn to the methods of problem solution according to the models offered.

4. Methods of solving the problems of portfolio analysis with fuzzy random data

The problem minimizing the possible risk under the preset level of profitability can be solved using Lagrangian multiplier method.

Omitting all the technical details, we introduce the formulae that can be used to calculate the optimal portfolio and the risk. We have

$$\varpi^* = \frac{[(m^T C^{-1} m) - E_*(e^T C^{-1} m)]C^{-1} e + [E_*(e^T C^{-1} e) - (m^T C^{-1} e)]C^{-1} m}{(e^T C^{-1} e)(m^T C^{-1} m) - (m^T C^{-1} e)^2},$$

where

$$C = \{C_{ip}\}_{i,p=1}^{n} = \{C_{pi}\}_{i,p=1}^{n} = \{Cov(X_i, X_p)\}_{i,p=1}^{n} = \{Cov(X_p, X_i)\}_{i,p=1}^{n}$$

is a covariance matrix of the fuzzy random variables.

$V(\varpi^*) = (\varpi^*, C\varpi^*)$.

It's easy to see, that the formula corresponds to the one of classical portfolio analysis, with the exception of the way of covariance coefficients calculation.

Under the presumptions made, when the possibilistic variables $X_i(\gamma)$ belong to the class of the symmetric triangle distributions $X_i(\gamma) \in Tr(a_i, d_i)$, the formula of covariance coefficients calculation is the following [6]:

$$Cov(X_i, X_j) = Cov(a_i, a_j) + \frac{1}{6} Cov(\sigma_i, \sigma_j).$$

Let's turn to the methods of solving the problem of maximizing the expected income under the preset level of the possible risk. Taking into consideration the results of paper [8], the following theorem can be proved.

Theorem. Let the fuzzy random variables have the following form $X_i(\omega,\gamma) = a_i(\omega) + \sigma_i(\omega) X_i(\gamma)$, $X_i(\gamma)$ are min-related and $X_i(\gamma) \in Tr(a_i, d_i)$, $i = 1,\ldots,n$.

Then the problem of maximizing the expected income under the preset level of the possible risk have the determined analog:

$$\sum_{i=1}^{n} R_i^+(\pi_0)\varpi_i \to \max,$$

$$\begin{cases} V(\varpi) \le r, \\ \sum_{i=1}^{n} \varpi_i = 1, \end{cases}$$

where $R_i^+(\pi_0) = a_i^0 + a_i\sigma_i^0 + (1-\pi_0)\dfrac{d_i\sigma_i^0}{2}$.

It's obvious the obtained analog is the problem of the quadratic programming. The problem can be solved by the standard methods.

Conclusion

In the paper more flexible mathematical portfolio analysis apparatus is presented. New variance and covariance definition is used. Under the approach the characteristics have no fuzziness. The formula of portfolio risk calculation with fuzzy random data is obtained. All the results are for non-symmetrical triangle fuzzy random variables.

In the paper more flexible mathematical portfolio analysis apparatus is presented. New variance and covariance definition is used. Under the approach the characteristics have no fuzziness. The formula of portfolio risk calculation with fuzzy random data is obtained. All the results are for non-symmetrical triangle fuzzy random variables.

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FUZZY ECONOMIC AND INFORMATION SYSTEMS

Plenary Report

Fuzzy Information Systems

Fuzzy Models of Economic Systems

Towards human-consistent data-driven decision support systems via fuzzy linguistic data summaries

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Abstract. We present the use of fuzzy logic for the derivation of linguistic summaries of large sets of data for business decision support. We discuss how to measure goodness of a linguistic summary, and how to embed data summarization within fuzzy querying, for an efficient computer implementation. We present an implementation for a sales database at a small-to-medium computer retailer.

1. INTRODUCTION

Decision making is becoming more and more sophisticated, time consuming and difficult for human beings who require some (scientific) support. Traditionally, different mathematical models, descriptive and prescriptive, with single and multiple criteria and decision makers, etc. were developed. Modern approaches to real world decision making go further. They speak about *good decisions* (not necessarily optimal) but above all about a *decision making process* that involves: use of own and external knowledge, involvement of various „actors" and aspects, individual habitual domains, non-trivial rationality, different paradigms, etc. A good example of such a decision making process is Peter Checkland's so-called *deliberative decision* making related to *soft approach to systems analysis*.

Deliberative (soft) decision making may be subsumed as: perceive the whole picture, observe it from all angles (actors, criteria,...), find a *good* decision using *knowledge* and *intuition*. The decision making process involves: recognition, deliberation and analysis, gestation and enlightenment (the „eureka!", „aha" effects), rationalization, implementation, and is always: heavily based on data, information and knowledge, and human specific characteristics (intuition, attitude, natural language for communication and articulation,...), needs number crunching but also more „delicate" and sophisticated „intelligent" analyses, heavily relies on computer systems, capable of a synergistic human-computer interaction. Some computerized systems, called *decision support* systems (DSSs) are clearly needed that should address: ill/semi/un-structured questions and problems, non-routine answers, a flexible combination of analytical models and data, various kinds of data (e.g. numeric, textual, verbal,...), interactive interface (e.g. GUI, LUI), iterative operation („what if"), various decision making styles, etc. All these phases are based on: *data* - raw facts; *information* - data in a context relevant to an individual, team or organization, *knowledge* - an individual's utilization of information and data complemented by an unarticulated expertise, skills, competencies, intuitions, experience and motivations. Knowledge is the most relevant, and it can be: *explicit* - expressed in words or numbers, and shared as data, equations, specifications, documents, and reports; can be transmitted between individuals and formally recorded, *tacit* - highly personal, hard to formalize, and difficult to communicate or share with others; technical (skills or crafts), and cognitive (perceptions, values, beliefs, and mental models).

Among the DSSs, one can basically distinguish: (1) data driven, (2) communication driven and group DSSs, (3) document driven, (4) model driven, (5) knowledge driven, (6) Web based and interorganizational. Data Driven DSSs emphasize access to and manipulation of internal company data and sometimes external data, and may be based –from the low to high level – first on simple file systems with query and retrieval tools, then data warehouses, and finally On-line Analytical Processing (OLAP) or data mining tools. Communications Driven DSSs use network and communications technologies to facilitate collaboration and communication, Group GDSSs are interactive systems that facilitate solution of unstructured problems by a set of decision-makers working together. Document Driven DSSs integrate a variety of storage and processing technologies for a complete document retrieval and analysis; documents may contain numbers, text, multimedia. Model Driven DSSs emphasize access to and manipulation of a model, e.g., statistical, financial, optimization; use data and parameters, but are not usually data intensive. Knowledge Driven DSSs are interactive systems with specialized problem-solving expertise about a particular domain, understanding of problems within that domain, and "skill" at solving some of these problems. Web based DSSs deliver decision support related information and/or tools to a manager/analyst using a "thin-client" Web browser (Explorer); TCP/IP protocol, etc.

Here we concentrate on the data driven DSSs, and show how the use of Zadeh's computing with words and perception paradigm (cf. Zadeh and Kacprzyk, 1999) through fuzzy linguistic database summaries, and indirectly fuzzy querying, can open new vistas in data driven DSSs (also, to some extent, in knowledge driven and Web based DSSs).

The role of a data driven DSS is to help decision makers make rational use of (vast) amounts of available data from which relevant, nontrivial dependencies should be found. One of promising approaches is to derive linguistic summaries of a set of data (database). We discuss linguistic summarization in the sense of Yager (1982, and later) [cf. Kacprzyk and Yager (2001), Kacprzyk, Yager and Zadrożny (2001)]. Linguistic summaries are derived as linguistically quantified propositions, e.g. for a personnel database by "most of the employees are young and well paid", with an associated degree of validity that was initially the degree of truth of a linguistically quantified proposition, then a weighted sum of 5 quality indicators as proposed in Kacprzyk and Yager (2001) and Kacprzyk, Yager and Zadrożny (2001).

We also follow Kacprzyk and Zadrożny's (1998 – 2000b), Kacprzyk's (1999a), and Zadrożny and Kacprzyk's (1999) idea of an interactive approach to linguistic summaries in which interaction with the user is used to obtain a class of summaries of interest via Kacprzyk and Zadrożny's (1994 - 1996) fuzzy querying add-on to Microsoft Access.

We show an implementation for a sales database of a computer retailer, and that the linguistic summaries may be very useful.

2. IDEA OF LINGUISTIC SUMMARIES USING FUZZY LOGIC WITH LINGUISTIC QUANTIFIERS

In the basic Yager's (1982) approach to linguistic summarization of sets of data, we have: (1) V is a quality (attribute) of interest, e.g. salary in a database of workers, (2) $Y = \{y_1, \ldots, y_n\}$ is a set of objects (records) that manifest quality V, e.g. the set of workers; $V(y_i)$ are values of quality V for object y_i, (3) $D = \{V(y_1), \ldots, V(y_n)\}$ is a set of data ("database"). Then, a *summary* of a data set consists of: (1) a summarizer S (e.g. young), (2) a quantity in agreement Q (e.g. most), (3) truth (validity) T - e.g. 0.7, as, e.g., T(*most* of employees are *young*)=0.7.

When we try to linguistically summarize data, the most interesting are non-trivial, *human-consistent* summarizers as, e.g.: productive workers, difficult orders, etc. involving complicated *combinations of attributes* like, e.g.: a hierarchy (attributes of various importance), attribute values are ANDed and/or ORed, *k* out of *n*, *most*, ... are to be accounted for, etc.

The calculation of truth (validity) of the basic type of a linguistic summary is equivalent to the calculation of the truth value (from [0,1]) of a linguistically quantified statement (e.g., "*most* of the employees are *young*"). This may be done by either Zadeh's (1983) calculus of linguistically quantified statements [cf. Zadeh and Kacprzyk (1992)] or Yager's (1988) OWA operators [cf. Yager and Kacprzyk (1997)].

A linguistically quantified statement, e.g. "most experts are convinced", may be written as "Qy's are F" where Q is a linguistic quantifier (e.g., most), $Y = \{y\}$ is a set of objects (e.g., experts), and F is a property (e.g., convinced). Importance B may be added, "QBy's are F", e.g., "most (Q) of the important (B) experts (y's) are convinced (F)". The problem is to find truth(Qy's are F) or truth(QBy's are F), here done via Zadeh's (1983) fuzzy-logic-based calculus of linguistically quantified propositions.

First, property F and importance B are fuzzy sets in Y, and a (proportional, nondecreasing) linguistic quantifier Q is assumed to be a fuzzy set in [0,1] as, e.g., for Q = "most"

$$\mu_Q(x) = \begin{cases} 1 & \text{for } x \geq 0.8 \\ 2x - 0.6 & \text{for } 0.3 < x < 0.8 \\ 0 & \text{for } x \leq 0.3 \end{cases} \tag{1}$$

Then, due to Zadeh (1983), we have:

$$\text{truth}(Qy'\text{s are } F) = \mu_Q[\tfrac{1}{n}\textstyle\sum_{i=1}^{n} \mu_F(y_i)] \tag{2}$$

$$\text{truth}(QBy'\text{s are } F) = \mu_Q[\textstyle\sum_{i=1}^{n} (\mu_B(y_i) \wedge \mu_F(y_i)) / \sum_{i=1}^{n} \mu_B(y_i)] \tag{3}$$

We can also use an OWA operator based calculus [Yager (1988); Yager and Kacprzyk (1997)] as the OWA operators may represent a wide array of aggregation operators (including linguistic quantifiers based), from that corresponding to "all", to "at least one", through all intermediate ones.

That basic validity criterion from Yager (1982) is widely employed but clearly not sufficient, and Kacprzyk and Yager (2001) and Kacprzyk, Yager and Zadrożny (2001) proposed other criteria: truth value, degree of imprecision (fuzziness), degree of covering, degree of appropriateness, and length of a summary, and the (total) degree of validity was defined as the weighted average of the above 5 degrees of validity. We seek an optimal summary maximizing that weighted sum.

3. DERIVATION OF LINGUISTIC SUMMARIES VIA A FUZZY LOGIC BASED DATABASE QUERYING INTERFACE

Our approach is rooted in our papers on fuzzy querying of databases [cf. Kacprzyk and Zadrożny (1994 - 1997c), Kacprzyk, Zadrożny and Ziółkowski (1989), Zadrożny and Kacprzyk (1995)] showing that a precise query is often difficult for the end user, and proposing the FQUERY for Access package [Kacprzyk and Zadrożny (1994 - 1997c), Zadrożny and Kacprzyk (1995)].

The same arguments apply while trying to summarize the content of a database in a short (linguistic) statement as, e.g., a summary like "*most* our customers are *reliable*" may very often be more useful than, say “65% of our customers have paid at least 70% of their duties in less than 10 days”.

We restate linguistic summarization it in the fuzzy querying context. First, (2) formally expresses a statement:

“Most records match query S” (4)

where S replaces F in (2) since we refer here directly to the summarizer.

We assume a standard meaning of the query as a set of conditions on the values of fields from the database tables, ANDed and ORed. We allow for fuzzy terms, hence the degree of matching is from [0,1] rather than {0,1}. So, a query S defines a fuzzy subset of records, and the membership of them is given by their matching degree with the query.

Similarly, (3) may be interpreted as:

“*Most records meeting conditions F match query S*” (5)

Thus, (5) says something only about a subset of records taken into account by (4). That is, in the database terminology, F corresponds to a *filter* and (5) claims that *most* records passing through F match query S. Moreover, a record may pass through it to a degree from [0,1].

We seek propositions (3), interpreted as (5) that are highly true, and they contain: a fuzzy filter F (optional), a query S, and a linguistic quantifier Q. We can assume various elements as known, and hence a classification of linguistic summaries may be as in Table 1, depending on what is given or what is sought with respect to the fuzzy filter F and query S.

The summaries of Type 1 and 3 have been implemented as an extension to our FQUERY for Access that makes it possible to use fuzzy terms in queries [cf. Kacprzyk and Zadrożny (1994 - 1997c), Zadrożny and

Kacprzyk (1995)], notably: fuzzy values (*low* in "profitability is *low*"), fuzzy relations (*much greater than* in "income is *much greater than* spending"), and linguistic quantifiers (*most* in "*most* conditions have to be met"). Fuzzy terms are defined via an interaction with the user.

Table 1. Classification of linguistic summaries

Type	Given	Sought	Remarks
1	S	Q	Simple summaries through ad-hoc queries
2	S B	Q	Conditional summaries through ad-hoc queries
3	Q $S^{structure}$	S^{value}	Simple value oriented summaries
4	Q $S^{structure}$ B	S^{value}	Conditional value oriented summaries
5	nothing	S B Q	General fuzzy rules

where $S^{structure}$ denotes that attributes and their connection in a summary are known, while S^{value} denotes constants in summarizer sought.

When the user initiates the execution of a query it is automatically transformed by appropriate FQUERY for Access's routines and then run as a native query of Access. The transformation is via calls to functions that secure a proper interpretation of fuzzy terms. Then, the query is run by Access as usually.

4. IMPLEMENTATION FOR A SALES DATABASE AT A COMPUTER RETAILER

The proposed data summarization procedure was implemented on a sales database of a computer retailer in Southern Poland [cf. Kacprzyk (1999), Kacprzyk and Strykowski (1999)].

Examples of linguistic summaries obtained are given in Tables 2 – 5. These are the most valid summaries, and they give the user much insight into relations between the attributes chosen, moreover they are simple and human consistent.

Notice that these summaries concern data from the company's own database. However, companies operate in an environment (economic, climatic, social, etc.), and aspects of this environment may be relevant because they may greatly influence the operation, economic results, etc. of a particular company. A notable example may here be the case of climatic data that can be fetched from some sources, for instance from paid or free climatic data services. The inclusion of such data may be implemented but its description is beyond the scope of this paper. We can just mention that one can obtain for instance the linguistic summaries as in Table 5 in the case when we are interested in relations between group of products, time of sale, temperature, precipitation, and type of customers.

Table 2. Linguistic summaries expressing relations between the group of products and commission

Summary
About ½ of sales of network elements is with a high commission
About ½ of sales of computers is with a medium commission
Much sales of accessories is with a high commission
Much sales of components is with a low commission
About ½ of sales of software is with a low commission
About ½ of sales of computers is with a low commission
A few sales of components is without commission
A few sales of computers is with a high commission
Very few sales of printers is with a high commission

Table 3. Linguistic summaries expressing relations between the groups of products and times of sale

Summary
About 1/3 of sales of computers is by the end of year
About ½ of sales in autumn is of accessories
About 1/3 of sales of network elements is in the beginning of year
Very few sales of network elements is by the end of year
Very few sales of software is in the beginning of year
About ½ of sales in the beginning of year is of accessories
About 1/3 of sales in the summer is of accessories
About 1/3 of sales of peripherals is in the spring period
About 1/3 of sales of software is by the end of year
About 1/3 of sales of network elements is in the spring period
About 1/3 of sales in the summer period is of components
Very few sales of network elements is in the autumn period
A few sales of software is in the summer period

It is easy to see that the contents of all the linguistic summaries obtained does give much insight to the user (analyst) in what is happening in the company and its operation, and can be very useful as a support for making decisions.

Table 4. Linguistic summaries expressing relations between the attributes: size of customer, regularity of customer (purchasing frequency), date of sale, time of sale, commission, group of product and day of sale

Summary
Much sales on Saturday is about noon with a low commission
Much sales on Saturday is about noon for bigger customers
Much sales on Saturday is about noon
Much sales on Saturday is about noon for regular customers
A few sales for regular customers is with a low commission
A few sales for small customers is with a low commission
A few sales for one-time customers is with a low commission
Much sales for small customers is for non-regular customers

Table 5. Linguistic summaries expressing relations between group of products, time of sale, temperature, precipitation, and type of customers.

Summary
Very few sales of software is in hot days to individual customers
About ½ of sales of accessories is in rainy days on weekends by the end of the year
About 1/3 of sales of computers is in rainy days to individual customers

5. CONCLUDING REMARKS

We presented how a fuzzy linguistic database summary (in the sense of Yager) can be a very powerful tool for gaining insight into what relations exist within data in a particular company. We have indicated that such relations that are derived just from data, can be valuable clues for the decision maker to make decisions pertaining to the operation of the company.

The philosophy and paradigm presented in this paper follow those of a data driven DSS, and one can see that fuzzy linguistic database summaries, or – more generally the computing with words and perception paradigm – can be a powerful tool that can help develop a new generation of human consistent, natural language based and easy to use data driven DSSs.

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Information Monitoring Systems as a Tool for Strategic Analysis and Simulation in Business.

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1. Introduction

The matter of work of a number of specialists in modern information society is gathering and analysis of information on some problem and tracking/monitoring the process of its development. The following examples can illustrate this idea:

– from business area: "Promoting of new goods on the market", "Expediency of new product's development", "Keeping of loyal customers", etc.; input information is results of market research, expert's evaluations, price fluctuations, legislation, models of technological and/or business processes, etc.
– from political/sociology science: "Ensuring of victory of candidate X on elections", "Ensuring of position of country U in region V", etc.; the input information is Gallup polls, statements of politicians, action of parties and/or governments, etc.
– from natural science/technologies: "Fuzzy and intelligent technologies", "Alternative energy source", etc.; the input information is books, articles, proceedings, reports, estimations of specialists, etc.

We can continue this list many times.

We shall name a task of evaluation of a current state of the problem/process and elaboration of the forecasts of its development as an information monitoring problem and human-computer systems ensuring support of a similar sort of information problems - information monitoring systems.

Information monitoring systems relate to a class of hierarchical fuzzy discrete dynamic systems. The theoretical base of such class of systems is made by the fuzzy sets theory, discrete mathematics, methods of the analysis of hierarchies which was developed independently in works of Zadeh [9, 10], Messarovich [2], Saaty [8] and others. IMS address to process uniformly diverse, multi-level, fragmentary, unreliable, and varying in time information about some problem/process. Based on this type of information IMS allow perform monitoring of the problem's/process' evolution and work out strategic plans of problem/process development. These capabilities open a broad area of applications. One of such applications is a system for monitoring and evaluation of state's nuclear activities (department of safeguards, IAEA) [3] – have been shortly described in the report.

2. Basic elements of IMSs and their characteristic

We shall name a task of evaluation of a current state of the problem/process and elaboration of the forecasts of its development as an information monitoring problem and human-computer systems ensuring support of a similar sort of information problems - information monitoring systems.

Basic elements of monitoring system at the top level are the information space, in which information about the state of the problem/process circulates, and expert (experts), working with this information and making conclusions about the state of the problem/process and forecasts of its development.

The information space represents a set of various information elements, which can be characterized as follows:

- d*iversity of the information carriers*, i.e. fixing of the information in the articles, newspapers, computer kind, audio- and video- information etc.;
- *fragmentariness.* The information more often concerns to any fragment of the problem, and the different fragments may be differently "covered" with the information*;*
- *multi-levels of the information.* The information can concern to the whole problem, to some its parts, to a particular element of the problem*;*
- v*arious degree of reliability.* The information can contain the particular data which has a various degree of reliability, indirect data, results of conclusions on the basis of the reliable information or indirect conclusions;
- p*ossible discrepancy*. The information from various sources can coincide, slightly to differ or in general to contradict one another;

- *varying in time*. The problem develops in time, therefore the information at different moments of the time about the same element of the problem may and should be differ;
- *possible bias*. The information reflects certain interests of the source of the information, therefore it can have tendentious character. In the specific case it may be misinformation (for example, for political problems or for problems, connected to competitiveness).

The experts are an active element of the monitoring system and, observing and studying elements of the information space, they make conclusions about the state of the problem and prospects of its development taking into account listed above properties of the information space.

3. Basic principles of information monitoring technology

Information monitoring systems allow:

– to process uniformly diverse, multi-level, fragmentary, unreliable, information varying in time;
– to receive evaluations of status of the whole problem/process and/or its particular aspects;
– to simulate various situations in the subject area;
– to reveal "critical ways" of the development of the problem/process. It means to reveal those elements of the problem, the small change of which status may qualitatively change the status of the problem/process as a whole.

Taking into account the given features of the information and specific methods of its processing, it is possible to declare the main features of the information monitoring technology as follows:

– The system provides the facility for taking into account data conveyed by different information vehicles (journals, video clips, newspapers, documents in electronic form etc.). Such a facility is provided by means of storage in a database of a system of references to an evaluated piece of information, if it is not a document in electronic form. If the information is a document in electronic form, then both the evaluated information (or part thereof) and a reference thereto are stored in the system. Thus the system makes it possible to take into account and use in an analysis all pieces of information which have a relationship to the subject area irrespective of the vehicles concerned.

- The system makes it possible to process fragmentary information. For this purpose a considerable part of the model is represented in the form of a tree. It is clear that for complex problems/process such representation of a model is some simplification. However in this way good presentation and simplicity of operation with the model is attained.
- Information with different degrees of reliability, some of it possibly tendentious, can be processed in the system. This is achieved by reflecting the influence of a particular piece of information on the status of the elements of the model of the problem with the aid of fuzzy linguistic values. It should be borne in mind that an evaluation of an element of the model.
- The information with a various degree of reliability, probably, biased can be processed in the system. For this purpose the description of the influence of the information received on a status of the model of a problem was done with use of fuzzy linguistic variable. It is necessary to take into account, that the evaluation of the element of model may both vary under influence of the information received and remain unchanged (i.e. be confirmed).
- Time is one of the parameters of the system. This makes it possible to have a complete picture of the variation of the status of the model with time.

Comparative analysis of information monitoring technology (IMT) and (the most similar to it) expert systems technology (EST) as well as properties of the “good” subject areas are presented in Table 1 and Fig.1.

Thus, the systems constructed on the basis of this technology allow having the model of the problem developing in time. It is supported by the references to all information materials, chosen by the analysts, with general and separate evaluations of the status of the problem/process. Use of the time as one of parameters of the system allows to conduct the retrospective analysis and to build the forecasts of development of the problem/process. There is the opportunity of allocation "of critical points", i.e. such element(s) of the model, the small change of which can cause significant changes in a status of the whole problem/process. The knowledge of such elements has large practical significance and allows to reveal "critical points" of the problem/process, to work out the measures on blocking undesirable situations or achievement desirable, i.e. somewhat operate the development of the problem/process in time in the desirable direction.

Table 1. Comparative analysis of IMT and EST.

Property	IMT	EST
Presence of knowledge base (model)	★★★	★★★
Presence of data base	★★★	★★
Possibility of coprocessing of data and knowledge	★★★	★
Presence of system explanation system		★★★
In-depth logical deduction	★	★★★
Decomposition's capabilities	★★★	★
Capability to process of fragmentary information	★★★	★
Capability to process of dynamic information	★★★	★
Capability to work with mistaken data	★★★	
Polylogicalness	★★★	★
Capability to work with fuzzy data	★★★	★

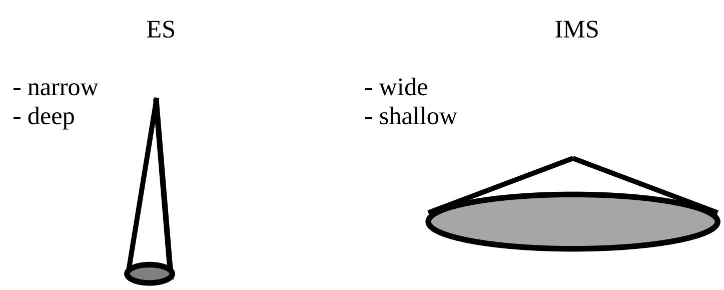

Fig. 1. Pithy analysis of characteristic of subject areas for IMS and ES.

4. Theoretical Basis

For effective practical application of the proposed technological solutions it is necessary to tackle a series of theoretical problems, the results of which are given below.

It is assumed that the expert describes the degree of inconsistency of the obtained information (for example, the readiness or potential for readiness of certain processes in a country [3]) in the form of linguistic values. The subjective degree of convenience of such a description depends on the selection and the composition of such linguistic values. Let us explain this on a model example.

Example [3]. Let it be required to evaluate the quantity of plutonium. Let us consider two extreme situations.

Situation 1. It is permitted to use only two values: “small” and “considerable quantity”.

Situation 2. It is permitted to use many values: “very small”, “not very considerable quantity”, ..., “not small and not considerable quantity”, ..., “considerable quantity”.

Situation 1 is inconvenient. In fact, for many situations both the permitted values may be unsuitable and, in describing them, we select between two “bad” values.

Situation 2 is also inconvenient. In fact, in describing a specific quantity of nuclear material, several of the permitted values may be suitable. We again experience a problem but now due to the fact that we are forced to select between two or more “good” values. Could a set of linguistic values be optimal in this case?

It is assumed that the system tracks the development of the problem, i.e. its variation with time. It is also assumed that it integrates the evaluations of different experts. This means that one object may be described by different experts. Therefore it is desirable to have assurances that the different experts describe one and the same object in the most “uniform” way.

On the basis of the above we may formulate the first problem as follows:

Problem 1. Is it possible, taking into account certain features of the man’s perception of objects of the real world and their description, to formulate a rule for selection of the optimum set of values of characteristics on the basis of which these objects may be described? Two optimality criteria are possible:

Criterion 1. We regard as optimum those sets of values through whose use man experiences the minimum uncertainty in describing objects.

Criterion 2. If the object is described by a certain number of experts, then we regard as optimum those sets of values which provide the minimum degree of divergence of the descriptions.

This problem may be reformulated as a problem of construction of an optimal information granulation procedure from point of view of criterion 1 and criterion 2.

It is shown that we can formulate a method of selecting the optimum set of values of qualitative indications (collection of granules [5]). Moreover, it is shown that such a method is stable, i.e. the natural small errors that may occur in constructing the membership functions do not have a significant influence on the selection of the optimum set of values. The sets which are optimal according to criteria 1 and 2 coincide. The results obtained are described in [7]. Following this method, we may describe objects with *minimum possible uncertainty*, i.e. *guarantee optimum* operation of the *information monitoring system* from this point of view.

Information monitoring technology assumes the storage of information material (or references to it) and their linguistic evaluations in the system database. In this connection the following problem arises.

Problem 2. Is it possible to define the indices of quality of information retrieval in fuzzy (linguistic) databases and to formulate a rule for the selection of such a set of linguistic values, use of which would provide the maximum indices of quality of information retrieval?

This problem may be reformulated as a problem of construction of an optimal information granulation procedure from point of view of information retrieval in fuzzy (linguistic) databases.

It is shown that it is possible to introduce indices of the quality of information retrieval in fuzzy (linguistic) databases and to formalize them. It is shown that it is possible to formulate a method of selecting the optimum set of values of qualitative indications (collection of granules [5]) which provides the maximum quality indices of information retrieval. Moreover, it is shown that such a method is stable, i.e. the natural small errors in the construction of the membership functions do not have a significant effect on the selection of the optimum set of values. The results obtained are shown in [4, 7]. It allows to approve that the offered methods can be used in *practical tasks* and *to guarantee optimum work of information monitoring systems*.

Because model of the problem/process have hierarchical stricture (see section 2), choice and selection (tuning) of aggregation operators for the nodes of the model is one more important issue in development IMS. We may formulate this problem as follows:

Problem 3. Is it possible to propose the procedures of information aggregation in fuzzy hierarchical dynamic systems which allow us to minimize contradictoriness in the model of problem/process in IMS?

It is shown that it is possible to propose the following approaches based on different interpretations of aggregation operators: geometrical, logical, and learning-based. The last one includes leaning based on genetic algorithms and learning based on neural networks. These approaches are described in details in [6].

5. Application's features

Some applied information monitoring systems based on described above technology have been developed. Based on this experience, we can formulate the following necessary stages of the development process:

- conceptual design;
- development of the demonstration prototype;
- development of a prototype of the system and its operational testing;
- development of the final system.

Volume of article does allow us to describe these items in details; therefore we can focus on a matter of principle only.

The most difficult point in development process is the elaboration of structure of the problem/process model. In some well-developed areas (marketing, medicine) we used descriptions of the process from professional books and references (like [1]) as a draft of the model, coordinated this draft with the professional experts (conceptual design and development of the demonstration prototype stages), and "tuned" this improved draft during testing of the system (development of a prototype stage).

Sometimes the problem/process for monitoring is formalized enough for application of information monitoring technology. An example of this situation is a state nuclear program evaluation procedure in IAEA [3]. Developed earlier so-called physical model of the nuclear fuel cycle was a good base for the model of the problem/process in information monitoring system. Based on this model a prototype of information monitoring system has been developed. This prototype allows [3]:

– Provide a tool for continuous monitoring of the status of the subject area.

- Provide IAEA expert with a tool to input into the system documents concerning the States' nuclear activities in textual format or references to documents in the form of hard copies, video topics, audio reports, etc.
- Produce an evaluation of the influence of obtained sign on the status of elements of the model and to change (confirm) their status accordingly.
- Provide a tool for examining the status of the subject area with several levels of detail.
- Detect inconsistencies between the declared States' capabilities for processing nuclear material and those capabilities as established by the Agency through analysis of information from other available sources.
- Assess the importance of any detected inconsistencies from the point of view of a change in the States' capabilities to produce HEU and Pu.
- Detect "critical points" important from the point of view of production of HEU and Pu, information about which is crucial for resolving an inconsistency between a country's declaration and its capabilities for processing nuclear material established by the Agency.
- Provide storage in its database of all the documents evaluated by the expert on references to them with linkage to specific elements of the model of the nuclear activity of a country.
- Provide IAEA expert with the tool for retrospective analysis of a change in the evaluations of each element of the model with the possibility of scanning the corresponding document or obtaining references to it.
- Record changes occurring in the system and provide the user with the tool for analyzing them.

6. Conclusion

IMS works with diverse, multi-level, fragmentary, unreliable, and varying in time information about some problem/process and allows performing monitoring of the problem/process evolution and working out strategic plans of the problem/process development.

The most difficult point in development process is the elaboration of structure of the problem/process model. Perspective way for automation of this process (development of the model of problem/process) is an application of advanced technologies like data mining. Our first experiments shown that data mining can be a good tool for this task especially if we have enough data on the problem/process.

Developed methods for problems 1-3 (section 4) allow us to guarantee optimum work of IMS.

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How to select a corporate information system using fuzzy sets

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The report demonstrates that corporate information system (CIS) should be selected basing on maximization of effect expected from implementation. The effect should be considered with fixed level of economic risk which is affected by implementation process. The following example [1] illustrates, that while making a decision about CIS implementation, it is necessary to take into account not only the most evident indicators, but also deep-laid strategic goals of CIS implementation in a company.

Problem definition of CIS implementation prospect evaluation

Company has to make a decision about ERP system implementation. System implementation cost consists of **1 million dollars** at the beginning and **250 000 dollars** of operational costs **each year** during next **2 years** (IT implementation project horizon is **5 years** or 6 time slices, including starting zero slice, with analysis interval equal to **1 year**).

The most important project goal defined by stakeholders is foreign investment market entry. At the same time, emission or credit parameter will be available only after results of IT implementation and depend on how painless the process of company restructuring and IT implementation will be. Company owners are ready to share control over the company in exchange for investment, reserving to themselves 25% of company plus one more vote (this is a blocking control; now they own 60% of stock).

Starting report forms (slice 0) are given in Tables 1 and Table 2.

Table 1. Company balance sheet (slice 0), million dollars

Assets		Liabilities	
Non-current assets	26	Capital	16
		Current liabilities	25
Current assets	19		
		Non-current liabilities	4
Total	45	Total	45

Table 2. Profit and Loss Statement (a year before slice 0), million dollars

Report item (form 2)	Value
Profit from operating activity	22
Operating activity expenses	14
Gross profit	8
Profit before Tax (EBIT)	7
Net profit	5

Special audit carried out before IT implementation allowed us to define two hierarchies of strategic indicators: effectiveness indicators and economic risks (Tables 3 and Table 4 correspondingly):

Table 3. Effectiveness indicators and their levels (slice 0; "*" – to be defined)

Indicator code	Indicator title	Indicator level
F_0	Company work effectiveness	*
F_1	Company finance level	*
$F_{1.1}$	Liquidity level	Low
$F_{1.2}$	Financial autonomy level	Medium
$F_{1.3}$	Profitability level	Medium
$F_{1.4}$	Assets turnover	Medium
F_2	Company management level	*
$F_{2.1}$	Top management level	Medium
$F_{2.2}$	Finance management level	High
$F_{2.3}$	Marketing and advertising departments level	Low
$F_{2.4}$	Distribution network and branches development level	High
F_3	Company transparency level	Very low
F_4	Brand value level	Low

There is a system of strategic indicators preference correlation below:

$$\begin{aligned} &F_1 \approx F_2 \approx F_3 \approx F_4; \\ &F_{1.1} \succ F_{1.2} \approx F_{1.3} \approx F_{1.4}; \\ &F_{2.1} \succ F_{2.2} \succ F_{2.3} \approx F_{2.4} \end{aligned} \quad (1)$$

where sign «$\succ$» means "preference", and sign «≈» means indifference.

Table 4. Economic risks indicators and their levels (slice 0)

Indicator code	Indicator title	Indicator level
R_0	Company economic risk	*
R_1	Risk of temporary lost paying capacity	Medium
R_2	Supplier relationship worsening risk	Medium
R_3	Customer relationship worsening risk	Medium
R_4	Staff demotivation risk	Low

And it is determined that:

$$R_3 \succ R_1 \approx R_2 \approx R_4. \quad (2)$$

We will also control strategic optional indicator **"Permissible size of foreign investment attraction"**, which is a very important goal for company owners.

Methodology for integral estimation of effectiveness and risk factors [2]

Let effectiveness or risk factors form hierarchies and all lower elements of these hierarchies are linguistic variables. There are also established preference systems for all elements on each level of hierarchies. We need to estimate integral qualitative level of company effectiveness and risk factors.

Let us take fuzzy granulator with trapezoid membership functions for each linguistic variable (Figure 1).

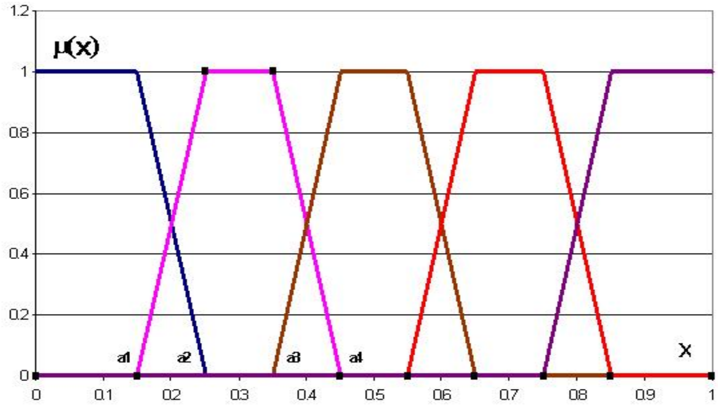

Fig. 1. System of trapezoid membership functions on 01 base

Basing on the system of indicators preference we also match each factor in the system with Fishborn weighting factor [3]. Fishborn fractions for all mixed preference correlation systems (for N=2, 3, 4) is presented in Table 5.

Table 5. System of Fishborn weights (N=2, 3 ,4)

N	Φ	p_1	p_2	p_3	p_4
2	$F_1 \approx F_2$	1/ 2	1/ 2	-	-
	$F_1 \succ F_2$	2/ 3	1/ 3	-	-
3	$F_1 \approx F_2 \approx F_3$	1/ 3	1/ 3	1/ 3	-
	$F_1 \succ F_2 \approx F_3$	2/ 4	1/ 4	1/ 4	-
	$F_1 \approx F_2 \succ F_3$	2/ 5	2/ 5	1/ 5	-
	$F_1 \succ F_2 \succ F_3$	3/ 6	2/ 6	1/ 6	-
4	$F_1 \approx F_2 \approx F_3 \approx F_4$	1/ 4	1/ 4	1/ 4	1/ 4
	$F_1 \succ F_2 \approx F_3 \approx F_4$	2/ 5	1/ 5	1/ 5	1/ 5
	$F_1 \approx F_2 \succ F_3 \approx F_4$	2/ 6	2/ 6	1/ 6	1/ 6
	$F_1 \approx F_2 \approx F_3 \succ= F_4$	2/ 7	2/ 7	2/ 7	1/ 7
	$F_1 \succ F_2 \succ F_3 \approx F_4$	3/ 7	2/ 7	1/ 7	1/ 7
	$F_1 \succ F_2 \approx F_3 \succ F_4$	3/ 8	2/ 8	2/ 8	1/ 8
	$F_1 \approx F_2 \succ F_3 \succ F_4$	3/ 9	3/ 9	2/ 9	1/ 9
	$F_1 \succ F_2 \succ F_3 \succ F_4$	4/10	3/10	2/10	1/10

There are 2^{N-1} preference systems for each N correlating alternatives.

Then we can offer a scheme for indicators qualitative levels aggregation using Yager OWR operator [4]. In such OWR convolution we use

Fishborn weights as weights. As for membership functions, we use membership functions from classifier (like in Figure 1) and these functions are chosen to correspond to qualitative level of linguistic variable. After transformations we get membership function for hierarchy top – integral indicator – and we can match it with qualitative description choosing the best approximation of this function with functions in Figure 1. As a result we get qualitative estimate for company integral effectiveness or risk factor.

Preliminary calculations

There are results calculated according to described methodology with data given in Table 3 and Table 4:

- Company work effectiveness – between "Low" and "Medium"
- Economic risk – "Medium"

In the course of audit there were performed quantitative and qualitative analysis of strategic indicators from Table 3 and Table 4. This analysis revealed some problem areas in the company, which were covered in a final report:

Positive:

- IT implementation can increase asset turnover (due to goods turnover optimization). With all this going on IT solution will not decrease company liquidity. Therefore CIS cost is not critical. And now it is important to understand how useful the system could be for the company.
- "Company transparency level" will grow to "High" level from the 3rd year of the project.
- CIS implementation leads "Brand value level" factor to "Medium" level on the 3rd year of the project and to "High" level on the 4th – 5th years (with simultaneous adding of the intangible asset "CIS ownership" at the cost of 5 million dollars to the company balance)
- "Customer relationship deterioration risk" will decrease according to the next scenario: "Low" (years 3 and 4) and "Very low" (the 5th year). It means that company will pass to the next qualitative level of outer world relationship management and contractors loyalty will increase.
- Liabilities restructuring (due to increased customer loyalty and deepened advance in payment structure (up to 50%)) will allow company to substitute 70% of current liabilities for non-current liabilities. As a result, "Risk of temporary lost paying capacity" will decrease to "Very

low" (from the 4th project year) and at the same time "Company liquidity level" will increase to level "High" (from the same project year).
- IT implementation will cause labor productivity rise in the fields of customer and supplier relationship management that will result in increased sales revenue (on 6 million dollars a year) and decreased goods cost (on 15%).
- Improved company position will allow it to make an intermediate emission on the 4th – 5th project year.

Negative:

- Simultaneous work with two system during the 1st and the 2nd project years will increase "Staff demotivation risk" from "Low" to "High" level (during first two years), then to "Medium" level (the 3rd year) and back to "Low" level (years 4 and 5).
- "Company transparency level" could also temporarily decrease (because of simultaneous work in two systems).

So, we need to define profitability of CIS implementation basing on dynamic of effectiveness and risk indicators.

Problem solution

Table 6 demonstrates results of scenario modeling for hierarchies through slices 1 – 5 (according to project period). The modeling is based on the result of high-level audit.

Table 6. "Effects" and "Risks" hierarchies evolution during 5 years of IT project

Indicator code	Indicator level for slices (years of IT project)				
	1	2	3	4	5
F_1	Medium	Medium	High	High	High
F_2	Medium	Medium	Medium	High	High
F_3	Very low	Low	High	High	High
F_4	Low	Low	Medium	High	High
F_0	Medium	Medium	Medium	High	High
R_1	Medium	Medium	Low	Very low	Very low
R_2	Medium	Medium	Low	Low	Very low
R_3	Medium	Medium	Low	Low	Very low
R_4	High	High	Medium	Low	Low
R_0	Medium	Medium	Low	Low	Very low

Two-dimensional dynamic "Effects – Risk" is shown on Figure 2.

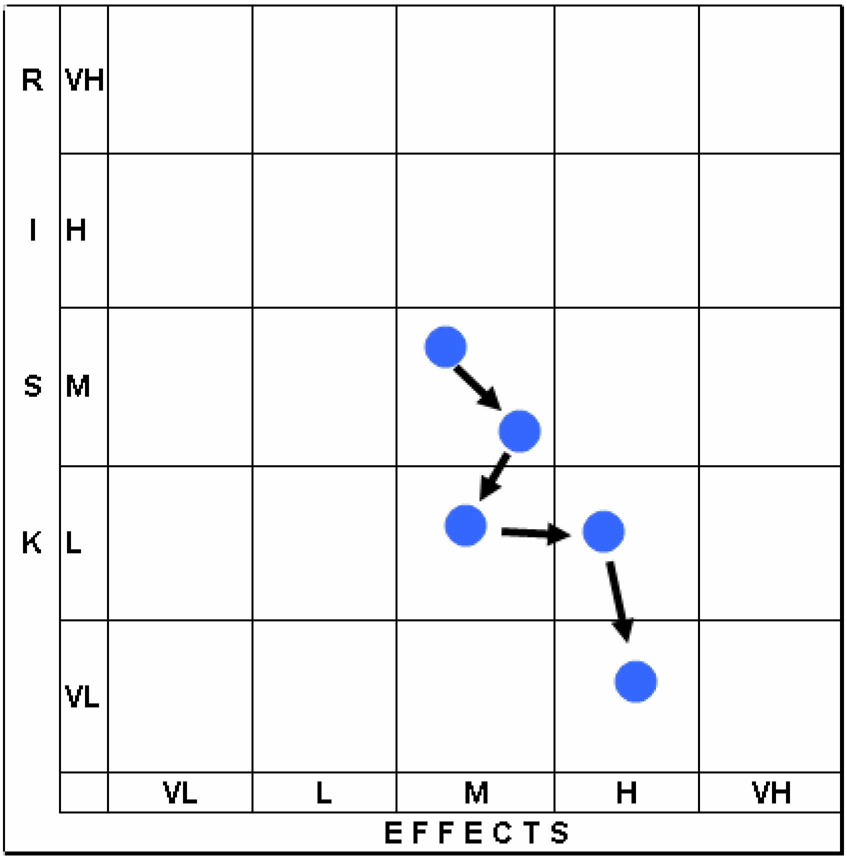

Fig. 2. Qualitative two-dimensional dynamic of aggregated strategic factors (effects and risks)

There is no doubt that this dynamic is positive: on the 5th project year company will get temporary monopoly as a result of establishment of exclusive relationship with contractors and getting qualitative competitive advantages from that relationship. At the same time we see considerable improvements in all company areas.

If we consider company strategic development not only basing on qualitative strategic indicators but also taking into account the balance sheet, connection between quantitative balance figures and qualitative strategic indicators levels will become more obvious. Table 7 and Table 8 present forecasted final report forms (slice 5):

Table 7. Company balance sheet (slice 5), million dollars

Assets		Liabilities	
Non-current assets	31	Capital	21
		Current liabilities	10
Current assets	20		
		Non-current liabilities	20
Total	51	Total	51

Table 8. Company Profit and Loss Statement (the 4th year before the 5th slice), million dollars

Report item (form 2)	Value
Profit from operating activity	28
Operating activity expenses	16
Gross profit	12
Profit before Tax (EBIT)	11
Net profit	7

It is clear that financial area has been improved and this improvement will allow stock emission to foreign market. Calculations made show that (with owners keeping 25% of shares plus one more vote and having blocking control) it will be possible to attract **15 million dollars** of share capital and **the same sum** of long-term credit capital for 10 years with interests of 7% a year. The result of these investments affect company balance sheet (Table 9):

Table 9. Company balance sheet after investment attraction, million dollars

Assets		Liabilities	
Non-current assets	31	Capital	41
		Current liabilities	10
Current assets	55		
		Non-current liabilities	35
Total	86	Total	86

Thus the company reached the goal of attraction foreign investments. After investment of 1.3 million dollars the company has got “cheap” financial resources of 30 million dollars. We don't need to calculate ROI to be sure that CIS implementation is a very profitable measure.

The main indicator **"Permissible size of foreign investment attraction"** has also been increasing from year to year due to company balance improvement and its financial strength growth. After CIS implementation the indicator has reached level "High". Thus the owners' goal which they wanted to be affected by CIS implementation has been achieved.

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Application of a fuzzy relational server to concurrent engineering

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Summary. We present the fuzzy relational data model implementation for Oracle8i which supports fuzzy queries of the type "find all records analogous to pattern" specified (possibly fuzzily) in query. In this paper, we focus on the new efficient methods for prototype searching of project archive for CAD-system.

1 Introduction

Now designers of complex projects are distributed over a territory and do their work using Wide Area Network. A project repository is a means to ensure data consistency. Usually, each designer group has an archive of previous projects, and the archive is available in the electronic form. The electronic archive can be treated as an active data warehouse. The international standard ISO-9000 requests to store every project in a project repository. Modern technology of data server is the basic principle of project repository construction. While designing a project, designers represent their current project decisions into the project repository through the data server. However, due to the fact that these decisions are usually incomplete and sometimes uncertain, they have to use fuzzy values for design parameters to represent their decisions. If a novel project is in progress, some parts of project information have to be fuzzy and incomplete. At early steps of designing, the user query is the Ad-Hoc Query and it contains the substantially fuzzy conditions. One of the most important types of query is the query of project by analogy where analogy is expressed by a fuzzy relation. The fuzzy query is the SQL-like query. There is a growing interest within the CE community in the extending of the relational data model to the level of fuzzy relational model for CAD-applications.

Dubois & Prade (1980) [2] developed the extension of relational model using Possibility Theory. Bosc & Kacprzyk (1995) [1] carried out research into fuzzy sets and possibility theory in DBMS. Chen & Kerre (1997) [3] edescribed the representation of fuzzy data in data base. But most of published research is theoretical nature. There are important causes for developing of fuzzy SQL server for industrial CAD-applications. This paper describes an application of fuzzy data representation by Data Base Management System (DBMS) Oracle means. Modern intellectual CAD-system can include a logical inference subsystem (may be Fuzzy Inference system FIS) and data base subsystem [5, 4]. Since these subsystems exchange data. DBMS should represent data containing fuzzy attributes.

2 Problem

Now relational data model is the most popular data model. But relational data model can store only crisp data. Modern repository of CAD-System should store multimedia objects, transaction history, workflows, knowledge and rules. But a modern project repository can not store fuzzy sets, membership functions, linguistic variables. DBMS can not represent an expert knowledge expressed in linguistic form. There is difficulty to create an effective archive of projects and organize search by analogy if your possibilities are limited relational DBMS only. The realization of fuzzy relational server will be useful.

This paper is devoted to the architecture of fuzzy SQl-server. For creation of this architecture we should carry out the following tasks.

1. The investigation of project information fuzziness.
2. Developing of extent fuzzy relational model.
3. Developing of operation algorithms for modified fuzzy relational algebra, developing of fuzzy subset for query language SQL (Fuzzy SQL).
4. Developing of metadata for fuzzy relational server.
5. Realization fuzzy subsystem of relational data server.
6. Developing CAD-application using fuzzy relational server.
7. Evaluating of efficiency.

3 Fuzzy relational data model

Firstly, let us consider basic definitions of the extended model. The fuzzy relational data model integrates relational model and fuzzy sets theory.

The relation scheem R is the finite set of attribute names $\{A_1, A_2, \ldots, A_n\}$. Each attribute name A_i corresponds to set D_i that is called domain of attribute A_i. The domains are nonempty finite sets. Let

$$D =< D_1, D_2, \ldots, D_n > \quad . \tag{1}$$

The attribute domain D_i is called fuzzy domain if the following conditions are held:

(i) the attribute name A_i is defined;

(ii) the universal set X is defined;
(iii) the set of terms T (the set of fuzzy labels) is defined.

The finite set of maps $\{t_1, t_2, \ldots, t_p\}$ from R to D is called fuzzy relation if mapping t_i exists and takes A_i to D_i and D_i is a fuzzy domain. A CAD-application can use fuzzy numbers. Thus the domain of fuzzy number is a set of real numbers. Fuzzy number has a membership function and a linguistic label.

The membership function defines a fuzzy number if the following conditions are held:

(i) the restriction condition, i.e. by definition

$$\max \mu(x) \leq 1, \quad \min \mu(x) \geq 0 \tag{2}$$

(ii) the correspondence condition, i.e. each set

$$\mu(x) = \{z\}, \quad x \in X \tag{3}$$

includes only one element
(iii) the continuity condition.

We use a piece-linear function approximation for the membership function representation. The number of pairs $\mu(x_i)/x_i$ in the membership function table is unrestricted. The set of these pairs defines a curve of membership function. The piece-linear representation is easily expressed in relational data base. Fuzzy data server should execute following operations: operations of one argument (defuzzyfication normalization, α-cut); operations of two and more arguments (negation, intersection, union); arithmetic operations (addition, subtraction, multiplication, division); fuzzy operation of comparison.

Operations of two and more arguments are defined by "min" and "max" functions. Arithmetic operations are determined as Zadeh's principle:

$$\begin{aligned} &C = A \bigtriangledown B = [\sup \min(\mu_A(a), \mu_B(b))]/[a \bigtriangledown b] \\ &a \in S_A, \quad b \in S_B \end{aligned} \tag{4}$$

the symbols S_A, S_B, A, B are the supports and membership functions correspondently for fuzzy numbers A and B, and symbol $\bigtriangledown$ denotes any arithmetic operation.

The comparison operation is very important in fuzzy relational model. This operation is included in the definition of relational operations of selecting, joining and natural joining.

Consider the following important assertions.

1. The comparison operation of fuzzy numbers gives fuzzy results. These results are fuzzy numbers equivalence degrees.
2. The intersection operation of fuzzy numbers may be used as a criterion of comparison. The degree of equivalence can be calculated as height of intersection. If intersection is empty then the degree of equivalence is zero.

Suppose A and B are two fuzzy numbers; then the degree of equivalence $E(A, B)$ is calculated as

$$E(A, B) = \max\left(\min(\mu_A(x), \mu_B(x))\right), \quad x \in \mathbf{R} \tag{5}$$

Consider the relational operations for fuzzy data. There are two types of relational operations. The first type of operations includes the comparison operation. The second type includes operations of projection, intersection and union. Fuzzy operations of second type are similar to relational operations for crisp data. Without loss of generality it can be assumed that any tuple satisfies the conditions of query with some degree.

Let $r(R)$ be a fuzzy relation, let A be a fuzzy attribute in $\mathbf{R}$, and $a \in$ dom (A). By definition of fuzzy attribute dom $(A) = D$. The universal set X is a set of real numbers. For any operation of selection such arguments are fuzzy numbers, hence

$$\begin{aligned} \sigma_{A=a}(r) = r' &= \{t \in r \mid \ t(A) = a\} \\ &= \{t \in r \mid \ E(t(A), a) \geq \mu_\theta\} \end{aligned}$$

here E is a degree of equivalence, μ_θ is a threshold. The value 0.5 is often used as the value of threshold on the assumption two fuzzy numbers are equal.

Consider relations $r(R)$, $s(S)$ and the operation of join $RS = T$. The join operation denoted $q(T)$ includes the tuples containing parts $t_r \in r$ $(t_r = t(R))$ and $t_s \in s$ $(t_s = t(S))$. It follows that $t_r(R \cap S) = t_s(R \cap S)$. Any tuple in q is the join of tuple part from r and the tuple part from s if attributes $(R \cap S)$ have equal values in both tuples. Then, $E(t_r(R \cap S), t_s(R \cap S)) \geq \mu_\theta$, here E is a degree of equivalence, μ_θ is a threshold (0.5). It results from this, that

$$\begin{aligned} r \bowtie s = q(T) = \{t = t_r t_s \mid \ & t_r \in r \ \& \ t_s \in s \ \text{ with } \ t_r \\ = t(R) \ \& \ t_s &= t(S), t_r(R \cap S) = t_s(R \cap S)\} \\ = \{t = t_r t_s \mid \ & t_r \in r \ \& \ t_s \in s \ \text{ with } \ t_r = t(R) \\ \& \ t_s = t(S), & E(t_r(R \cap S), t_s(R \cap S)) \geq \mu_\theta\} \ . \end{aligned}$$

The values of $(R \cap S)$ are included in relation q. The value $t_r(R \cap S)$ is not equal to $t_s(R \cap S)$. What value t_r or t_s should we include in q? Let us use a union of fuzzy values for attributes from $(R \cap S)$. The concepts of functional dependency and normal forms are important for relations constructing. If K is a primary key of relation then the subset K does not include a fuzzy attribute. Since the result of comparison operation depends on the threshold; we choose Proposition 2. Let t_1 and t_2 be tuples, let K be the primary key of relation $r(R)$, $K \subseteq R$. We can not assume that $t_1(K) \neq t_2(K)$. It follows that functional dependency will be defined by a new definition. Relational algebra generates relational calculus and query language. The result of any query is a relation, too. A query language is a formal system. In the first steps of project developing, designers search for the old project by analogy. The old project is represented as the set of parameters. The set of parameters are stored in data base as tuples. Let us use fuzzy comparison as a degree of analogy of two projects:

$$p \sim q = C_E(x_i(p), x_i(q)) \tag{6}$$

here C_E is fuzzy comparison operation, p and q are the tuples containing parameters $x_i(p)$ and $x_i(q)$.

4 Fuzzy data manager

We realized Fuzzy Data Manager by DBMS Oracle8i using data scheme for fuzzy data representing. We used procedures and triggers of server Oracle8i. The data schema determines the set of special tables for membership functions representing and fuzzy linguistic labels. The set of this tables is called the fuzzy repository. The repository extends data dictionary. The data dictionary stores tables, views, and indexes. All mechanisms of fuzzy data processing are collected in four packets. The packet is data base object. The packet contains types, objects, and procedures PL/SQL. PL/SQL is the procedural extension of query language SQL. These four packets are used for fuzzy relations processing. The fuzzy attribute is represented as a symbol string in tuples. The special trigger inserts, modifies, and deletes data in fuzzy relation. The users can use procedures and functions of Fuzzy Data Manager. We developed special fuzzy client using language Java.

5 Using fuzzy relational data base in CAD-application

We used Fuzzy Data Manager to perform the tasks traditional for CAD. For example, the search for optimal corps dimension. We used special decision making system (DMS). The DMS includes three parts: DBMS; Fuzzy inference system (FIS); System of multi-criterion analyses (SMA). The DMS uses Mamdani's fuzzy inference (Dubois et al. 1980). The FIS gets fuzzy data from the data base. It follows that DBMS should store and process fuzzy data. We integrated DBMS and FIS. Fuzzy DBMS can store crisp technical parameters, fuzzy data, and expert preferences. The users of DMS can use fuzzy search. We developed data schema and we inserted the corps dimension and the radio-elements information old projects to data base. We defined the fuzzy attributes (membership functions and fuzzy labels). DMS can work in two regimes: automatic and interactive. The FIS and DBMS interact automatically. Users work with a data base in interactive regime. They formulate a query using SQL language.

We used Fuzzy Data Manager for representation of different products. Main part of this system is the data base “Product structure”. The fuzzy search by analogy is realized for CAD-application. The data schema “Product structure” is changed dynamically. If the project is developed then the quantity of project parameters grows. We cannot use the structure of a certain product as the structure of a table. We used complex data schema. The first table stores all objects, the second tables stores all possible parameters, the third table stores the values of parameters, and the fourth table describes the structure of certain products. This data schema of data base "Structure of product" allows usto represent crisp and fuzzy attributes, create a unique set of parameters for each object, modify the set of parameters, and save the structure of tables and queries. There are two condition of analogy between two certain products. Firstly, if the parameters of products are analogous then products are analogous. Secondly, if two products have a lot of common elements then we make a conclusion about analogy of the products. The algorithm of fuzzy search by analogy has the following steps.

1. We describe new product as set of parameters and we determine the fuzzy structure of a product.
2. We insert data about the new product in the data base. We can use fuzzy value of parameters.
3. If we find analogous product by parameters then we create a special query
4. The program compares the new product with all old projects.
   a) The value of each parameter is compared with the corresponding parameter value of old products.
   b) The degree of equivalence for each parameter is calculated.
   c) The aggregated equivalence degree of products is calculated.
5. If we find analogous products by structure then we create a special query.
   a) Program intersects the set of elements for the new product with the set of elements for the old product.
   b) The degree of analogy is calculated as cardinality of intersection.

The system chooses product with the maximal degree of analogy. Consider the example. The designer evaluates the quantity of elements on front wainscot to choose a corp dimension. The designer uses linguistic labels. Fuzzy Data Manager finds the records about the corps which meet expert conditions. This example finds the old project with quantity of elements on the front wainscot similar to the new product.

Thus, we obtain a set of analogs. We used one parameter for searching. Let us get the analogs by other parameters.

As a result so, we obtain the second set of analogs. The special SQL-query can intersect these sets of analogs. To consider Fuzzy Data Manager efficiency, we used a big data base and we found the records which satisfies special criterion. We measured time of selecting and quantity of selected elements. We created the table from 90000 records. Each record contains three attributes: integer number from 1 to 90000; the name of parameter (length of string equals from 1 to 26 symbols); the value of parameter (integer number from interval [-21475; 21474]. We selected elements with parameter value "about one hundred". We created the membership function for fuzzy label "about one

```
create view o_by_q as
   select id_parent, sum(q) as sq
      from a_depends
      group by id_parent;
select id_obj, name
   from a_object, o_by_q where
   id_obj=id_parent and name=
   'Front.Wainscot'
and
      fuzzy_math.equal('quality.little',
      sq )>0.6;
```

Fig. 1. Pseudocode sketching "Choose product with small quantity of elements on front wainscot".

Table 1. The analysis of Fuzzy Data Manager efficiency

Total number of records	Time (s)	Number of relevant records	Number of records selected of fuzzy query		
			$\mu_\theta =$ 0.50	$\mu_\theta =$ 0.75	$\mu_\theta =$ 1.00
10,000	17	5	4	3	2
20,000	34	14	10	7	5
30,000	50	25	18	13	9
40,000	67	36	26	20	14
50,000	85	44	31	24	17
60,000	100	47	32	25	18
70,000	116	52	36	27	18

hundred". Then we deleted from our data base 10000 elements and repeated our experiment.

We used computer HP9000 K360. If threshold value be 0.50 then we obtained 71% from relevant elements. If threshold value be 0.75 then we had 53% of relevant elements and if the value of threshold be 1.00 then the answer of data base contained 37% of relevant elements. The elements were sorted according to relevance degree. Table 1 shows that new functional possibilities of fuzzy data server request more time for searching.

6 Conclusion

Our main results are the following.

1. We researched fuzzy relational data model for representation data with fuzzy attributes. Piece-linear approximation usage of membership function distinquishes our model.
2. The extended fuzzy relational data model executes operations (projection, selection, and joining) using fuzzy attributes. Our model allows us to use fuzzy relation and quantifiers in a fuzzy query. We defined predicate of equivalence as membership functions operands intersection height.
3. We developed architecture of Fuzzy Data Manager as a set of the procedures and triggers for Data server Oracle 8i.

Our practical results can be summarized as follows.

1. Fuzzy Data Manager can be used in CAD system. Fuzzy Data Manager is an important part of project repository. Thus, Fuzzy Data Manager can represent the incomplete current project decision. We can find analogous decisions in the repository of old projects by fuzzy queries.
2. Fuzzy Data Manager can be used for doing research on extended fuzzy relational data model. The functional dependencies and normal forms of new data model should be researched. But this will be the topic of another paper.

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Toward Problem of Information Retrieval from Internet

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Abstract: The paper considers some aspects of problem of information retrieval from Internet; the mathematical model of information retrieval systems (IRS) is formulated. The analysis of given model is carried out and inherent lacks of that model are specified. The possible approaches to improvement of efficiency of search are offered.

Keywords: information retrieval, Internet, IRS, agent, fuzzy logic, MAS.

1 Introduction

During many centuries, the Mankind solved the problem of information retrieval. More advanced search methods and techniques are developed with the exponential growth of the amount of data, which was potentially accessible for one person.

Some aspects of the problem of information retrieval from Internet are considered in this paper.

The opportunity of effective search and extraction of the information has the major value for use of Internet's potential. Development of World Wide Web and growth of data level contained in Internet makes a problem of information retrieval more actual.

By some ratings, there are more 4 billion pages in Internet on 2001 [1]; the traffic increased each month by 30%. As predicted Computer Industry Almanac Inc., number of users will increase up to 945 million [2] by 2005.

Observed avalanche growth the Internet has resulted to that existing information retrieval services and systems (IRS) actually did not consult with a problem of information retrieval. Therefore, on some researches, any of the global IRS does not cover more than 16 percent from pages located in World Wide Web. Furthermore, the efficiency of search is very poor.

First of all, it relates with organization of the searching process at IRS [6, 10, 11, 13, 14, 15, 16]. The informational retrieval from Internet is particular case of the problem of informational retrieval. This problem is defined by totality of retrieval objects (for example, books, magazines and newspapers in library; the tables in database) and totality of retrieval actors (for example, library visitor), who looking for necessary data. In the context of the problem of information retrieval from Internet the single document, which is described by unique Internet-address (URL), will be understood as retrieval object, and user, who is carrying out request for retrieval of concrete subset of documents, will be understood as retrieval actor.
The rest of the paper is organized as follows. In the next section, the model of the problem of informational retrieval from Internet is introduced. In Section 3, the analysis of model is carried out. In Section 4, the increase of efficiency of search is considered. In Section 4.1 and 4.2, the possibility of application of Fuzzy Logic and multi-agents systems for the problem of information retrieval from Internet is discussed. The paper is concluded in Section 5.

2 Model of the problem of information retrieval

Consider principles of operation of hypothetical IRS and formalize the problem of information retrieval from Internet on basis of principles analysis.
Any information retrieval system consists of following parts [17]: information retrieval language (IRL); translation rules from natural language to IRL; search algorithms; technical retrievers; document database based on some medium.

Denote all set of documents located in Internet as $\overline{UD} = \{Ud^i\}_{i=1}^{\overline{N}}$, where $\overline{N}$ – quantity of documents.
Information retrieval is based on indexing of documents located in Internet; it is the process of creation of document's image by association of descriptors of the contents (so named terms) with document. Later on, terms are used for document's relevance estimation and received estimations have significant importance for efficiency of IRS.

Document's relevance is similarity of user request and semantic content of document [17].

As been mentioned above, none of the global IRS covers more than 16 percent from pages located in Internet. For concerned hypothetical system denote set of indexing documents as $UD = \{Ud^i\}_{i=1}^{N}$, where $UD \subset \overline{UD}; N = \alpha * \overline{N}, 0 < \alpha \le 1$.

Two types of terms are distinguished: objective and non-objective. Objective terms are inherent the semantic content of document and its selection is trivial. There are author's name, documents URL and date of publication among objective terms. Non-objective terms reflect inherit content of document, also name "the terms of content" is used. Today unambiguous selection rules of non-objective terms and degree of conformity rules are not proposed.

Let $TOb = \{Tob^i\}_{i=1}^{Nob}$ be a set of objective terms and $TSub = \{Tsub^i\}_{i=1}^{Nsub}$ be a set of non-objective terms. Here Nob и $Nsub$ – set's dimensions.

Usually document is associated with the non-objective terms. Such kind of association could be complemented by weight that defines a degree of term's data reflection of the content of document.

Lets $T = Tob \cup Tsub$.

For considered IRS we define indexing operator $Ind : Ud \to T$, which puts in conformity a vector

$$T^i = (Tob^i, Tsub^i) = (Tob_1^i, \ldots, Tob_{Mob}^i, Tsub_{Mob+1}^i, \ldots, Tsub_{Mob+Msub}^i) = (Tob_1^i, \ldots, Tsub_M^i)$$

for any document Ud^i, where $Tob_j^i \subset Tob$, $Tsub_j^i \subset Tsub$; Mob и $Msub$ – quantity of terms those were associated with document, $M = Mob + Msub$.

Furthermore, during indexing the matrix of relation between terms and documents $W = \{w_{ij}\}_{N*M}$ is created. The element of a matrix w_{ij} determines weight of the term j in the document i – the more term's rate in document i and less quantity of documents with term j the more weight of the term j in the document i. Presence or absence of the term is reflected by equalities $w_{ij} = 1$ и $w_{ij} = 0$ accordingly.

Two important parameters affect on efficiency of the system of the indexing. The Completeness of the Indexing (CI) indicates the

degree of the recognition of the document's subject matter. Clearly the full system of the indexing generates set of all terms, which reflect all aspects of the document's themes. The second parameter – Specificity of Terms (ST) – characterizes as far as the spectrum of concepts covered by terms is wide. As result of searching by "broad" terms we will receive many relevant documents along with a significant amount of the irrelevant information. If we use more specific ("narrow") terms we will receive smaller amount of documents and it can be accompanied by missing of some relevant information.
Search process occurs in the following way. User formulates his request, as a rule on natural language. Then this request is converted to terms set, so without loss of generality we can suggest vector Q, which consists of elements of set T, is in parameter of IRS. The vector $R = \{r^i\}_1^N$ is generated as result of search procedure of considered hypothetical IRS. The value r^i registers so far as the document i conforms user's request. Then the document's list $Rud = \{Ud^j \subset Ud \,|\, r^j \geq \gamma\}$ ordered by average weighted accordance coefficient is given to user. The value $0 < \gamma \leq 1$ determines the threshold of average weighted accordance coefficient.
So, we can denote search process as follows:

$$W \times Q = R \qquad (1)$$

Beforehand, vector Q is changed in accordance with matrix W for multiplication. The document's list for user is formed on basis of vector R.
It is obvious, if document contains request's terms and its average weighted accordance coefficient is greater than threshold value γ, then this document is conform to user's request formally. However, this coincidence does not mean that the inherit content of obtained document conforms to user's request. Such situation is known as information noise. When a part of relevant documents does not include in total selection is said about losses of information. The information noise and losses of information could be estimated quantitatively by factors of technical efficiency of IRS – Search Completeness (SC) and Search Precision (SP) [6, 17].

Search Completeness is rate of found relevant documents to common amount of relevant documents in researched space. Search Precision is rate of found relevant documents to common amount of the received documents.

Lets for given query Q operation $rel()$ determine a subset of relevant documents for set $Ud_0 = \{Ud^i | rel(Ud^i, Q) = 1\}$ and for search result $Rud_0 = \{Rud^i | rel(Rud^i, Q) = 1\}$.

In that way, parameters of search completeness and search precision can be calculated as follows:

$$SC = \frac{|Rud_0|}{|Ud_0|};\ SP = \frac{|Rud_0|}{|Rud|} \quad (2)$$

Ideally, value of both parameters should come nearer to 1, but actually it is necessary to make a compromise. Indexing by wide terms gives higher fullness of searching due to loss of accuracy and on the contrary, increase of accuracy of search due to use of "narrow" terms results in reduction of fullness of searching. By this reason, efficiency of many IRS is rated by SP at various values of SC.

In that way, mathematical model considered hypothetical IRS could be denote as set:

$$ISS = \{UD, Ind, T, Q, R\} \quad (3)$$

The most of modern information retrieval system could be described by formulated model (3); search process in those IRS could be described by (1).

3 The analysis of model

Considered model (3) has several essential lacks. First, it is a rather difficult formalization of operation $rel()$. As been mentioned above, practically, the existing informational retrieval systems decide about only formal relevance of the documents for user's query based on some normalized coefficient of agreement. If coefficient's value surpasses the given threshold, the document is admitted relevant and is included to resulting sample. There are various methods of definition of the given coefficient [6]; usually it is calculated as the average sum of the terms taken with weights. However, only user, who initiated search, decides about relevance of the document.

Due to using of parameters of fullness and accuracy of search (2) for estimation of efficiency of IRS, we can consider a problem of improvement of quality of indexing instead of the problem of information retrieval. Indexing always is one of the biggest problems at creation of IRS. The problem of automatic correlation the «high quality» terms with documents is studied many years but remain unsolved yet. High quality search will be impossible until given problem is decided.

Due to the exponential growth of the Internet creation and supporting of an exhaustive index of all contents is not impossible. Moreover, increasing of the sizes of an index will result in significant deterioration of efficiency of search.

Also, one of lack of considered model it is practically full absence of feedback with a user. User operates only one entrance parameter of model (3) – a vector of inquiries (1). Usually, information retrieval starts since inexact and incomplete request, which is gradually specified by a method of iterations. Thus, for increase of efficiency of search user should be able formulate correctly query in language used in IRS. This is so named IRL – information retrieval language.

4 Increase of efficiency of information retrieval

The lacks of modern informational retrieval systems and rapid growth of data level contained in Internet makes the problem of information retrieval more actual.

Since today existing global IRS practically single information retrieval from Internet facility, it is necessary to provide following conditions for increase of effectiveness:

- Maximum full use the possibilities of existing IRS;
- Minimize the losses caused by inherent IRS lacks.

As been mentioned above, it is impossible to obtain high values of fullness and precision of search at the same time. One of the possible variant of decision this problem is generation knowledge base of existing IRS, which description the pair of “search completeness-search precision” for concrete application domain (for example, “Informatics”, “soft computing”).

The reduction of the information retrieval losses can be achieved due to increase of accuracy of translating document's content to IRL [17] (i.e. the problem of increase of indexing accuracy) and in case of similarity of user request and descriptor of content. As a decision of given problem use of "intellectual" assistances between user and IRS may be offered. The assistant must provide the translation user's query on natural language to IRL, performance of grammatical and syntactical checking; replacement of synonyms; thesaurus database maintenance. Some of these abilities are realized in modern information retrieval systems.

The problem of document's relevance estimation relates to problem of increase of indexing accuracy. Every information retrieval system uses its own algorithm of document's relevance estimation and for different IRS those estimations differ from each other for the same document. In that case, as a decision of given problem use of "intellectual" assistances may be offered once again. The assistance must regularize the request results by postulated criteria. Furthermore, it is necessary to take account of human element – so, by some studies [3], most users explore three first search results only. At present some researchers thinks that decision of considered problem should be solved within the framework of two rather new directions: Fuzzy Logic and paradigms of multi-agents systems (MAS) [7].

4.1 Application of Fuzzy Logic for the problem of information retrieval from Internet

Fuzzy logic [4] –a direction of the scientific researches which begun still works of Lotfi A. Zadeh, being an expansion of classical (Boolean) logic and based on the concept of the partial truth. Practically, it is a methodological expansion any specific theory received by fuzzification of its basic objects – the suggestion that characteristic function can take on any values on segment [0; 1], not only 0 or 1.

Fuzziness is shown during informational retrieval from Internet at follow. Firstly, user, when he formulates his request, handles with fuzzy concepts due to polysemy of words. Secondly, relevance of the received result is usually expressed by user in fuzzy ratings, which

are depended on from contents of found documents and preliminary rating of its contents.
The use of Fuzzy Logic possibilities is limited by expansion of binary logic model of information retrieval at modern IRS. However, efficiency of given application is very low [6].
The most obvious application of methods of Fuzzy Logic within the framework of decision of problem of information retrieval is translation of units of natural language to terms of IRL. L.A. Zadeh assumes that modern information retrieval systems should turn to the intellectual systems having ability to deduce new knowledge on the basis of available data in further [18, L.A. Zadeh]. In his opinion, the main problem is concluded by particularity of representations of the available knowledge, in their immanent fuzziness. The qualitative improvement IRS becomes possible only when the problem of integration of information elements will be solved [18, G. Korotkikh]. Today some researchers try to increase efficiency of IRS by development of user's feedback on the basis of methods of Fuzzy Logic [18, R.I John].

4.2 Application of MAS for the problem of information retrieval from Internet

The term "multi-agents systems" is used for a designation of the systems consisting of set of independent essences – agents [5]. Practically in all works, where the term "agent" and his basic properties [8, 9] are defined, a platitude became the remark about absence of a common opinion on this issue. Such kind agents as independent agents, mobile agents, personal assistants, intellectual agents, social agents and etc has occurred during development and realization of systems within the framework of the given direction due to the variety of treatments of the term "agent". Thus, instead of unique definition of the basic agent, there is a set of definitions of derivative types. By analogy with object oriented programming the term "agent" can be interpreted as virtual class (the collection of the virtual methods and properties) [8]; the different realizations of agent are generated on its basis as descendant classes.
Actually, definition of the agent is set by the description of his properties. Usually, agent has the following properties [5]:
- Adaptability – the agent has ability to be trained;

- Autonomy – the agent has an opportunity of independent actions, formulating for itself the purpose and carrying out actions for achievement of objects in view;
- Cooperation – the agent can cooperate with other agents in several ways, for example, playing a role of the supplier/consumer of the information or simultaneously both these roles;
- Communicativeness – agents can communicate with other agents;
- Ability to discourses – agents can have partial knowledge or mechanisms of a conclusion, for example, knowledge how to cite the data from various sources to one kind. Agents can specialize on a concrete subject domain;
- Mobility – the agent has ability to transfer an agent's code from one server on another.

Existing information retrieval systems (for example, Yandex, Rambler, Aport, Google – the most popular IRS in Runet [11]) in the context of a decided problem of information retrieval can be considered as prototypes of search's agents. They have properties of autonomy, cooperation, communicativeness and mobility. The agents of search formed based on one of prototypes, also should have property of adaptability and ability to discourses.

For development MAS, it is necessary to solve the following questions [5, 9]:

- What class of the agent's architecture to use?
- What class of language for agent's communication to use?

There are agents-coordinators (coordination of interactions of other agents), agents-interface (the organization of user's interaction with MAS, MAS with agents), agents-experts (the analysis of a subject domain, a filtration of the received data) and agents of information's extraction (extraction of the data, an estimation and an explanation of relevance) should be realized in MAS with search agents. Itself MAC should have open architecture (should allow to incorporate in process of search heterogeneous agents) and to be scaled.

Thus, among functions MAC it is possible to allocate the following:

- Coordination of works the information retrieval systems;
- Formulation of inquiries in view of specificity of a concrete IRS;
- Maintenance of feedback mechanisms with user;
- Estimation of result's relevance;
- Filtration of the information;
- Creation and storage of the specialized indexes.

5 Conclusions

Today Internet is the huge storage of distributed digitized data; and it is necessary to have effective tools of information retrieval and data processing for optimal using of Internet's opportunity. Low efficiency of modern information retrieval services and systems is main problem for the transformation Internet in an intellectual network. Various methods and approaches within the framework of decision of given problem are offered [12], prototypes of "intellectual" informational retrieval systems [18, Y. Tang and Y. Zhang] are developed. Common factors in these attempts and approaches are using agents and multi-agents systems, application of Fuzzy Logic and technology of distributed calculations.

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New Method for Interval Extension of Leontief's Input-Output Model Using Parallel Programming

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Abstract. New method for interval extension of the sound Leontief Input-Output model used now in more than 70 countries over the world for manufacturing processes optimization, economy condition improvement and intersectors' costs allocation analysis is presented. The method is based on a new approach to the solution of linear equations and systems of equations with interval parameters. An interesting theoretical result is a triangular fuzzy number obtained as a solution of interval equation. In practical calculations such fuzzy solutions may be defuzzified to obtain the resulting crisp interval. It is shown that the proposed method considerably reduces a width of the resulting output interval. The problem is solved with use of parallel algorithms to reduce computational time in complicated input-output analysis. The parallel algorithm was implemented on the base of the effective MPI (Message Passing Interface) library and is tested on so-called supercomputer (cluster), which was built at Technical University of Czestochowa.

Keywords. Input-Output Analysis, IOA, Interval Number, Fuzzy Number, Modified Gauss Procedure, Interval Input-Output Model, Interval Input-Output Table, Parallel Programming, MPI

1. Introduction

Input-output analysis (IOA) formulated by V. Leontief is an extremely effective tool used in more than 70 countries over the world for manufacturing processes optimization, economy condition improvement and intersectors' costs allocation analysis. Firstly IOA had been exploited in United States economy analysis [1-3]. Practical applications of input-output analysis [4-6] have proved its exceptional usefulness in the cost control and in the cost management domain. Leontief's model have been successfully used for costs management in global enterprises' business planning [6]. Nevertheless, the model originally proposed by V. Leontief does not take into account the natural uncertainty of such complicated mathematical descriptions as real-world economy models. Because of disadvantages revealed when using conventional classical stochastic approaches [7,8], the present paper is focused on interval and fuzzy extensions of the classical input-output method.

The idea is not new. The fuzzy extension of Leontief model in a spirit of L. Zadeh [9] was proposed by J. Buckley in [10].

However there are some obstacles in a way of practical embodiment of this approach. The dimensionality problem arises additionally complicated by the interval extension effect that usually leads to final intervals or fuzzy intervals so wide that it makes them virtually senseless. Thus two main problems must be resolved: the resulting intervals (fuzzy intervals) width restriction and the computation speed increasing.

The new method for solving the first problem is represented in Section 2. Section 3 is devoted to the second problem, some numerical examples are considered as well.

2. The fundamentals of "interval extended zero" method

In general case, the effect of interval width increasing cannot be eliminated completely. Nevertheless, it does not mean that we could not attempt to improve interval arithmetic rules to reduce the width of resulting intervals to the maximum possible extent.

The second methodological problem of interval equations solution, which is not widely discussed in scientific literature, we refer to as "interval equation's right hand side problem". Suppose there exists some basic non-interval algebraic equation $f(x) = 0$. Its natural interval extension can be obtained as $[f]([x]) = 0$. Obviously, if $[f]([x]) = [\underline{f}, \overline{f}]$, then equation $[f]([x]) = 0$ is true only when $\underline{f} = \overline{f} = 0$. It is easy to show that equation $[f]([x])=0$ often can be verified only for the inversed interval [x], i.e. when $\overline{x} < \underline{x}$. At present time inversed intervals are analyzed in the framework of the Modal Interval Arithmetic [11], but it is very hard and perhaps even impossible to meet a practice situation when notation $\overline{x} < \underline{x}$ has a meaning.

Let us consider interval extension of simplest linear equation *ax-b=0*

$$[\underline{a}, \overline{a}] \cdot [\underline{x}, \overline{x}] - [\underline{b}, \overline{b}] = 0 \tag{1}$$

Obviously, its solution may be presented as [x]=[b]/[a] and in accordance with the basic definitions of interval arithmetic [12]:

$$\underline{x} = \frac{\underline{b}}{\overline{a}}, \overline{x} = \frac{\overline{b}}{\underline{a}} \tag{2}$$

Formally, when extending equation *ax-b=0* one have to obtain not only interval on its left side but interval zero on the right side, which cannot be the degenerated interval [0,0] in general case. Any zero containing interval may be assumed as such interval zero. It is easy to see that for any interval *[a]* from basic definitions [12] we obtain $[\underline{a}, \overline{a}] - [\underline{a}, \overline{a}] = [\underline{a} - \overline{a}, \overline{a} - \underline{a}] = [-(\overline{a} - \underline{a}), \overline{a} - \underline{a}]$. Thus, if we want to treat a result of subtraction of two identical intervals as an interval zero, the most general definition of such "zero" will be interval symmetrical with respect to 0.It must be emphasized that introduced definition states nothing about the width of interval zero. Hence as a result of the interval extension of *ax-b=0* in general case we obtain

$$[\underline{a}, \overline{a}] * [\underline{x}, \overline{x}] - [\underline{b}, \overline{b}] = [-y, y] \tag{3}$$

In fact, right side of (3) is some interval extension of zero. This is the reason for us to call our approach the "interval extended zero" method. Of course, the value y in (3) is not yet defined which is quite natural since the values $\underline{x}, \overline{x}$ are also not defined.

For the sake of simplicity let us consider only the case of positive interval numbers *[a], [b]* , i.e. $\underline{a}, \overline{a}, \underline{b}, \overline{b} > 0$. Then from (3) we obtain

$$\underline{a} * \underline{x} - \overline{b} = -y, \ \overline{a} * \overline{x} - \underline{b} = y. \tag{4}$$

And finally from (4) we get

$$\underline{a} * \underline{x} - \overline{b} + \overline{a} * \overline{x} - \underline{b} = 0 \tag{5}$$

If there are some restrictions on the values of unknown variables, the Eq (5) along with these restrictions may be considered as so-called Constraint Satisfaction Problem [13] and interval solution may be obtained. The first restriction on variables $\underline{x}$ and $\overline{x}$ is a solution of (5) assuming $\underline{x}=\overline{x}$. In this degenerated case we obtain the solution as $x_m = \frac{\underline{b}+\overline{b}}{\underline{a}+\overline{a}}$.

It is easy to see that x_m is an upper bound for $\underline{x}$ and a lower bound for $\overline{x}$: if $\underline{x}>x_m$ or $\overline{x}<x_m$ we obtain degenerated solution, i.e. $\underline{x}>\overline{x}$.

The natural lower bound for $\underline{x}$ and upper bond for $\overline{x}$ may be derived from basic definitions of interval arithmetic [12] as $\underline{x}>\frac{\underline{b}}{\overline{a}}$, $\overline{x}<\frac{\overline{b}}{\underline{a}}$. Equation (5) with all defined above restrictions on $\underline{x}$ and $\overline{x}$ is a typical Constraint Satisfaction Problem [13]. Its interval solution is

$$[\underline{x}] = [\frac{\underline{b}}{\overline{a}}, \frac{\underline{b}+\overline{b}}{\underline{a}+\overline{a}}],\ [\overline{x}] = [\frac{\underline{b}+\overline{b}}{\underline{a}+\overline{a}}, \frac{\underline{b}+\overline{b}}{\overline{a}} - \frac{\underline{a}\cdot\underline{b}}{\overline{a}^2}] \qquad (6)$$

Expressions (6) define all possible real-valued solutions of Eq. (3). The widest one characterizing by $\underline{x}_{min}=\frac{\underline{b}}{\overline{a}}, \overline{x}_{max}=\frac{\underline{b}+\overline{b}}{\overline{a}}-\frac{\underline{a}\cdot\underline{b}}{\overline{a}^2}$ constitutes interval $[\underline{x},\overline{x}]=[\frac{\underline{b}}{\overline{a}},\frac{\underline{b}+\overline{b}}{\overline{a}}-\frac{\underline{a}\cdot\underline{b}}{\overline{a}^2}]$, which derives the widest interval zero after its substitution in (3). In other words, the maximum interval solution's width, $W_{max}=[\frac{\underline{b}}{\overline{a}}-\frac{\underline{a}\cdot\underline{b}}{\overline{a}^2}]$, corresponds to the maximum value of y: $y_{max}=\overline{b}-\frac{\underline{a}\cdot\underline{b}}{\overline{a}}$. Substituting degenerated solution $\underline{x}=\overline{x}=x^*$ in (3) we obtain the minimum value y: $y_{min}=\frac{\overline{a}\cdot\overline{b}-\underline{a}\cdot\underline{b}}{\underline{a}+\overline{a}}$. It easy to see that for any permissible solution $\underline{x}' > \underline{x}_{min}$ we have corresponding $\overline{x}' < \overline{x}_{max}$, for each $\underline{x}'' > \underline{x}'$ the inequality $\overline{x}'' < \overline{x}'$ and $y'' < y'$. We can see that values y characterize the closeness of right side of Eq.(3) to degenerated zero and that minimum value y_{min} is defined exclusively by interval parameters $[a]$ and $[b]$. Hence the values of y may be considered in a certain sense as a measure of the uncertainty of the interval solution coursed by the initial uncertainty of Eq. (3). Therefore introducing $\alpha = 1-(y-y_{\min})/(y_{\max}-y_{\min})$ we can connect α with each permissible solution of (3), $[\underline{x},\overline{x}]$. The values of α may be treated as the labels of α-cuts representing some fuzzy solution of interval equation (4). Finally, we obtain the solution as a triangular fuzzy number $\tilde{x} = \{\frac{\underline{b}}{\overline{a}}, \frac{\underline{b}+\overline{b}}{\underline{a}+\overline{a}}, \frac{\underline{b}+\overline{b}}{\overline{a}} - \frac{\underline{a}\cdot\underline{b}}{\overline{a}^2}\}$

We can reduce resulting fuzzy solution to the interval one using well known defuzzification procedure. In our case defuzzified left and right boundaries of the solution can be represented as

$$\underline{x}_{def} = \frac{\int_o^1 \underline{x}(\alpha)d\alpha}{\int_0^1 d\alpha}, \quad \overline{x}_{def} = \frac{\int_o^1 \overline{x}(\alpha)d\alpha}{\int_0^1 d\alpha} \tag{7}$$

Since $\underline{x}(\alpha)$ and $\overline{x}(\alpha)$ may be expressed analytically, from (7) we have

$$\underline{x}_{def} = \frac{\overline{b}}{\underline{a}} - \frac{y_{max}-y_{min}}{2\underline{a}}, \quad \overline{x}_{def} = \frac{\underline{b}}{\overline{a}} + \frac{y_{max}+y_{min}}{2\overline{a}}.$$

It is easy to prove that $[\underline{x}_{def}, \overline{x}_{def}] \subset [\underline{x}_{min}, \overline{x_{max}}]$.

The mode $x_m = \frac{\underline{b}+\overline{b}}{\underline{a}+\overline{a}}$ of fuzzy solution $\tilde{x}$ will be useful for our further analysis.

As it has been shown above, x_m is a degenerated solution of Eq. (3), i.e. $\underline{x} = \overline{x} = x_m$. On the other hand, x_m is an asymptotical solution when intervals $[a],[b]$ are contracting to points. Thus x_m plays a role in fuzzy solution similar to a middle point of usual crisp interval.

To illustrate, let us consider simple example: *[a]=[14,60], [b]=[25,99]*. Conventional interval arithmetic (2) results in $[\underline{x},\overline{x}] = [0.42, 7.07]$.

Using new method we obtain $[\underline{x}_{min}, \overline{x_{max}}] = [0.42, 1.97]$ and $[\underline{x}_{def}, \overline{x}_{def}] = [1.04, 1.82]$.

It is easy to see that $[\underline{x}_{def}, \overline{x}_{def}] \subset [\underline{x}_{min}, \overline{x_{max}}] \subset [\underline{x}, \overline{x}]$.

Moreover, the width of $[\underline{x}, \overline{x}]$ is in 4.3 times greater than that of $[\underline{x}_{min}, \overline{x_{max}}]$ and in 8.5 times greater than that of $[\underline{x}_{def}, \overline{x}_{def}]$.

Described above approach was employed for elaboration of the new interval Gauss elimination algorithm for solving of interval extended Input – Output problem without drastic increasing of resulting intervals.
The general representation of the Leontief Input-Output model is

$$xout = (I - A)^{-1} \times f \tag{8}$$

where $(I - A)^{-1}$ is known as Leontief inverse matrix, I is an interval identity matrix, A is a main interval technological coefficients matrix, f is an interval vector of final outputs (usually sales and inventory), $xout$ is a total output products vector.

In our approach mathematically equivalent mode of Eq. (8) is used:

$$(I - A) \times xout = f \tag{9}$$

The method for solving this problem with all the parameters of Input-Output model being intervals is constructed on the basis of classical interval Gauss elimination procedure with the cardinal modification of its backward stage using new "interval extended zero" method.

3. Parallel programming using of Cluster for interval IOA implementation

To solve the problem of interval IOA the effective software was elaborated based on new mathematical tools and the parallel programming technique [14]. An effective library MPI (Message Passing Interface) [15] was used for that purpose. The "master-worker" standard model was used for organization of computations and messages passing. This standard allows to load the balancing of used processors practically uniformly and makes it possible to lower the computation time. The granularity of algorithm and the good efficiency of all nodes of cluster utilization are guaranteed as well. Moreover, only small chunks of main matrix are sent to individual nodes of cluster instead of large data portions. Such organization of Cluster work causes desirable movement of small data packets in fast communication network connecting all nodes of Cluster. Further, the movement of small data packages causes an effect of the adaptation of the amount of executed work to efficiency of individual nodes of cluster, because faster nodes execute more portions of operations. The tests and researches are conducted using computational ACCORD cluster (Academic Cluster of Czestochowa for Research and Education) with distributed memory, built at Technical University of Czestochowa under the management of professor Roman Wyrzykowski. ACCORD cluster is constructed with:

a) nine Intel ISP2150G server platforms. Each of them contains two Pentium III 750MHz processors, 512MB RAM memory and 9GB SCSI UW2 disk memory. One of the nodes plays a role of administration server and is equipped with 1GB RAM memory and 30GB disk memory. Nodes are connected with high-efficiency Myrinet network, which have capacity 2 + 2 Gbps in Full Duplex mode.

b) sixteen nodes with AMD processors. Each node contains two Athlon 1200 MHz MP processors (a half of nodes) or Athlon 2100 + MP processors, 512MB DDRAM memory and 60GB IDE HDD memory. Nodes are connected with Fast Ethernet network.

Let us consider the numerical example of Interval IOA. The data were adopted from work [16] where Interval IOA of real textile-dyeing factory was considered. The selected firm whose capital is close to 150 million NT$ is mainly organized into seven different sectors: two production sectors, finance, managerial, engineering, research and development (R&D), administration.

To estimate the quality of obtained results we propose the special relative index of uncertainty, *RIU*:

$$RIU = \left(\frac{max((x_m - \underline{x}),(\overline{x} - x_m))}{x_m} \right) \cdot 100\% ,$$

where $x_m = (\underline{x} + \overline{x})/2$.

In our case study the maximal *RIU* of the input interval matrix A is about 5% whereas the maximal *RIU* of the output vector is only *5.7%*.

Obviously, such "uncertainty extension" is practically negligible. This is the main advantage of proposed method.

The embodiment of proposed method using parallel algorithms realized on the elaborated Cluster (transputer) allows to reduce the computation time at least in 32 times in comparison with time expenses needed using usual single-processor computer.

Hence, proposed new mathematical method realized on the elaborated modern transputer system may be used as a practical tool for the decision of complicated real-world economy problems on the base of Leontief Input-Output models in uncertain setting.

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The solution of transport problem in fuzzy statement on the basis of platform AnyLogic

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Abstract

The problem of definition of an optimal route for transportation of cargoes in which time of lorry on a way is specified by triangular fuzzy number is put and solved. The optimal solutions make Pareto set in "risk to wreck expenses plan – average expenses for a route" coordinates. The problem was modeled on program platform AnyLogic.

1 Introduction

One of the most frequently meeting problems in real life is the transport problem with fixed schedule of transportation of cargoes from addressees to receivers (from warehouse – to the end user). There are many classical algorithms how to solve transport problem, but they operate with precise (determined) values of parameters. In real tasks it's very often that time of transport on way is not a fixed known value. In such cases it is very hard to bring the task to standard transport algorithms or timetable scheduling algorithms.

At the same time there is a problem for owner of transport networks how to optimize in accordance with two criteria:

- Expected expenses for realization of the business-process of transportation should be minimal;
- Risk to wreck planed task by expenses should be minimal too.

Simultaneous achievement of an optimum by these two criteria is impossible since at the minimal risk the maximal expenses can be reached and vice versa. That's why the solution of optimization problem of cargo transportation is Pareto set of not dominated alternatives of the routes, which does not archive a simultaneous minimum of expenses and risk but which simultaneously dominate all other routes by one of the chosen criteria (expenses or risk).

There are two opened questions: a) how to represent random time for the transportation in mathematical model b) how to estimate the risk. We shall discuss in order.

The time of transport on the way is not classical stochastic variable because there is no classical distribution for events of traffic congestions. The delay time in traffic congestions cannot be described by stochastic variable also. At the same time practice of regular gaugings of time on the way allows to specify three parameters: a) average time on the way; b) maximal time on the way when simultaneously there are some adverse events on a route, causing transport congestions; c) minimal time on the way when transport congestions is not present almost. Such data allow to set the time on the way as triangular fuzzy number [3].

To estimate the risk of wrecking the expenses plan next method is used. On the basis of a transport network modeling the fuzzy number of expenses for a route (which as we show further, has an any kind) is restored. We are set by the specification of the maximal expenses for a route, which acceptable for the owner of a transport network. Then there is a nonzero opportunity that in real expenses will be below the specification. This opportunity can be appreciated on the basis of a method from [1].

At modelling of transportation can be effectively used the model, which was built in the plaform AnyLogic[2]. AnyLogic is a new technology simulation tool. It is based on the latest advances in complex systems modeling theory and working standards in system design. AnyLogic is very different to traditional tools that were developing each around its own paradigm matching a specific narrow application field.

The innovative core technology together with the remarkable set of features makes AnyLogic the best-in-class predictive technology solution for a broad range of real world systems of very diverse nature, including:

- Enterprise-wide process modeling
- System and business dynamics
- Telecom, network and computer systems
- Dynamic systems and control algorithms
- Traffic, transportation, pedestrian movement

- Various military applications

Allowing quick start for new users, having excellent animation capabilities, and including a lot of classical models, AnyLogic is being used in teaching and research by a large number of universities worldwide.

2 Description of business process

The business process of cargo transportation is given on Fig. 1 (in IDEF3 notation), and transport network connecting n of warehouses with m of reception points is represented on Fig. 2. Each warehouse has the schedule of sending cargoes to reception points. There is a common machines park, which at the start moment is distributed among warehouses. If there is no lorry at the moment when according to the schedule it's necessary to send cargo to reception point, the lorry is rented for one run from outside company for the certain payment. After arriving of lorry to destination point, it can be sent to any warehouse, if the lorry was not rented. Rent for lorries is calculated from spent time.

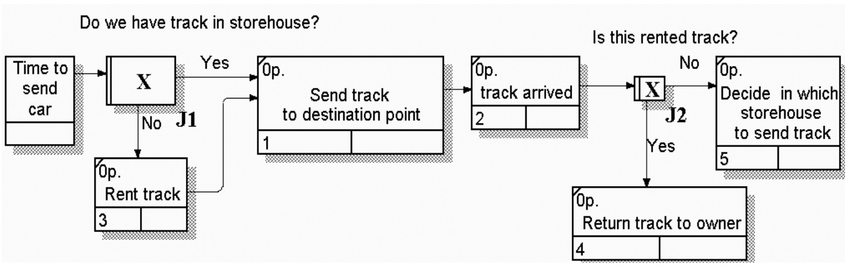

Fig. 1. Business process diagram in IDEF3 notation

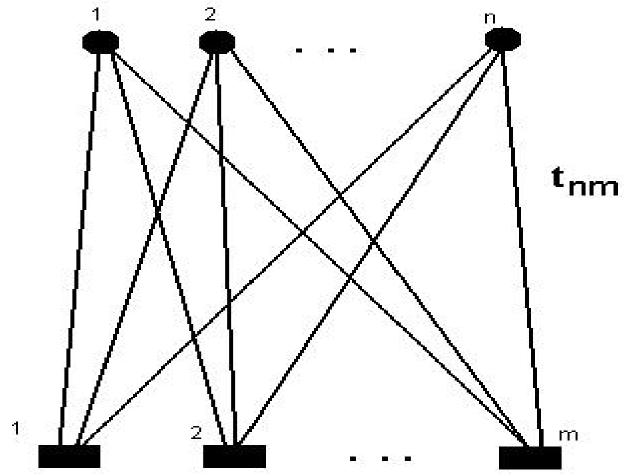

Fig. 2. Task view

3 Mathematical model of process

For the model building it was decided to use AnyLogic platform (the interface of platform is shown on Fig. 4), which allows to build dynamic systems. For simplification of the model it was decided that amount of warehouses equals five and amount of reception points equals four. It was decided that lorry's rent does not depended from length of way on which lorry was rented and rent is calculated according to amount of time on which lorry was rented. The time from warehouse "i" to reception point "j" (tij) is defined by membership function of the following kind (Fig. 3).

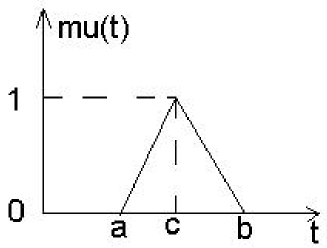

Fig. 3. Membership function of time on way

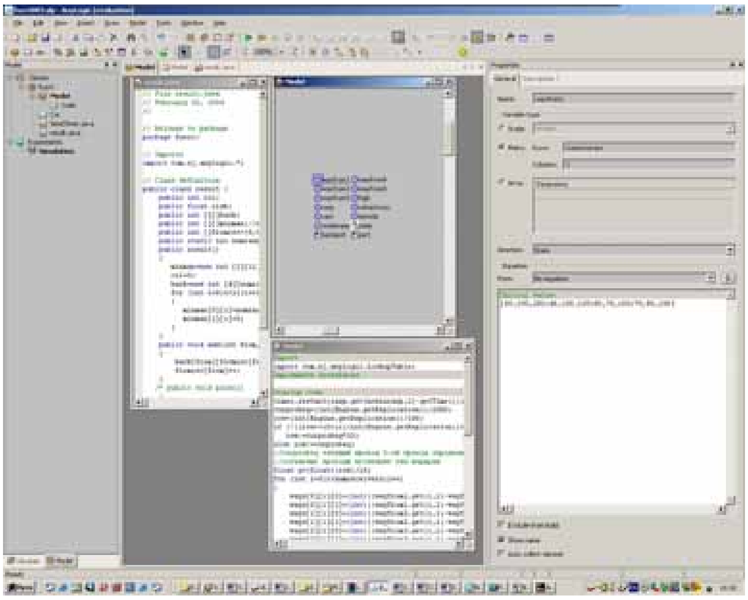

Fig. 4. Model in AnyLogic platform

Since the analytical decision of this problem is impossible, was decided to simulate work of dispatching office. First thousand runs of the common sendings schedule is used to generate a group of routes which are considered as the best according to heuristic algorithm. During these runs time spent on road, is considered as stochastic variable of uniform distribution in appropriate membership interval (it was decided to take 11 levels, see Fig. 5 since in our task last interval degenerates in a point, so only one run is spent on it).

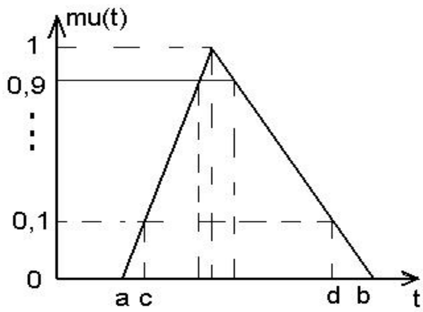

Fig. 5. Membership function divided in levels

At the second stage the model builds membership functions of rent and counts up risk for 50 routes most frequently met at the first stage. For this purpose the model is run 1000 times for each route , in result we receive the minimal and maximal value of a rent for each level of membership function. The fuzzy number of expenses for rent is restored by this way(see Fig. 6).

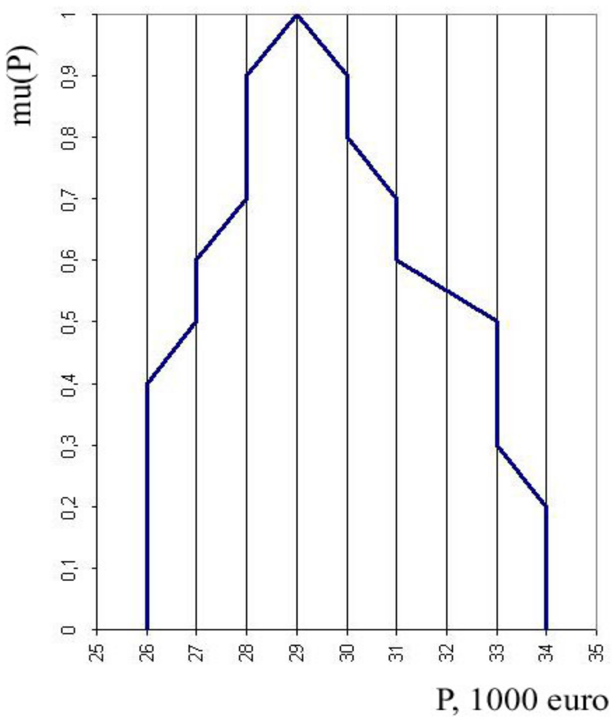

Fig. 6. Membership function of rent payment

4 Statement of optimization problem

It is required to determine all routes in a transport network for which the minimum of expenses for rent of transport is reached at the fixed risk level of wrecking a expences plan. The solution of optimization problem of cargo transportation is Pareto set of not dominated alternatives of the routes, which does not archive a simultaneous minimum of expenses and risk but which simultaneously dominate all other routes by one of the chosen criteria (expenses or risk).

5 Expenses for the process of transportation as fuzzy number

Estimation of expenses

It's expenses membership function for route which was received by its runs for 11 levels of rent membership function, in result segment interval of expenses for each level is defined.

Rent expenses calculates by the following formula:

$$P = t_{rent} * K, \tag{1}$$

where K is rent for one hour, t_{rent} is time of rent in ours(fuzzy number determined in result of modelling)

Estimation of risk to exceed budgetary restriction

The risk of such event, that the expected size of expenses for cargo transportation appears above the budgetary restriction, calculates by the approached formula [1]:

$$Risk = \Delta\alpha \sum_{i=0}^{9} * \left(p_{\max}(i) - p_{\lim it}\right) / \left(p_{\max}(i) - p_{\min}(i)\right), \tag{2}$$

where $P_{max,min}(i)$ is the borders of segment interval of expenses on level „i“, P_{limit} is maximum of of expenses budget, $\Delta\alpha$=0.1, $P_{max}(i) > P_{limit.}$

6 Allocation of Pareto set of optimal solutions

As a result of work we receive set of routes for each of which average expenses and risk to excess of the certain level of expenses are known. On the basis of these data it is possible to construct Pareto set (see Fig. 7).

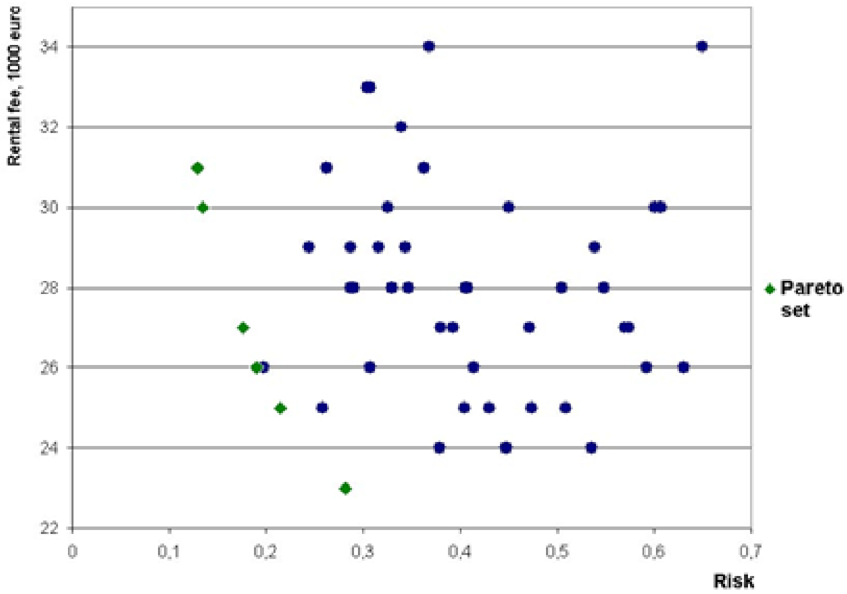

Fig. 7. Pareto set

7 Conclusion

In situations where essential informational uncertainty of non-statistical nature takes place, application of interval and fuzzy estimations of the initial data is the most justified way. And at the decision of a transport problem in fuzzy statement, it is possible to optimize a transport route by allocating Pareto set of decisions which are characterized by minimum levels of expenses for a route at the fixed risk level to excess budgetary restriction on the size of expenses. Imitation of transport streams is convenient to make by creating models on platform AnyLogic.

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On the Application of Fuzzy Sets Theory to Russian Banking System Fragility Monitoring

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Abstract: There is an extensive work that has been done last two decades in monitoring and prediction of macroeconomic systems stability. These problems require taking a proper account of many factors affecting the trends of social-economic development. In our research we develop some fuzzy indicators to monitor the fragility of the Russian banking system. A fuzzy method for group expert estimation is also discussed.

1 Introduction

The problem of financial systems fragility has become an important issue since several economic crises has occurred in second half of the last century (Sweden, Norway, 1990; Mexico, 1994; Jamaica, 1996; Indonesia, Thailand, the Philippines, South Korea, 1997; Russia, 1998). Some aspects of financial systems fragility modeling are discussed in comprehensive studies by Chari (1989), Demirguc-Kunt, Detragiache (1999), Barth et al. (2000), Bertrand (2000), Kaufman, Seelig (2000), etc. One of the most important parts of these studies is systemic risks assessment. The variety of risks includes credit solvency, devaluation of the national currency, liquidity crises and many others. Combining different risk factors into integral indicator often is a problem of aggregation of quantitative information that may have different units of measure thus making the usual multivariate regression less substantial. This problem may be solved by methods of fuzzy sets theory.

2 Procedure

We apply the approach described in (Nedosekin, 2000) to construct the integral rate of Russian banking system fragility. The factors of this indicator are:

- volatility rate of average margin of the banking system's consolidated portfolio (interest rate risk);
- volatility of the RUR to USD exchange rate (currency devaluation risk);
- insolvent credits ratio (credit risk);
- short-term liquidity gap coefficient (liquidity risk).

The full set of banking system states is divided into five groups (the so called linguistic pentascale):

- stable (risk level is very low);
- close to stable (risk level is low);
- transitional (risk level is moderate);
- fragile (risk level is high);
- crisis (risk level is very high).

For every factor we define the function of membership to every level of pentascale. First of all we define scalar nodal points, for example, USD/RUR exchange rate volatility of 1% per month corresponds to a low level of risk. Then we construct the membership functions so that they conform to a Pospelov's grey scale (see (Pospelov, 1994) for details). For trapezoid functions one can use the algorithm described in (Nedosekin, Frolov). The symmetric Gaussian functions also allow constructing a grey scale. We have to define the standard deviation of the first level Gaussian, other standard deviations can be derived from recurrent formula:

$$\sigma_{k+1} = \frac{\mu_{k+1} - \mu_k}{\sqrt{2\ln(2)}} - \sigma_k \ ,$$

where μ_k - nodal point, σ_k - standard deviation of the k-th function.

The recurrent formula can be obtained from the equation describing the intersection point of the neighbor membership functions:

$$G(x_k; \mu_k, \sigma_k) = G(x_k; \mu_{k+1}, \sigma_{k+1}) = 0.5 \, ,$$

where $G(x; \mu, \sigma)$ is a Gaussian with mean μ and standard deviation σ.

The intersection point x_k is derived as $x_k = \dfrac{\mu_k \sigma_{k+1}^2 - \mu_{k+1}\sigma_k^2}{\sigma_{k+1}^2 - \sigma_k^2}$, thus,

$$\exp\{-\frac{\mu_+ - \mu_{k+1}}{\sigma_{k+1}^2 - \sigma_k^2}\} = 0.5 \, .$$

For example let the nodal points for insolvent credits ratio be defined as: 1% for very low risk, 2% for low risk, 5% for moderate risk, 10% for high risk, 15% for very high risk. With a given standard deviation of the 1st level Gaussian as 0.004, calculated standard deviations for other levels will be 0.0045 for low risk, 0.0209 for moderate risk, 0.0215 for high risk, 0.0209 for very high risk (see Figure 1).

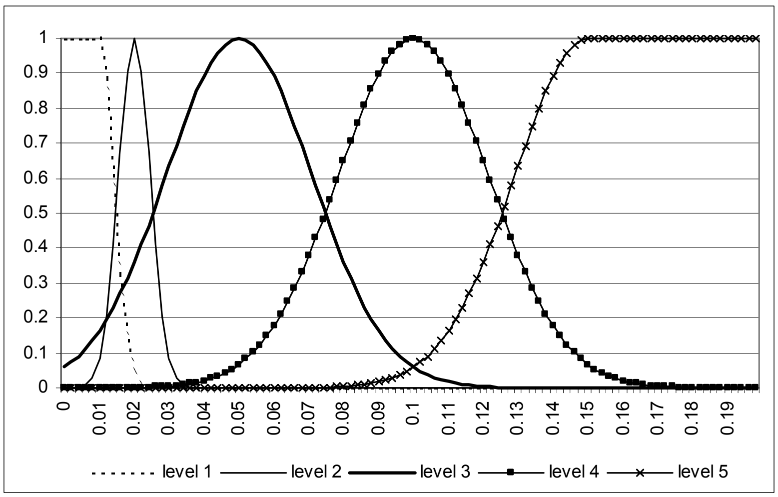

Figure 1. Pospelov's grey scale for Gaussian membership functions

On the next step of the procedure we calculate the scalar values of membership functions for the actual values of components. These values are used when fragility rate is calculated as weighted average of equiprobable fuzzy sets. The result is fuzzy set, which is defined on [0,100] interval, where 0 stands for the lowest and 100 for the highest risk level. The example of fuzzy fragility rate for Russian banking system in the second fall of 2003 is shown on Figure 2. Monitoring the scalar center or the degree of membership to risk levels allows to make judgment on fragility level's slight increase during 2003 mainly due to USD exchange rate volatility.

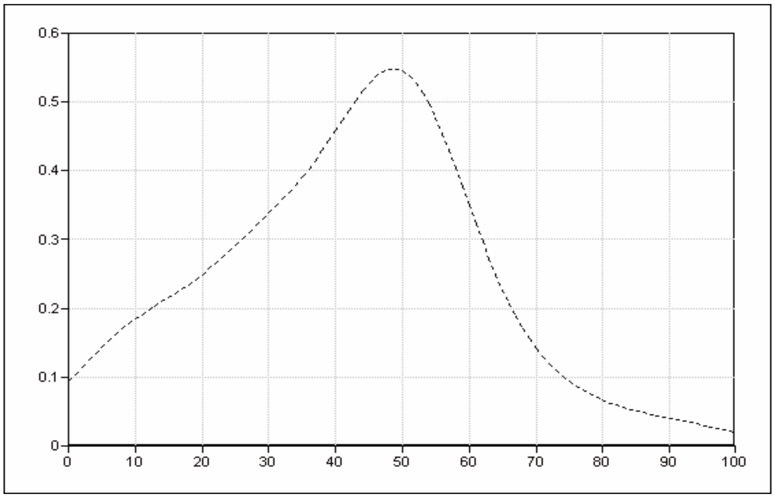

Figure 2. Russian banking system fuzzy fragility rate in 2003

This risk assessment is based on the official banks accounting data thus "looks into the past". The problem of fragility prediction is much the problem of factors forecast. Numerous methods exist to deal with this uncertainty. Some of them are based on expert estimations. The usual quantitative expertise procedure includes gathering expert judgments (usually as a scalar or interval numbers), aggregation of variables and analysis of concordance between experts. This procedure can involve fuzzy theory methods that extend the scalar/interval approach.

Let $\mu_{A_i}(x)$ stand for membership function of fuzzy estimation A_i given by i-th expert for a quantitative factor $x \in B \subset R$, $\mu_{A_i}(x) : B \to [0,1]$. Average expert estimation Q is convex combination of A_i: $\mu_Q(x) = \omega_1 \mu_{A_1}(x) + ... + \omega_n \mu_{A_n}(x)$, where ω_i stands for i-th expert weight ($\sum_{i=1}^{n} \omega_i = 1$).

We developed iterative procedure that updates weights of the experts (originally equal) based on the accuracy of their estimates and other criteria when factual data become available.

Scalar average estimation can be derived as a scalar center of Q, which for a trapezoid membership is equal to: $\hat{Q} = \frac{1}{4} \sum_{i=1}^{n} \omega_i \left(a_i^1 + a_i^2 + a_i^3 + a_i^4 \right)$, where a_i^j stands for the nodal points of i-th expert trapezoid estimation.

After that one can ascertain distant estimations by computing the Hemming or other distance between A_i and Q. Most distant estimations are left for reconsideration or excluded from average estimation. Average expert estimation for economic factor can then be used in a fuzzy fragility rate calculation procedure.

3 Results

The described procedures are implemented in a software system developed for The Central Bank of Russian Federation to monitor the fragility rate of Russian banking system on a regular basis. Extensive calculations have showed that the approach requires accurate expert parameters estimation as results may vary greatly depending on the parameters of membership functions. Further research is being carried out towards correct parameters estimation based on macroeconomic data of countries that experienced crises.

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A fuzzy model of productivity control

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Summary:

There is a problem with the optimal use of factors influencing a productive final effect based on the assumption that a network of devices (divisions) contains nodes, the capacity of which can be controlled. In the simplified variant it is assumed that only an association of the human factor (in this case influencing a production directly) and the equipment factor are effective in the productivity result. In this example, the human factor can also be used in other nodes (the condition: an excess of human resources and an existence of suitable equipment reserves in the “new” node), and the equipment factor can be used for “processing” of other products (the condition: an excess of equipment resources in the “old” node and an existence of the suitable human resource reserves). Of course, arguments relating to concrete resources and reserves apply only to the chosen moment, which will be called *the observation moment (monitoring)*.

Key words: fuzzy models, productivity, control.

Monitored input data for productivity control

The monitoring system assumes the determination of the constant observation step Δt (observation unit), for example, in a time dimension. The demand for manufactured products, human and equipment resources and reserves will be observed (see Figure 1). Discrete data are subjected to linearization to obtain a greater legibility. It will be possible to determine the productivity relating to the people and equipment on the basis of this information (Figure 2). The observation of the time period of an effective work *t1*-*t8* was included (see Figure 1). The first job order *s1* at the *t1* moment is received. A member of personnel begins the realization of the order at the moment *t1*. Suitable devices, which are operated by “new” workers (that is seen in diagram **T** in the time period *t1*-*t2*), are successively set in motion as successive processing stages are started. Similarly, the end of the order realization is connected with “releasing” the devices and workers operating them. At the moment *t3* (i.e. before the end of the order realization *s1*) the next order *s3* appears. However, the personnel size increases stepwise earlier, i.e. at the moment *t2*, which also causes a step both in the diagram of recourses and in the diagram of reserves (**R** and **T**). The personnel size reduces itself nonlinearly in the period time *t5*-*t7* which obviously influences in a non-linear way the current reserves relating only to the staff (**T**), because the production can not slow down from the assumption. The order realization *s3* finishes at the moment of *t4* which causes the human and equipment resources and reserves to become equal (*Rp* = *Tp*, *Rh* = *Th*).

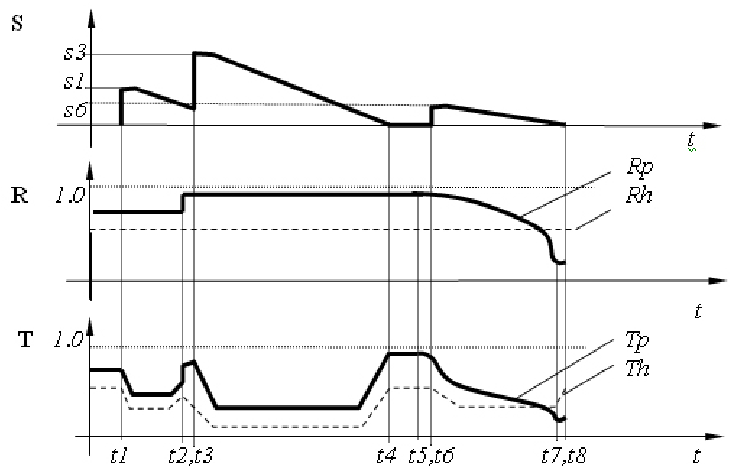

Fig. 1. An exemplifying diagram of the demand variation on products (S), resources (R) and reserves (T) of equipment (Rh,Th) and humans (Rp,Tp) (here: the product making reduces the demand, as that satisfies it)

The diagrams of resources and reserves are presented in the scale from *0* to *1*, which makes the direct evaluation of the surplus of human or equipment resources impossible. The specific character of the *Rp* curve may be well founded by the distribution of the order appearing in the time period *t1-t8*. Nevertheless, as can be seen in **T** diagrams**,** the degree of resources use is not high. The total “allocation” of the equipment (lack of reserves) and staff ($Tp \cong 0$, $Th \cong 0$) is an ideal situation. The higher level of the equipment use (smaller reserves) in comparison with human resources use shows in the surpluses of human resources. Similar arguments concern the use of the staff and equipment resources. The use of statistical and averaging parameters appears to be a purposeful procedure and well founded, because by operating the human and equipment resources it is possible to increase the efficiency of their use.

Algorithmization of productivity parameters

Usually equipment and human productivity are treated independently although, as is well known, they can be easily combined each other. Considering the concrete order one becomes aware that at a certain level of employment and equipment highest efficiency with reference to the time or to obtained financial effects will be reached. Determining the value of orders on a certain level, a description of the equipment and employees productivity can be completed in the following way:

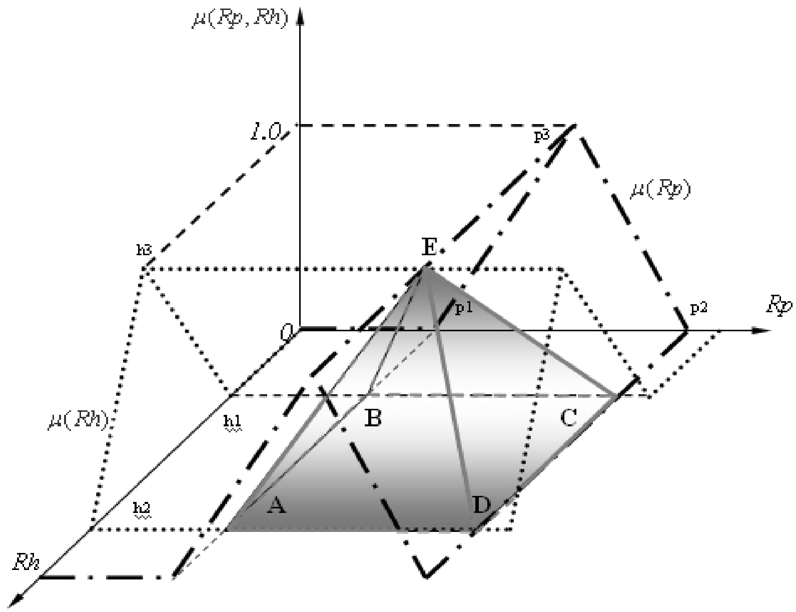

Fig. 2. Dependence of the averaging relative productivity $\mu(Rp, Rh)$ on the human (Rp) and equipment (Rh) resources for the fixed orders level

The situation presented in Figure 2 can be described as follows: beginning from a certain level of employment (point p1) and equipment resources (point h1) it is possible to start the production process. Reaching the optimal productivity (point **E**), which may have a linear or non-linear course, the human and equipment resources are increasing. The resources surplus begins to appear after the projection of the level of the average optimal relative productivity (points p3 and h3). Acceptation that at points p2 and h2 zero level will be reached is an obvious simplification. The obtained 0,p1,p3,p2 and 0,h1,h3,h2 diagrams can be interpreted as fuzzy sets which membership functions of the triangular type class and can be described as follows [8]:

$$\Lambda\, p_{p1,p3,p2}(Rp) = \begin{cases} 0 & \text{for } (Rp <= p1) \text{ or } (Rp >= p2) \\ \dfrac{Rp - p1}{p3 - p1} & \text{for } (Rp > p1) \text{ and } (Rp <= p3) \\ \dfrac{p2 - Rp}{p2 - p3} & \text{for } (Rp > p3) \text{ and } (Rp <= p2) \end{cases} \qquad (1)$$

$$\Lambda\, h_{h1,h3,h2}(Rh) = \begin{cases} 0 & \text{for } (Rh <= h1) \text{ or } (Rh >= h2) \\ \dfrac{Rh - h1}{h3 - h1} & \text{for } (Rh > h1) \text{ and } (Rh <= h3) \\ \dfrac{h2 - Rh}{h2 - h3} & \text{for } (Rh > h3) \text{ and } (Rh <= h2) \end{cases} \quad (2)$$

Denote by $\mu(Rp)$ and $\mu(Rh)$ the productivities with reference to the equipment and staff (see Figure 2) and by $\mu(Rp, Rh)$ - the interrelationship between these characteristics. The connection diagram of these productivities assumes the pyramid shape with the **A,B,C,D** base and vertex **E**. This graph is obtained by the use of cylindrical extensions and projections [6]. The estimation of the current value of the mutually related characteristics is shown in Figure 3.

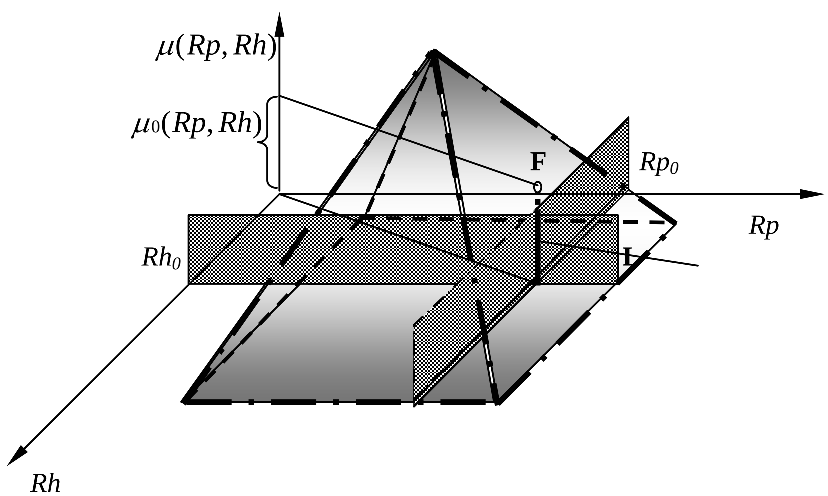

Fig. 3. Determination of the values of the mutually related productivity characteristics $\mu_0(Rp, Rh)$ relating to the workers and equipment, where Rp_0 is the current level of human resources and Rh_0 is the current level of equipment resources

The planes determined by the levels Rp_0 and Rh_0 intersect along the straight line **L** which intersects the “productivity” pyramid at point **F** lying on the one of the lateral faces (see Figure 3). This point determines the value of the mutually related productivity characteristics $\mu_0(Rp, Rh)$. Thus, one of two possible situations always occurs:

$$\mu_0(Rp, Rh) = \mu(Rp_0) = \Lambda\, p_{p1,p3,p2}(Rp_0) \quad \text{or}$$
$$\mu_0(Rp, Rh) = \mu(Rh_0) = \Lambda\, h_{h1,h3,h2}(Rh_0) \tag{3}$$

The determination of which becomes punctured by the straight line **L** is a trivial problem and it can be reduced to the observation from the top in which part the considered point **F** is located. A clearer picture of this situation can be obtained from the top (see Figure 4).

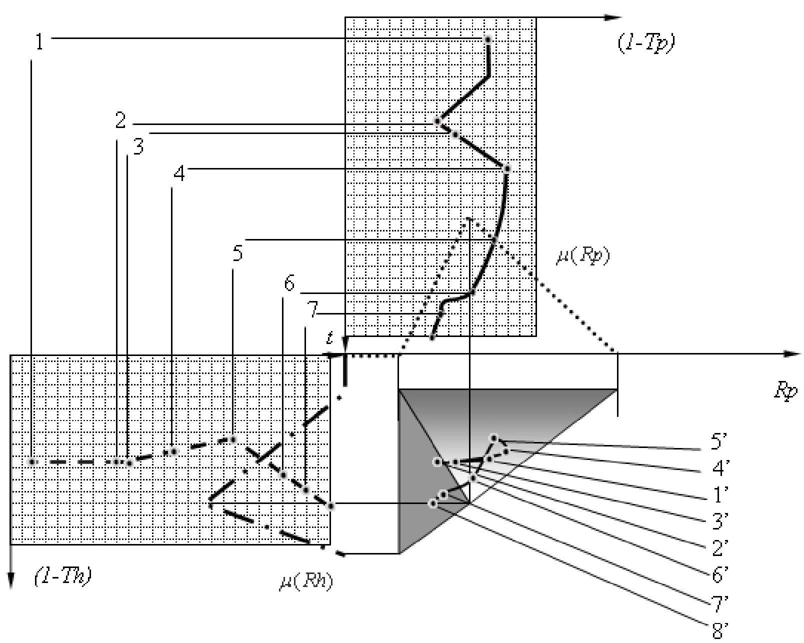

Fig. 4. A variation course in time of the mutually related productivity characteristics on the basis of the data on the current use of the staff and equipment: (*1- Tp*), (*1 – Th*). The observation time - 30 o. u. (temporal observation units)

The analysis results and values of the $\mu_0(Rp, Rh)$ function are presented in Table 1.

The **ABE** or **CDE** plane determines the function description of the mutually related productivity characteristics depending on the place of coordinates intersection relating to the level of equipment and staff use (Figure 2). In order to obtain a more real description of the situation instead of using the static data *Rp* and *Rh* (which, to a large degree, can pass even as fictional and not fulfilling the current situation) it is necessary to refer to the actual information on the staff and equipment involved at a given moment: *(1 - Tp)* and *(1–Th).* Thanks to this, it is possible to obtain the actual values of the $\mu_0((1-Tp),(1-Th))$ function instead of the $\mu_0(Rp, Rh)$ (which notation was used only for the simplification and sequencing of this argument).

Table 1. The analysis results and values of the $\mu_0(Rp, Rh)$ function

	$\mu(Rp)$	$\mu(Rh)$	$\mu(Rp)$	$\mu(Rh)$	$\mu(Rp)$	$\mu(Rh)$	$\mu(Rp)$	$\mu(Rh)$	$\mu(Rp)$	$\mu(Rh)$	$\mu(Rp)$	$\mu(Rh)$	$\mu(Rp)$	$\mu(Rh)$	$\mu(Rp)$	$\mu(Rh)$
1	**X**	**0.57**														
2			**0.46**	**X**												
3					**X**	**0.57**										
4							**X**	**0.51**								
5									**X**	**0.38**						
6											**X**	**0.74**				
7													**0.61**	**X**		
8															**0.45**	**X**

The input data are diagrams of the staff and equipment variation in time: (*1-Tp(t)) (1-Th(t)).* They are equipped with graphic networks giving facilities for the reading out of the time at the monitoring moment and the engagement function values. The numbers from 1 to 8 are denotations of the chosen or characteristic points (e.g. break down of a line) that are used for further productivity analysis. The denotations from 1' to 8' are the analysed points representation on the "pyramid productivity".

Table 1 shows the concrete values of the productivity function with the simultaneous mark from which pair of the planes they come, i.e. which productivity (equipment or personal: $\mu(1-Tp), \mu(1-Th)$) will be used as the formulating or predominant.

Inference elements on the basis of fuzzy conception of productivity evaluation

An inference in the aspect of the productivity control will concern the selection of personnel and equipment resources (as well as even other, e.g. organizational, psychological) that guarantee the maximal productivity. A defusification procedure can be helpful [8]. This is the sharpening procedure thanks to which the concrete parameter values determining optimal productivity will be obtained. There are many defuzzification methods [8], from which it is necessary to select this one, as it is most commensurate to the considered situation. The maximum centre method or the centre of gravity method can be proposed, too. The selection of this or another method should be preceded by practical tests appointing the most favourable for the sake of solution efficiency. Applying the procedures presented above it is possible to determine the membership functions of parameters influencing the productivity: $\mu(Rp)$ and $\mu(Rh)$.

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Regression Prediction Models with Fuzzy Structure of Adaptive Mechanism

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Introduction

Using in forecasting practice models have, as a rule, a rigid structure of adaptive mechanism. This fact limits opportunities of a model's adjustment to actual data and doesn't guarantee adequate accuracy of prognosis. To improve adaptive properties authors suggest a specific method, which allows constructing a regression prediction model with fuzzy structure of adaptive mechanism.

1. Adaptive Regression

Adaptive mechanisms in formation of multifactor prediction models are needed when factors' effect on a problem variable depends on time, i.e. when a model with coefficients changing with time is required for a real process for reaching adequacy. In a general case such a model can be written as

$$y_t = \mathbf{x}_t \mathbf{B}(t) + \varepsilon_t \tag{1}$$

where

y_t – dependent value at the moment t;

$\mathbf{x}_t = (x_{1t}, x_{2t}, \cdots, x_{mt}) - m$ - vector $1 \times m$ of independent variables (factors) at the moment t;

$\mathbf{B}(t) = (b_{1t}, b_{2t}, \cdots, b_{mt}) - m$ - vector $m \times 1$ of estimated model coefficients changing their values with time according to an unknown law;

ε_t – unobserved random error.

For convenience sake a free model member is not separately isolated but it is implicitly considered that the first component of the vector-line of independent variables is equal to unity, i.e. represents a dummy variable $(x_t = 1, \quad t = 1, 2, \ldots, T)$.

To take into account a changing character of coefficients we suppose that each set of current values of these coefficients provides a high approximation precision only to current values and close to them ones leaving it low for previous values. Actually, in terms of the theory of static regression analysis this supposition is equivalent to heteroscedasticity, i.e. to the case when, unlike homoscedasticity, the ratio

$$M(\varepsilon_i \varepsilon_j) = \begin{cases} \sigma_i^2, \; i = j \\ 0, \; i \neq j \end{cases} \tag{2}$$

determines a covariance matrix, though diagonal by itself, but having unequal to each other diagonal elements with $\sigma_i^2 > \sigma_j^2$, if $i < j$. To align the approximation precision over the whole considered period of time and thus reducing the task to an accepted method of weighted least squares we introduce a weight function

$$\mathrm{f}(t, j, \alpha) = \alpha^{t-j}, \quad 0 < \alpha \leq 1, \tag{3}$$

which facilitates the exponential smoothing of deviations.

With unequal results and given values of the weight function the estimation vector $\hat{\mathbf{B}}(t, \alpha)$ of the model (1) coefficients for any $t \in [T_1, T]$, if $T_1 < m$, can be obtained as a solution of an extremum problem

$$\hat{\mathbf{B}}(t, \alpha) = Arg\min \sum_{j=1}^{t} \alpha^{t-j} [y_j - \mathbf{x}_j \mathbf{B}(t, \alpha)]^2, \tag{4}$$

the functional of which is an exponentially weighted sum of deviations of calculated values from actual ones.

Because of the approximation precision shift to the results with high weight coefficients the model with coefficients obtained in a solution of

(4) makes it possible to consider in calculation of predictable values mainly the tendencies revealed in last values. If such a model is formed successively for each $t \in [T_1, T]$ we deal with the current regression analysis, which helps to trace the dynamics of coefficients of regression and understand its character, which is generally the aim of solution of a dynamic sum. However, it's not suitable for calculations as it implies their full each step reiteration. Much more preferable for these purposes is a recurrent least squares method which excludes unwanted reiterations in calculations and actually presents a basis for the formation of adaptive mechanism of a regression model. It's convenient to use the recurrent method of ordinary least squares when solution of the extremum problem (4) is given in vector type

$$\hat{\mathbf{B}}(t,\alpha) = \left(\mathbf{X}_t'\mathbf{L}_t\mathbf{X}_t\right)^{-1}\mathbf{X}_t'\mathbf{L}_t\mathbf{y}_t\,, \tag{5}$$

where

$\mathbf{y}_t$ – vector-column with components equal to predictable values in successive points of time;

$\mathbf{X}_t$ – matrix with elements equal to factor values in successive points of time;

$\mathbf{L}_t$ – diagonal matrix of weight coefficients.

To express current estimations of $\hat{\mathbf{B}}(t,\alpha)$ through values of a previous period $\hat{\mathbf{B}}(t-1,\alpha)$ and a current value presented by values of the dependent variable y_t and vector of independent variables $\mathbf{x}_t$ at the point of time t. We offer solution (5) in a form convenient for the recurrent scheme

$$\begin{aligned}\hat{\mathbf{B}}(t,\alpha) = [\alpha\mathbf{X}_{t-1}'\mathbf{L}_{t-1}\mathbf{X}_{t-1} + \mathbf{x}_t'\mathbf{x}_t]\times \\ \times[\alpha\mathbf{X}_{t-1}'\mathbf{L}_{t-1}\mathbf{y}_{t-1} + \mathbf{x}_t'y_t].\end{aligned} \tag{6}$$

Later, introducing value

$$\mathbf{C}_{t-1} = \mathbf{X}_{t-1}'\mathbf{L}_{t-1}\mathbf{X}_{t-1} \tag{7}$$

and using the Sherman – Morrison formula for recurrent inversion of matrixes [2]

$$\left(\mathbf{C}_{t-1} + \mathbf{x}_t'\mathbf{x}_t\right) = \mathbf{C}_{t-1}^{-1} - \frac{\mathbf{C}_{t-1}^{-1}\mathbf{x}_t'\mathbf{x}_t\mathbf{C}_{t-1}^{-1}}{\mathbf{x}_t\mathbf{C}_{t-1}^{-1}\mathbf{x}_t + 1}, \tag{8}$$

we obtain the recurrent scheme of estimation of current model coefficients (1) which essentially represents the adaptive mechanism of regression model. On the whole a regression model with such an adaptive mechanism can be written as

$$\hat{y}_t = \mathbf{x}_t \hat{\mathbf{B}}(t-1,\alpha) \tag{9}$$

$$\hat{\mathbf{B}}(t,\alpha) = \hat{\mathbf{B}}(t-1,\alpha) + \frac{\mathbf{C}_{t-1}^{-1}\mathbf{x}_t'}{\mathbf{x}_t \mathbf{C}_{t-1}^{-1}\mathbf{x}_t' + \alpha}\left[y_t - \hat{y}_t\right] \tag{10}$$

$$\mathbf{C}_t^{-1} = \frac{1}{\alpha}\left[\mathbf{C}_{t-1}^{-1} - \frac{\mathbf{C}_{t-1}^{-1}\mathbf{x}_t'\mathbf{x}_t\mathbf{C}_{t-1}^{-1}}{\mathbf{x}_t \mathbf{C}_{t-1}^{-1}\mathbf{x}_t' + \alpha}\right]. \tag{11}$$

With given initial values $\hat{\mathbf{B}}(0,\alpha)$, $\mathbf{C}_0^{-1}$ (usually determined with OLS) and known parameter of smoothing $\alpha = \alpha^*$ the model (9)-(11) with the access of “fresh” data makes it possible to renovate coefficients and, considering this renovation, continue respective calculations of values $\hat{y}_t$.

2. An Adaptive Regression Model with Vaguely Defined Response Level

Alongside with acknowledged advantages the adaptive regression analysis given by (9)-(11) has a number of drawbacks the main of which being that generated by the adaptive mechanism model response often aims at higher approximation precision which is damaging for reliability of prediction calculations. Externally this drawback is revealed in an excessively high level of response

$$u_t = \mathbf{x}_t \hat{\mathbf{B}}(t,\alpha) - \mathbf{x}_t \hat{\mathbf{B}}(t-1,\alpha) \tag{12}$$

to any external effects $\mathbf{x}_t$ and a vibrating model performance. It naturally reduces reliability of prediction estimations and therefore requires a reasonable modification of an adaptive mechanism.

To stabilize the effect we introduce into the functional of (4) an additional component decreasing the level of response. Reasonably, the sum of response squares can be used as such a component. This modification leads to a situation when estimations $\hat{\mathbf{B}}(t,\alpha,\gamma)$ can be obtained as the solution of (4) with combined functional

$$\hat{\mathbf{B}}(t) = Arg\min\{(1-\gamma)\sum_{j=1}^{t}\alpha^{t-j}[y_j - \mathbf{x}_j\mathbf{B}(t)]^2 + \\ +\gamma\sum_{j=1}^{t}\alpha^{t-j}[\mathbf{x}_j\hat{\mathbf{B}}(t-1) - \mathbf{x}_j\mathbf{B}(t)]^2 . \tag{13}$$

Parameter $0 \le \gamma \le 1$ regulates the contribution of stabilizing component into a joint criterion of estimation of adaptive model coefficients. A recurrent process of weighted least squares in a considered case can be given by

$$\hat{\mathbf{B}}(t) = \hat{\mathbf{B}}(t-1) + \frac{\mathbf{C}_{t-1}^{-1}\mathbf{x}_t'}{\mathbf{x}_t\mathbf{C}_{t-1}^{-1}\mathbf{x}_t' + \alpha}(1-\gamma)[y_t - \hat{y}_t], \tag{14}$$

in which parameter γ decreases the response level. The model with such an adaptive mechanism naturally provides a higher reliability of prediction calculations for which we need an optimal value of γ. Used for this purposes process of selection from postprediction values enables us to determine the value of γ^* which essentially can be considered only approximately optimal. Such uncertainly in defining an optimal value of the parameter is caused by the indefiniteness of future. Most probably the true optimal parameter value will be different from the one, defined by postprediction calculations. It's a serious argument for the fact that parameter γ can be a fuzzy number of a triangular type which can be verified both by the way of experiment and by any other way. So, the model whose coefficients are corrected by the adaptive mechanism (14) with vaguely determined parameter γ will be called an *adaptive regression model with vaguely defined response level.*

3. A Model with Fuzzy Structure of Adaptive Mechanism

This adaptive regression model can be further improved by the introduction into the functional one more stabilizing member, protecting the model from oversensitivity to peak values [1]. The model coefficients in this case are determined as solution of the extremum problem

$$\hat{\mathbf{B}}(t) = Arg \min\{(1-\gamma-\beta)\sum_{j=1}^{t}\alpha^{t-j}[y_j - \mathbf{x}_j\mathbf{B}(t)]^2 +$$
$$+\gamma\sum_{j=1}^{t}\alpha^{t-j}[\mathbf{x}_j\hat{\mathbf{B}}(t-1) - \mathbf{x}_j\mathbf{B}(t)]^2 + \tag{15}$$
$$+\beta\sum_{j=1}^{t}\alpha^{t-j}[\mathbf{x}_j\hat{\mathbf{B}}(t-2) - 2\mathbf{x}_j\hat{\mathbf{B}}(t-1) + \mathbf{x}_j\mathbf{B}(t)]^2\}.$$

Probable values of a smoothing parameter α in this expression remain in the former range $0 < \alpha \le 1$ and parameter values γ и β defining the share of each components participation in a joint criterion remain *fuzzy* and satisfy the following inequations: $\gamma \ge 0$, $\beta \ge 0$, $\gamma + \beta \le 1$.

Affected by fuzziness of parameters γ и β the functional of (15) is also fuzzy or vaguely defined which intrinsically leads to *a model with a fuzzy or vaguely defined structure of adaptive mechanism*. Actually, using as we did before the recurrent method of estimation we get a formula for correction of current parameter values

$$\hat{\mathbf{B}}(t) = \hat{\mathbf{B}}(t-1) + \beta[\hat{\mathbf{B}}(t-1) - \hat{\mathbf{B}}(t-2)] +$$
$$+\frac{\mathbf{C}_{t-1}^{-1}\mathbf{x}_t'}{\mathbf{x}_t\mathbf{C}_{t-1}^{-1}\mathbf{x}_t' + \alpha}(1-\gamma-\beta)[y_t - \mathbf{x}_t\hat{\mathbf{B}}(t-1)], \tag{16}$$

which, depending on fuzzy parameter values can change not only the level of response but the very principle of this level formation. This possibility is caused by a multivariant structure of an adaptive mechanism. All structure variants can be easily isolated with defining membership function in case we consider all possible combinations (Cartesian product) of a multitude of fuzzy parameters. All multivariance of probable values of the adaptive mechanism can be given in form of a table. We introduce indices

$$\Delta\mathbf{B}_t(\alpha) = \frac{\mathbf{C}_{t-1}^{-1}\mathbf{x}_t'}{\mathbf{x}_t\mathbf{C}_{t-1}^{-1}\mathbf{x}_t' + \alpha}, \tag{17}$$

$$e_t = y_t - \mathbf{x}_t\mathbf{B}(t-1, \alpha, \gamma). \tag{18}$$

Using these indices we enter the corresponding variables into the table 1.

Table 1. Variants of adaptive mechanism structure

Parame-ters	$\gamma = 0$	$0 < \gamma < 1$	$\gamma = 1$
$\beta = 0$	$\hat{\mathbf{B}}_t = \hat{\mathbf{B}}_{t-1} + \Delta\mathbf{B}_t(\alpha)e_t$	$\hat{\mathbf{B}}_t = \hat{\mathbf{B}}_{t-1} + (1-\gamma)\Delta\mathbf{B}_t(\alpha)e_t$	$\hat{\mathbf{B}}_t = \hat{\mathbf{B}}_{t-1}$
$0 < \beta < 1$	$\hat{\mathbf{B}}_t = \hat{\mathbf{B}}_{t-1} + \beta[\hat{\mathbf{B}}_{t-1} - \hat{\mathbf{B}}_{t-2}] + (1-\beta)\Delta\mathbf{B}_t(\alpha)e_t$	$\hat{\mathbf{B}}_t = \hat{\mathbf{B}}_{t-1} + \beta[\hat{\mathbf{B}}_{t-1} - \hat{\mathbf{B}}_{t-2}] + (1-\gamma-\beta)\Delta\mathbf{B}_t(\alpha)e_t$	-
$\beta = 0$	$\hat{\mathbf{B}}_t = \hat{\mathbf{B}}_{t-1} + [\hat{\mathbf{B}}_{t-1} - \hat{\mathbf{B}}_{t-2}]$	-	-

The obtained table clearly demonstrates the essence of a fuzzy adaptive mechanism, the model characterized such qualities being called a regression prediction model with unclearly defined structure of an adaptive mechanism.

Conclusion

Thus, a fuzzy structure of adaptive mechanism ensures a reliability growth of model's adjustment thanks to enhancement of adaptive capabilities. This type of structure is arousing from inclusions fuzzy parameters of adaptive mechanism in functional of optimization problem. These parameters permit to change a model's reaction during prediction calculations depending on a pattern of dynamics of investigated economic process. Different kinds of combination of significant points of fuzzy parameters generate seven versions of fuzzy adaptive mechanism. Such prediction models provide an adequate mapping of actual economic processes and demand reliability of forecast evaluations.

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