FINANCIAL MANAGEMENT WITH FUZZY SETS

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FIRST WORD

"Financial management with fuzzy sets." It flatly contradicts the desirable expectations of a manager dealing with the finances. Some confusion among the experts can be anticipated, for there can be no uncertainty or vagueness when dealing with money! It seems, any uncertainty in decisions contradicts an essence of the financial responsibility, and the title of the book is at least misleading.

There exists a proverb: "As you name the ship, so it shall sail" and one could predict a sad fate for the book, based on the first impression, if not for one circumstance.

"Financial management with fuzzy sets" is an honest title. The fuzziness is the synonym of uncertainty which envelopes financial activity, like a fog. The monograph could be called "Financial Management Under Uncertain Conditions," and this name would not bring an unfavorable criticism (and I think there are already some books with similar titles). But the author digs deeper: not only would he like to designate uncertainty, but he also chooses the methods of the theory of fuzzy sets to solve the problem.

A.O.Nedosekin began his scientific career in the 80^{th} as a disciple of the probability theory. He started researching systems reliability where the application of probabilities is undisputable. However, he solved problems of technical stability where the probabilities of external adverse events could not be of a classical nature, and it was necessary to stipulate, on the one hand, a subjective origin of probabilities, and, on the other hand, hypothetical nature of the probabilistic rule of influences. In spite of the perfect probabilistic scenarios based on this axiom, the author worried about insuperable break between unpredictability of the events of nature and the probabilistic scenarios based only on the subjective reasoning of an expert. Finally, because of the break between the theory and the practice the author has changed the methodology replacing probabilities with the fuzzy sets.

This change of the paradigm worked like a charm when the author started the economic research. While it is possible somehow to put a hardly predictable tomorrow in probability terms, how would you describe almost indistinguishable states and situations of today? Where is the border between high and low levels of factors, how would you make the qualitative interpretation of the quantitative processes on the basis of probabilities? The probabilistic methods do not answer these questions, while the fuzzy sets seem to fit the purpose perfectly. Establishing the connection between the quantitative and qualitative models is their main application.

It turns out the qualitative models in the economy are the most natural ones in the majority of the cases. An expert observes statistics represented by a relatively small set of data. In addition, the data are gathered at different times; therefore, they are obviously non-uniform. The data cannot be in any way interpreted by means of the classical probabilistic distribution. An expert is left with only an approximate (fuzzy) classification of these factors by levels (for example low, average, high). After the problem of the classification has been solved, this classification of separate factor can be used for further aggregation of separate factors at higher level of the data hierarchy.

The author analyzes the risk of bankruptcy of an enterprise in just such a way. There is a set of separate financial factors ***X1, X2***..., for each of which the fuzzy sets are formed. The author offers the aggregation scenario for these classifications to form a new uniform classification based on the synthetic integrated factors, i.e., he actually solves the problem of the complex financial analysis in fuzzy formulation. Thus, the author manages to form the following system of uncertain knowledge: «If ***X1*** is a low level, and ***X2*** is an average level, and so on...., the integral level of the financial status of an enterprise **Y** is average». That is, the conclusion about the financial status of an enterprise is made not on the basis of the Altman's non-transparent formula (only the lazy have not yet criticized it), but on the basis of satisfactory knowledge of the status of separate factors in the structure of complex evaluation.

The stated matrix scenario developed in 1999 by Nedosekin and Maksimov happened to be also rather suitable in the complex evaluation of the investment attractiveness of shares, and in the evaluation of the competitiveness of business during strategic planning. This is discussed by the author in detail in chapters 5 and 8.

The author has found another application for the formalism of theories of fuzzy sets describing fuzzy sequences, such as parameters of the business plan. Many scientists had dealt with the fuzzy budgeting, for example, Koffman and Hill Aluja, whose monograph the author cites with pleasure. But Nedosekin went further and, having received the representation of the resulting project factors in the form of the triangular fuzzy number, he formed the formula for evaluating the risk of project inefficiency. Looking at this formula now, it is hard to understand why it was not obtained before, as it is quite obvious and logical. Later, Nedosekin expanded the assumptions about the triangular resulting factor. He composed the formula for the risk of a project with the arbitrary NPV.

It has become possible to apply the Nedosekin-Voronov formula (named after the authors who published it in their article in 1999) not only to the investment projects, but also to the general financial planning, including calculations of pension plans, discussed by the author in Chapter 9.

The methods of funds management have a special place in the book. Alongside with the aforementioned methods of complex evaluation of the quality of shares, the author offers the method of funds portfolio optimization. Thus, he rehabilitates the Markovitz' method neglected by financial science because of the discrepancy of some method assumptions with the market realities. Having written the Markovitz' method in fuzzy formulation, the author offers an opportunity to solve the optimization problem by the means of the portfolio with fuzzy limits between the shares.

The author similarly solves the problem of the funds indices forecast. He observes the connection between the macroeconomic factors (which act in his model as the exogenous parameters) and the funds market indicators relying on the hypothesis of rational investment preferences, modeling them in the form of fuzzy analytical dependences. Finally, the forecasts are made with the help of the indices in the triangular fuzzy form.

Even five years ago, it would have been impossible to claim that the investments to the global funds market were of rational nature (just the opposite was true). However, the author, having twice successfully predicted the fall of American funds indices (confirmed in multiple periodicals) came to the strong conclusion that the funds portfolio practice starts to consider the rational market proportions and the fundamental factors. These very reasons form the basis of the forecast method offered by the author. Having said so, the author honestly admits that his model will be valid for the next 5-10 years, and then it will need some correction, because the paradigm of the funds market will change again, and the market will pass through the phase of the next re-adjustment.

Speaking about the software of financial management in fuzzy formulation, the author presents the system of the funds portfolio optimization implemented in managing the Pension Fund of the Russian Federation. It's a pity he does not mention the system of strategic planning implemented by the Russian regional company Siemens that he directly helped to create. Although, I am informed that the author intends to write another monograph to address the problems of strategic planning in fuzzy formulation.

The book is first of all intended for a foreign reader. Therefore, there should not be a surprise that it does not refer to any Russian sources. These references can be found in his previous work "Funds Portfolio Management Under Uncertain Conditions" which is accessible on-line on several sites, including the personal site of the author http://sedok.narod.ru.

In conclusion, the financial management is uncertain because of both the environment of decision making, and the essence of these decisions (for example, the portfolios with the fuzzy limits). It is impossible to overcome the market uncertainty, but it is possible to deal with it. This monograph, in my opinion, demonstrates some very successful ways to do it.

This book may be useful to study:

Financial management;

Corporate Finance;

Management information systems;

Fuzzy sets;

Applied mathematics.

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> For now we see through a glass, darkly; but then face to face: now I know in part; but then shall I know even as also I am known.
>
> ***1 Corinthians 13:12.***

About fifteen years ago my friend and I saw an excellent performance in the Kabuki theater for (this theater comes to Russia but once every 25 years). I can still see this play. Here is the plot of one of the segments. An artist is in the jungle, suddenly appears a tiger. The artist understands that the only way to get out of danger is to draw the tiger. The artist begins to draw the tiger on a vertical stone wall. Spectators cannot see his painting. And suddenly the image of the tiger appears slowly on the opposite side of the wall! The artist enchanted the tiger, and the tiger disappears.

I mention this allegory for a reason. In the finance, the tiger is an unrelenting threat to lose money, the jungle is the world of tomorrow, full of uncertainty and surprises, and the artist is a financier. To draw a tiger means to get to know the tiger, to make him apparent, to unmask him.

This tiger is immortal. We cannot kill him, nor can we domesticate this animal, or chain or cage him. The tiger is chaos. But any chaos suggests the order, so that when a system undergoes chaos and withstands its pressure, it assumes the new degree of order. Such is the law of evolution, we did not come up with it, and we cannot cancel it.

To draw the tiger means to plan his contours, to describe his habits, to know his daily routine, his diet. Then we will, probably, manage to escape from the paws of the predator. We can neither cage the tiger, nor be protected from it by a fence and to live inside such a fence is to cage ourselves. The jungle is the rules of the game, an extraordinary gamble. The danger is inescapable. The life itself is dangerous. But if we are more resolved and not afraid of pain, if we are not afraid of the future, we will see that the jungle is the most fascinating adventure. We should construct a hut in the jungle, wade a river, meet a tiger and enchant it to stay alive. The fate of a scout is an honorable one. This is our common fate, the fate of a person and humankind, and we have no other.

There is nobody, who has never lost money, at least once. To some extent, at least once we were all mugged. America has an old history of the financial pyramids. Russia faced them for the first time in the 90th. But losing money is equally annoying and hurts just the same whether we are familiar with the pyramids or not. However, the pain and disappointment are the great teachers. They teach patience, care, and right arrangement of emphases, help to get rid of greed and make the person wiser in the end.

I was present one time at a criminal trial of the pyramids case. This process took some years (because 8,000 people suffered from the pyramid, and the court considered necessary to analyze this case to the last dollar). At the first sessions there were no limits to the anger of conned people. They demanded the most severe punishment for the swindlers. Someone was even ready (given a revolver and the right to judge) to lynch the defendants. At the end of the trial when passions died out one of the victims suddenly said “It’s our own fault.” It sounded as a lesson gained by suffering. It took years to live with a grudge to finally understand that either everybody is guilty, or nobody at all, that the pyramid is only the result of greed and fear, it is a materialized image of an illness and simultaneously a bitter medicine for it.

Today’s slogan is to get wiser, to learn to live in the jungle, without turning into a greedy and cowardly wild animal along the way. In terms of money it means to be an investor, to learn to estimate the risks and to deal with them.

Investing under uncertainty is an art that takes decades to perfect. The investment is a gamble. Not everybody can take it. But everything connected with money, is necessarily hazardous, because money is extremely fluid. To keep money under the control is much more difficult, than to earn money. Therefore, we can evaluate all weight of losses from financial cataclysms borne upon the shoulders of pensioners. The blown off bubble of the “new economy” had hit them most of all, because they had already made their investment choice and can now only get the results of this choice, sometimes rather regrettable and negative, in the form of the loss of capital.

Everything tells us that it is necessary to learn to invest under uncertainty of the unknown future and rather vague present. This key reason determined the title of this book. Investing under uncertainty, we must base our decisions only on the most stable ideas proven by the thousands of years of managing material assets. The golden rule of the investment postulates that the greater expected gain comes at the price of the greater risk of loss. The diversification teaches us not to put all eggs in one basket. There are some other similar reasons, from which an experienced person will draw conclusions. For example, we shall cite Ecclesiastes 4:6: “Better is a handful with quietness, than both the hands full with toil and vexation of spirit.” I wonder if it was said about the investments into the state bonds.

All of us – the sellers, the buyers, and the experts – are doomed to be active. But when making the market decisions we face one common problem – the uncertainty of tomorrow which creates vague investment environment. Everyone aspires to make this world more predictable, causing the demand for planning, forecasting, and evaluation of the market risk. We generate the potential scenarios based on the changes of the prices, volumes of production output and sales, changes of macro-parameters of economic environment (such as the levels of taxation, the rates on short-term liabilities, the rate of inflation, etc.), and then analyze the reaction of the corporate finances in that hypothetical situation. The optimistic scenarios improve the financial status of the corporation and its market position, and the pessimistic ones worsen it, leading the corporation on the verge of bankruptcy.

The central question is about the expectancy of those or other scenarios in perspective development of corporation. And here the researchers start to introduce the weights of scenarios into the integrated picture, and these weights have the probabilistic sense. Thus, the following two questions arise at once:

What is basis for these weights to be established?

Were all potential scenarios of the development of the corporation and its environment taken into account in the integrated picture?

An honest answer to these two questions is not consoling: there are no bases for the weights in the convolution of the scenarios, not all the scenarios are taken into account, and it seems impossible to consider them all.

We could switch from the discrete space of scenarios to the continuous one by replacing the discrete weight distribution of factors with the continuous density of the distribution. Having such distributions at the input of the model, it is possible to restore precisely or approximately the distribution of the output parameters of the model (for example, the financial factors). Unfortunately, while doing so does relieve the problem of limitation of scenarios, it does not eliminate the problem of the unfounded nature of modeling probabilistic distributions.

Considering the classical understanding of the probability, first of all we have to note that such probability is introduced as a frequency of homogeneous events occurring under the constant external conditions. In real economy there is neither uniformity nor invariance of the conditions. Even two enterprises belonging to the same industry and working in the same market, develop differently by virtue of their internal structures. So, successful management of one such company results in its success, and unsuccessful management results in its bankruptcy. At the level of "black boxes" both companies can look identical and homogeneous but this uniformity disappears when the detailed information about these companies is considered.

The time wise uniformity does not exist either. For example the American mutual funds market of 2002 is not at all the same as it was in 1999 (before the recession). All the macroeconomic parameters are different. It is clear that the market before the crisis and after it corresponds to two quite different probability models; the scenarios will be different as well as their weights.

It took much effort for science to depart from classical understanding of probabilities. With the transition from the classical probability to the axiological (subjective) one, the role of an expert setting the probabilistic weights increased as well as the influence of his or her subjective preferences on their evaluation. Accordingly, the more subjective becomes the probability, the less scientific it appears.

The appearance of the subjective probabilities in the economic analysis is not incidental. It marks the **first strategic retreat** of the science in the face of uncertainty of ineradicable nature. Not only is such an uncertainty ineradicable, but it is also "nasty" in the sense that it does not have a structure which could be once and for all modeled with the probabilities and the probabilistic processes. The methods successfully used in the theory of mass services and in statistics as a science of the behavior of large number of homogeneous (belonging to one modeling class) objects are not at all suitable for the models of financial management. Researchers deal with the limited set of events, diverse by origin, and it is difficult for them to make conclusions based on the existing information.

Thus, the experts themselves, their scientific activities, their preferences become the object of the scientific research. The confidence (or uncertainty) of an expert in evaluation of results get a quantitative value and there is no place for probabilities here anymore. We can compare it to a situation when a doctor who treated a patient, now needs a treatment himself. The object of the scientific research is augmented. Earlier, it only included the economic entity (a corporation, an industry, an economic region, a country). In modern financial management, the object of scientific research is supplemented with a decision maker (DM). The decision maker acts both as a financial manager, and as a financial analyst preparing the decisions for the manager. The activity of both these types of a DM is a subject of detailed study.

The most important part of such a formulation of a scientific problem is to learn to model the activity of a subject. In particular, it is important to understand, the criteria a decision maker uses to recognize the current economic situation, the status of an object of the research, and the field for decision-making. The information is usually insufficient, and it is not of a high quality. Accordingly, the decision maker departs consciously or subconsciously from the discrete numerical estimation, replacing it with the qualitative situation characteristics expressed in a natural language (for example, "a high/low level of a factor," "a big/small/insignificant size of cash flow," "acceptable/unbounded risk," etc.). Until the terms of natural language cannot be evaluated quantitatively, they can be freely interpreted. However, if such an evaluation takes place as a conventional model formed by a consensus of opinions and preferences of many experts observing approximately the same economic reality, it becomes significant for modeling of an economic object alongside with the information about this object.

I deem it both convenient and useful to apply the formalisms of the theory of fuzzy sets in the financial analysis to model financial activity under a considerable uncertainty. I began my scientific career in the field of the probability theory so it was even harder for me to admit that the probabilities are absolutely inferior to the fuzzy sets when it comes to their ability to describe the very essence of subjective activity of the person who is learning the world and making decisions. To fight the uncertainty we must learn to model it, differentiating between what is cognizable and what is not. We will have achieved the humanly possible maximum if we can find the formula for the boundary between non-cognizable and cognizable, between certain and indiscernible. The rest is up to the higher forces (though, the human thirst for knowledge is certainly infinite).

What does "the high credit interest rate" mean today? We will not know anything, unless we poll a group of enterprises using bank credit resources. All these enterprises get different credit interest rates: the more reliable is the borrower, the smaller interest is charged. All the borrowers are different, but the comprehensive research reveals a more complete picture (usually interpreted as a tests histogram). It becomes possible to determine a certain average rate of credit interest around which all other rates are grouped. The farther we move to the right from a certain mean value along the X axis (the level of the interest rate), the more justification we have to declare, that the given rate is "high." So we can allocate the following three groups of rates – "high," "average," and "low" and to categorize all available rates by the selected classes (clusters) in one of the two following ways. We can do it quite precisely (though roughly), by establishing the corresponding intervals on the axis X. Then belonging to an interval will stipulate an unequivocal verbal evaluation. To do it more carefully, however, will require describing our trust in classification. Then the precise sets of intervals will be transformed to fuzzy subsets with dim borders, and the extent of belonging of this or another interest rate to a given subset is determined by the function of belonging constructed by special rules.

Thus, in course of research of uncertainty in the economy the ways of the **second strategic retreat** of science were outlined. If earlier scientists had to reject classical probability for the benefit of probability subjective, now the latter itself does not satisfy a researcher, because it contains too much subjective expert evaluation and too little information on how this evaluation has been obtained.

We don't expect the third strategic retreat because we have reached the limit. We retreat because we want to keep the adequacy of the used models and the required extent of their authenticity. We want to be fair so we gradually replace subjective probabilities with fuzzy sets. And here we have an opportunity for regrouping and strategic assault on uncertainty. There are several reasons for this:

Fuzzy sets ideally describe the subjective activity of a decision maker.

Fuzzy numbers (the type of fuzzy sets) are ideally suitable for planning factors in future when their evaluation is complicated (it is dim and has no sufficient probabilistic basis). Thus, all scenarios under certain separate factors can be merged in a summary scenario in a form of a triangular number with three factor values: the least probable, the most expected, and the greatest possible. Thus, the weights of separate scenarios in the structure of a summary scenario are formalized as a triangular function of belonging of a factor level to the "approximately equal to average" fuzzy set.

We can formalize within the limits of one model both the particulars of an economic object, and the cognitive abilities a manager and an analyst connected with this object.

We can still use probabilistic descriptions as probabilistic distributions with fuzzy parameters [Nedosekin]. The vagueness of parameters of distribution is caused by the lack of classical statistical sample of observation, and for the analysis we use the scientific category of quasi-statistics (which I introduced in [Nedosekin]). Such approach sets the triangular parameters of distribution based on the procedure of establishing the extent of plausibility. Thus, the way for synthesis of probabilities and fuzzy sets is outlined.

Actually, the given monograph is devoted to validation of applicability of fuzzy set descriptions in financial management. The book was preceded by five years of scientific research [My Internet site] on application of the theory of fuzzy sets in the financial and investment analysis. The probability as a tool for modeling financial processes has been used in economic analysis for a relatively long time (for more than half a century). Fuzzy sets are rather an unusual and a new tool of economic research, and this is true not only in Russia (where the market economy exists for only about 20 years), but also for the rest of the world.

Publications on fuzzy sets application in economic and financial analysis have become an avalanche. The International Association for Fuzzy Set Management & Economy [SIGEF] regularly approves new results in the field of fuzzy sets economic research. The researchers had written hundreds of monographs on this subject. In Russia this process is speeding up. I provide a collection the papers on the "Fuzzy sets in the economy and finance" on my Internet web page. There are just a few works published at this time including my own works (less then a hundred), but as our first president Gorbachev used to say "the process had begun."

At the end of the introduction I want to express my gratitude.

I thank God for everything.

My mother Tatiana and my father Oleg for the opportunity they gave me to participate in the affairs of this world.

My wife Nonna for her patience, compassion, and enormous help.

My teacher, the member of the Russian academy of safety, Doctor of Science (Engineering), Professor G.N.Cherkesov – for leading me into the world of scientific research.

American scientists, Professors James Buckley [Buckley homepage], Richard Hoppe [Hoppe homepage] and the author of the world famous technique of the evaluation of the corporation bankruptcy risk Edward Altman [Altman homepage] – for assistance to my scientific research.

The Artificial Life, Inc. [Alife Homepage] because the nature of my job there determined the direction of all my research in the field of funds management.

The Siemens Business Services [SBS homepage] because the methods I developed constitute the basis of the SBS Russia software used in portfolio management of pensions accumulative component for the Pension fund of the Russian Federation;

The Academician N.P.Fedorenko's international scientific fund for economic researches – for financial support of my research within the framework of the grant.

I. UNCERTAINTY AND FUZZY SETS

1. UNCERTAINTY: ONE MUST FIGHT, BUT ONE CAN'T WIN

1.1. To recognize a situation

Everything can be learned by comparison, and one quantitative example easily illustrates this notion.

Let's look at the "Price-to-Earnings" ratio (the P/E ratio) for a number of anonymous companies in Technology Sector, Office Equipment Industry (USA) (the measurements were carried out in January, 2003, see Table 1.1). This factor is very important and characterizes the investment attractiveness of companies, industries, sectors, and the USA funds market as a whole.

Table 1.1 P/E ratios of the companies comprising the industry

Sector: Technology		**34.75**
Industry: Office Equipment		**19.36**
1	Corp. 1	8.26
2	Corp. 2	8.55
3	Corp. 3	12.05
4	Corp. 4	12.08
5	Corp. 5	13.94
6	Corp. 6	15.68
7	Corp. 7	18.27
8	Corp. 8	**21.04**
9	Corp. 9	22.63
10	Corp. 10	23.84
11	Corp. 11	36.89
12	Corp. 12	41.12

Let's look at any company (for example, Corp. 8) and ask ourselves: is its P/E ratio *large or small*? There are two possible answers to this question, and both are well-founded.

The ratio is average for it is close to the industry ratio: ($21.5 \approx 19.36$); based on the industry ration, the P/E ratio of, let's say, the Corp. 1 is low.

The ratio is low, in comparison with the sector ratio ($21.5 < 34.75$).

So, which answer is correct? Let us continue the analysis and see, what the chosen industry looks like comparing to the others in this sector. The P/E ratios for industries in the technology sector are shown in Table 1.2.

Table 1.2 P/E ratios for industries comprising the sector

Sector: Technology		34.75
1	Office Equipment	19.36
2	Computer Services	24.29
3	Computer Storage Devices	24.46
4	Computer Hardware	28.95
5	Scientific and Technical Instr.	29.49
6	Electronic Instruments and Controls	29.97
7	Computer Peripherals	30.39
8	Software and Programming	33.03
9	Computer Networks	37.06
10	Communications Equipment	46.23
11	Semi-conductors	46.52

Now we can see that the Office Equipment industry is in the best position among the industries of this sector (based on the chosen factor). And from this standpoint, we can re-enforce our notion that the P/E ratio for Corp. 8 is low after all. Subsequent manipulations, however, will sober us up. Let us see now how the technology sector is positioned relative to other sectors of the American economy (Table 1.3).

Table 1.3 P/E ratios of the sectors of the USA economy

1	Utilities	14.48
2	Consumer Cyclical	16.36
3	Conglomerates	17.95
4	Financial	18.18
5	Capital Goods	18.55
6	Consumer Non-Cyclical	22.61
7	Services	24.93
8	Healthcare	25.25
9	Transportation	26.15
10	Energy	27.00
11	Fixed assets	32.97
12	**Technology**	**34.75**

Now we see that the technology sector value is the worst of all, and the national average varies between 20 and 25. Based on this information, the value of the P/E ratio for the Corp. 8 becomes average again. This answer could have satisfied us, but for one more reason.

In all cases we have evaluated the factor qualitatively comparing this factor with those of the industry, the sector, or the economy as a whole. Anyhow, we based the evaluation on the *relative* foundation. Should the investment decision-making be feasible based on the P/E ratio alone and should we firmly decide to invest a certain portion of money in the shares of a certain sector or industry, then our relative evaluation would be exhaustive. We would simply compare the given level with the industry average and the process of decision-making would be completed.

But, if we want to make a decision based on the absolute foundation, we must ask an unconditional question such as whether or not to invest in Corp 8. Then the comparative evaluation appears insufficient. We must understand what the P/E ratio should be principally used to make the shares attractive. We also are compelled to move one level higher to look at the share market from the macro-economic positions.

Dr. Robert Shiller [Robert Shiller homepage] has been observing the dynamics of the P/E ratio on the leading index of American shares S&P500 for a long time (Fig. 1.1).

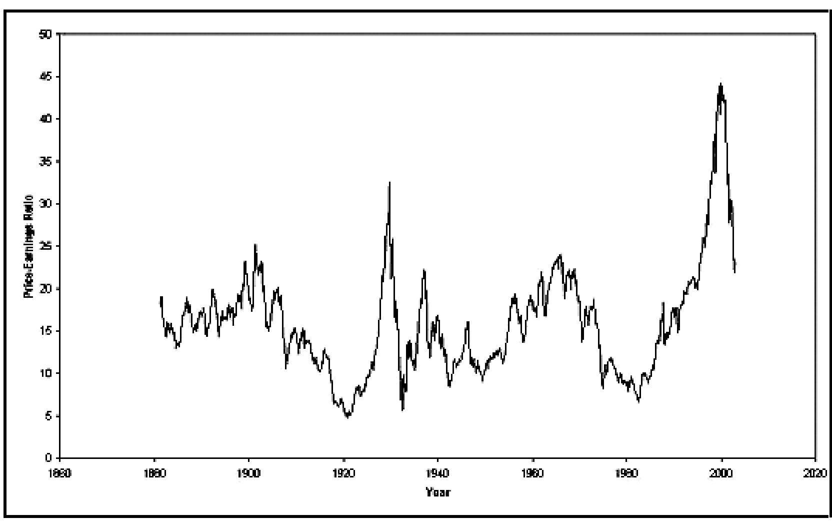

Figure 1.1 S&P500 P/E. *Source:* [Robert Shiller homepage]

The last peak on the diagram in Fig. 1.1 is a 5-year bubble of the new economy, which blew off (it might not be final, but probably is irreversible). The “normal” level of the examined factor measured for the last 120 years, is between 10 and 20. This corresponds to the equilibrium level of profitability of state bonds from 3 to 7 percent annually (we will consider this point in detail in the chapter 7 of this monograph). The low level of the factor corresponds to the inflation, and the high level – to the deflation or as the FRB chairman Alan Greenspan calls it, "irrational exuberance." This exuberance spreading on the markets, heated them to transcendental heights, but now the times are different, and that exuberance had disappeared without a trace.

Let's go back to the Corp 8. Its P/E ratio remains average, but a time may come, when this ratio will also seem high. Therefore, its qualitative evaluation should be made from different points of view, yet dynamically, taking into account the macro-economic tendencies.

Certainly, it is completely insufficient to take into account only one factor to make a conclusion about investment attractiveness of a stock. However, even the elementary analysis based on just one factor shows us how difficult it is to recognize a situation, to make estimation, and to come to a quality conclusion based on the isolated and incomplete quantitative data.

Corporate financial analysts have absolutely analogous problems. It is necessary to conduct both the vertical and the horizontal analyses considering the corporation's statistics for a comparatively long period of time to make the conclusions about its financial indicators based on financial data for a quarter or a year. At the same time, an analyst must relate this data with corresponding numbers for the competition within the industry as well as the industry within the sector, and the sector within the whole country.

We could come up with more examples. It is clear, however, that the present is vague and ambiguous. Therefore, it is always necessary to consider the person who makes the judgments. We need to know whether the forecaster is an expert or a novice as well as the contents of initial data for which such judgments are made.

1.2 Predicting tomorrow

When the leading American market indices are falling and the euro presses the dollar all over the world, many people ask what is going to happen next. The world financial crises had taught us very well and we understand that our world is interconnected, and there are no more distances between the financial markets. Whatever happens in the USA, must concern Russia. Too big a country undergoes a crisis, and not just any country, but the issuer of the world reserve currency and the generator of the uniform measure of prices.

So, let's look at the approaching future conjecturally, "through a dim glass."

Regretfully forecasting can only partly be considered a science. To the extent the future is not determined by the present (and it is often so, indeed), the **forecasting is impossible**. Otherwise our world would be easily described by formulae, and the mechanistic outlook in the spirit of Newton-Laplas would prevail. However, the universe presents itself as a rather strange place, where the majority of events are unpredictable. In this philosophical sense we are indeterminists rather than fatalists, and it forces us to use the categories of *randomness, chance, probability, and expectancy* in the scientific analysis.

The mere fact of the recognition of the limited capabilities of forecasting as a tool to predict the future is a scientific validation of our research. The alternative point of view that total predictability of the future using the data of the past is possible, seems to us absolutely unscientific and moreover doomed to extinction. It is strange and ridiculous that a number of authors subconsciously adhere to similar mechanistic view on a matter. Most of these scientists are the developers and researchers of dynamic adaptive systems, including the ones that include human participation. Life however is more than just mechanics and technology, and what is suitable for the behavior forecasting of inanimate objects, always stumbles when analyzing live ones and in particular with the economic analysis, because the economy is first of all people who generate a certain way of production and distribution of material wealth. The economy is often irrational, because it is driven by irrational motives such as greed and foolishness. There is no such thing as disinterested economy; it is based on the consolidation of selfish interests, but sometimes several interests of the same persons contradict each other. Thus, foolishness and imprudence that often accompany greed, which is the basis money-grubbing, will harm the effort to grow rich.

The phenomenon of Cassandra who predicted the fall of Troy even before the landing of the Greek armies on the coast of Asia Minor is also unscientific; anyway, it is not on that level of the modern science development. Otherwise, it would make sense to go to fortunetellers and read horoscopes. With all due respect to the horoscopes coming from professional and skillful astrologists, we are morally unprepared to involve these methods into the scientific practice. Although, we shall note, that the kings did not disdain the advice of the astrologers, and it often saved their lives (see "Quentin Durward" by Sir Walter Scott). After all the intuition is the mother of luck (gamblers understand it well, and as we agreed life is a gamble).

Unlike Cassandra, to forecast we must establish strict scientific relationships between the causes and the effects even if these relationships are expressed in the language of probabilities and fuzzy sets. In application to our problem, it means that the qualitative expert model of the stock market and its macroeconomic environment should precede the quantitative forecasts of this market. The correct understanding of processes at the qualitative level leads to reliable quantitative evaluations; while a well understood and correctly evaluated initial uncertainty can be converted into objective evaluation of the dispersion of the stock market forecasted parameters.

Without going into details we can describe current tendencies of the world financial market, and what most likely to expect in the next 5-10 years.

The bubble of the "new economy" blew off, the USA experienced the recession and the threat of deflation. An adequate response to these processes is the expansion of the USA into the third world countries, sometimes by military action.

The world stock markets lost their stability and the reference points. As a matter of fact, the American market today is controlled by the news and opinions rather than objective quantitative factors. As for the markets of the developing countries, they are vulnerable to the fluctuations of cross-countries rates of world currencies as never before.

In the last two years the world private pension systems lost billions of dollars (because of the investments in the superheated stock). Combined with relentless aging of the population it causes the world pension crisis.

US dollar gradually devalues relative to the euro. In the first place, the weakening of the US national currency is caused by revision of the role of the USA in the system of international investments. There is a tendency of lessening of attractiveness of such investments in the US economy.

Investors change. They correct their investment preferences, rationalizing them. The Chairman Mao Tse-Tung noted that each generation should have their own war. Paraphrasing Mao, we can say each generation gets its experience of the optimum investment in funds from the ground zero and it will learn nothing without pain. Were the lessons of history learned, the story with the NASDAQ would have never occurred, because the history of the Panama Canal would have been remembered.

There is a crisis in the science of funds management. The theories of Markowitz, Sharpe-Littner, Black-Sholes, Bollerslev (GARCH – forecasting) are being criticized (Markowitz, Sharpe-Littner, Black-Sholes). The new paradigm of the stock market calls for a new scientific paradigm. There is a demand for new theories of market evaluation.

It is obvious that America enters the time of uncertainty followed by the rest of the world. This new time doesn't have the clear contours yet, the strategic investment targets are still unformed. Until the new rational investment paradigm (instead of the paradigm of irrational exuberance) is not final, the traditional resources such as gold and oil will remain overpriced. At the same time we have some ideas and a number of forecasts that have already come true [Nedosekin]. We will discuss it in more detail in Chapter 7.

1.3 An expert and his cognitive activity

Until now, the subjective factor of making financial decisions had no satisfactory theory for quantitative evaluation. At the same time the vagueness accompanying the financial decisions constantly gives rise to the uncertainty of the person making this decision, creates risk of wrong interpretation of initial information for decision-making. The capability to measure this uncertainty is long overdue.

The uncertainty of decision maker in evaluation of a situation generates qualitative statements in terms of natural language. For example, looking at the fundamental characteristics of a security an investor considers the current value of the P/E ratio which is equal to 20. Is it "large" or "small?" This question has already been discussed in 1.1. The investor can use a financial consultant. The exact answer to this question is a histogram, where the X axis represents the values of the P/E ratio and on the Y axis represents the relative frequency of the factor for enterprises of the same industry, as the object of analysis.

Analyzing the histogram, an investor may ask why some companies can get away with the large P/E ratio, and others can't, and what level of the P/E ratio should be deemed objective. The investor consults with his analyst again, and it turns out that the profitability of a security is inversely proportional to its reliability, and people frequently buy highly capitalized companies' stock, because the low risk of default is more important to them than the profits. As for the objective level, it depends on the time of the analysis. For example, for hi-tech companies in 1999-2000 the characteristic level of the P/E ratio was some tens units. Today the typical value is 10-15, because there was a correction.

At last the investor is ready to make a decision. He thinks: "Today the price of a share of company X is $20 and its P/E ratio is 41. Its capitalization is 100 billion dollars; however, I think that the company is overvalued, and its P/E ratio is too high. I think for this company the acceptable range of the P/E ratio is about 30-35. The price of the company shares grows today, but I still find this growth unreliable and I think the price can go down again. To match my expectations I shall buy these shares at a target price of $15-$17."

Thus, the investor made an independent evaluation of the situation and came to the decision. We can see that this decision was based on the following.

- The expectations connected with the prospects of the growth of given shares.
- Fuzzy classification when the investor compared the current capitalization of the company with its P/E and analyzed the level of the factor.

Everything the investor put in words can be transformed into mathematical descriptions. Then the expectations, preferences and fuzzy evaluations made by the investor, will be the initial information for modeling qualifications for making investment decision.

Evaluating the shares, the investor can also make macro-economic assessments, for example, the prospects of some industry or even the whole national economy. The statement that the USA is in the phase of a recession alone contains huge amount of information, which should be taken into account for decision-making. Section 5 of this book considers it in detail. For now, let's just remark that the recession puts some industries in preferential position relative to the others. It means there is an inter-industry redistribution of investment risks, which should be taken into account.

The investor, buying or selling shares, should understand whether the market is bearish or bullish. This understanding lets him know that "in the bear market the over-valued assets *will most likely fall* while the under-valued assets might fall *but not as much*. In the bull market, however, the under-valued assets *will most likely grow* while the over-valued might grow *but insignificantly*." Everything marked by italics in this quote represents a subject of the investor's evaluation of the current status of market and its prospects.

Using this example of investment decisions we can conclude that a huge amount of information is contained in difficult to formalize intuitive preferences of the decision maker. When these preferences and assumptions of a decision maker are in the verbal form, they can at once get a quantitative evaluation on the base of formalisms of the theory of fuzzy sets and make a detached content of initial information within the framework of the financial model. We can refer to this detached content as an **expert model**.

The information contained in the expert model forms an informational situation referring to the level of the input uncertainty of the financial model. It acts as a filter for initial evaluation of parameters, transforming them from the number of quasi-statistical observations (see 1.4) to the function of belonging of the corresponding carrier of the parameter to some fuzzy set clusters (the states of the parameter level). Thus, after a number of transformations we can go from fuzzy evaluation of input parameters to fuzzy evaluation of financial results and evaluate the risk of not achieving these results within the framework of planning of financial decisions.

1.4 Statistics and quasi-statistics

Uncertainty is an ineradicable quality of market environment caused by the market conditions simultaneously influencing immeasurable number of factors of various nature and orientation that are not a subject to cumulative evaluation. Even if all market factors were taken into account in the model (which is quite unrealistic), ineradicable uncertainty of market reactions to the different influences would remain.

The market uncertainty is legitimately considered to be "bad", i.e. it is not statistical by nature. The economy continuously changes conditions of management, it is ruled by the laws of cyclic development and the economic cycles are not fully reproducible because the cyclic dynamics of the macro-economic factors are superimpose the dynamics of scientific and technical progress. This superimposition results in the unique market paradigm. It follows that it is impossible to get a sample of statistically homogeneous events observable in the unvarying external conditions of observation from their general set. That is, there is no classically understood statistics here.

In all definitions of the term "statistics" (the extensive list of such definitions is found in [Nedosekin]) there is a common grain, which actually is related to the statistics in the most general sense of the word, and this grain is as follows. We have a certain set of observations on an object or on a set of the objects. We assume that the random sample of observations taken from their hypothetical general set hides a certain fundamental law of distribution, which will remain true for some period of time, and this will allow us to predict the trend of future observations and the calculated range of deviations of these observations from the calculated expected trend values.

Having agreed that all observations are made in the constant and homogeneous external conditions and/or on objects with identical properties, such as the same reason for their occurrences was observed, we estimate and confirm the distribution by the means of the frequency method. Breaking the whole allowable range of the observable parameter into a number of equal intervals, we can count the number of observations in each interval to construct a histogram. Using the known methods we can transform the histogram into the density of probabilistic distribution with optimal parameters. Thus, the identification of a statistical law is completed.

When dealing with "bad" uncertainty, when we do not have enough observations to correctly confirm one or another law of distribution, or we observe the objects which, strictly speaking, cannot be referred to as homogeneous, then, there is no classical statistical sample.

At the same time, even without sufficient number of observations, we are inclined to suspect that these observations hide some statistical law. We cannot evaluate the parameters of this law precisely, but we can come to a certain agreement on the kind of this law and on the range of the scattering of the key parameters that form its mathematical description. It is pertinent to introduce the concept of quasi-statistics here [Nedosekin]:

Quasi-statistics is a sampling from the general set of observations considered to be insufficient for identification of probabilistic law of distribution with precise parameters, but is sufficient to prove to some subjective extent of authenticity the law of observation in probabilistic or any other form, and the parameters of this law will be set by special rules to meet the required authenticity of identification of the law of observation.

This definition of quasi-statistics gives a broader understanding of probabilistic law, when it has not only the frequency, but also subjectively-axiological sense. Here the contours of the synthesis of probability in the classical sense and the one understood as a structural characteristic of cognitive activity of an expert-researcher are marked.

This definition also marks a wide field of compromise between what is considered a sufficient sampling, and what is not. For example, an expert estimating the financial positions of the mechanical engineering companies understands that each enterprise is unique and occupies its own market niche, etc. Consequently there is no classical statistics here even if the sampling includes hundreds of companies. Nevertheless, the expert, studying a sample of a certain parameter, notices, that for the majority of the companies the value of the given parameter are grouped within some calculated range, closer to the most expected, typical values of the factors. This pattern enables the expert to assert that the law of distribution exists, and then the expert can look for a probabilistic or a fuzzy sets form of this law.

The similar reasoning can be used when the expert observes one parameter of one company temporally. Clearly in this case the statistical uniformity of observation is absent, because the market environment, the conditions of the company management, production factors, etc. continuously vary. Nevertheless, an expert, evaluating a considerable number of observations can say that "this state of the parameter is typical for the firm, this one is extraordinary, and here I have doubts about the classification". Thus, the expert expresses the rule of the distribution of the parameter by classifying all the observations vaguely, linguistically, and this in itself is a fact of generation of information important for a decision-making. Hence, an expert used quasi-statistics to formulate the law of distribution.

The concept of quasi-statistics gives a wide open space for application of fuzzy sets for modeling laws of behavior of a certain set of observations. Strictly speaking, without postulating quasi-statistics, it is impossible to create scientifically proven models of non-uniform and observation-limited processes of the stock market and the economy as a whole, it is impossible to take into account the uncertainty accompanying the process of making financial decisions.

It is time to introduce the formalisms of fuzzy sets used in the course of this monograph. A part of them is offered by the founder of the theory of fuzzy sets – professor Lofti A.Zadeh [Zadeh homepage], the rest is new and is found in [Nedosekin].

2. FUZZY SETS COME TO THE AID

2.1 The Carrier

The carrier U is a universal set that contains all the results of observations within the framework of the evaluated quasi-statistics. For example, if we observe the age of people occupied in certain industries of the economy the carrier is a segment of real axis [16, 70], where the unit of measurement is the years of human life.

2.2 Fuzzy set

The fuzzy set A is a set of values of the carrier, such, that to each value of the carrier is assigned the extent of belonging of this value to the set A. For example, the letters of Latin alphabet X, Y, Z certainly belong to the set Alphabet = {A, B, C, X, Y, Z}, and from this point of view the set Alphabet is precise. However, analyzing the set "The optimal age of a worker" we can see that the age 50 years old belongs to this fuzzy set only to some degree μ referred to as a function of belonging.

2.3 The Function of Belonging

The function of belonging $\boldsymbol{\mu_A(u)}$ is a function with carrier U as its domain, $u \in U$, and the interval [0,1] as its range. The higher $\mu_A(u)$, the higher the extent of belonging of an element of the carrier u to the set A. For example, Fig. 2.1 illustrates the function of belonging of the fuzzy set **"The optimal age of a worker"** generated on the basis of polling of experts.

It is obvious that experts consider an age between 20 and 35 undoubtedly optimal, and 60 and older undoubtedly not optimal. In the interval between 35 and 60 the experts show uncertainty in the classification, and the structure of this uncertainty is shown by the diagram of the function of belonging.

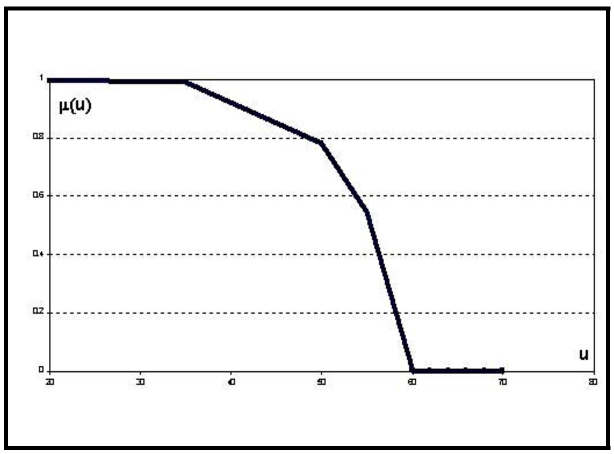

Figure 2.1 The function of belonging of the fuzzy sub-set "The optimal age of a worker"

2.4 The Linguistic Variable

Zadeh [Zadeh] determines the linguistic variable as follows:

$$\Omega = \langle \omega, T(\omega), U, G, M \rangle, \qquad (2.1)$$

where

- ω is the name of variable;
- T is the term-set of values, i.e., the set of its linguistic values;
- U is a carrier;
- G is the syntactic rule generating the terms of the set T;
- M is a semantic rule, which gives each linguistic value ω its corresponding sense $M(\omega)$, and $M(\omega)$ designates the fuzzy sub-set of the carrier U.

For example, let's set the linguistic variable **Ω = "An age of a worker"**.

We shall determine the syntactic rule G as a definition of "the optimal", imposed on the variable Ω. Then, full term-set of values ***T = {T_1 = The optimal age of a worker, T_2 = Not the optimal age of a worker}***. The carrier ***U*** is the interval [20, 70] measured in the years of human life. And on this carrier the following two functions of belonging are defined: for the value T_1 – $\mu_{T1}(u)$ (shown on Fig. 2.1), and for T_2 – $\mu_{T2}(u)$, and they correspond to fuzzy sub-sets M_1 and M_2, respectively. Thus, the constructive description of the linguistic variable is completed.

2.5 Operations on fuzzy sub-sets

For classical sets the following operations are introduced:

intersection of the sets is the operation on the sets A and B which results in the set $\boldsymbol{C = A \cap B}$ that contains only those elements belonging both to the set ***A*** and the set ***B***;

union of the sets is the operation on the sets ***A*** and ***B*** which results in the set $\boldsymbol{C = A \cup B}$ that contains those elements belonging to the set ***A*** or to the set ***B*** or to the both sets;

complement of the set is an operation on the set A which results in the set $\boldsymbol{C = \neg A}$ that contains all elements belonging to the universal set, but the elements of the set ***A***.

Zadeh offered a collection of the similar operations on fuzzy sets through operations on functions of belonging of these sets. So, if the set ***A*** is defined by the function $\mu_A(u)$, and the set ***B*** is defined by the function $\mu_B(u)$, the result of an operation is the set ***C*** with the function of belonging $\mu_C(u)$, and:

if $\mathbf{C = A \cap B}$, then $\boldsymbol{\mu_C(u) = \min(\mu_A(u), \mu_B(u))}$; (2.2)

if $\mathbf{C = A \cup B}$, then $\boldsymbol{\mu_C(u) = \max(\mu_A(u), \mu_B(u))}$; (2.3)

if $\mathbf{C = \neg A}$, then $\boldsymbol{\mu_C(u) = 1 - \mu_A(u)}$. (2.4)

2.6 Fuzzy numbers and operations on them

The fuzzy number is a fuzzy sub-set of the universal set of real numbers that has *normal* and *convex* function of belonging, that is, such a function of belonging that a) there is such a value of the carrier, which function of belonging is equal to 1, and also b) moving from the maximum to the left or to the right the value of function of belonging decreases.

Let's consider the following two types of fuzzy numbers: trapezoid and triangular.

2.6.1 Trapezoid fuzzy numbers

Let's consider some quasi-statistics and set the linguistic variable **Ω = "The value of parameter U"**, where U is a set of values of the carrier of quasi-statistics. Let's allocate two terms-sets of values***:*** $\boldsymbol{T_1}$ ***= "U y lies in the interval approximately between a and b"*** with the fuzzy sub-set $\mathbf{M_1}$ and unnamed value $\mathbf{T_2}$ with the fuzzy sub-set $\boldsymbol{M_2}$, where $\boldsymbol{M_2 = \neg M_1}$. Then, the function of belonging μ_{T1} (u) has a trapezoid appearance, as shown in Fig. 2.2.

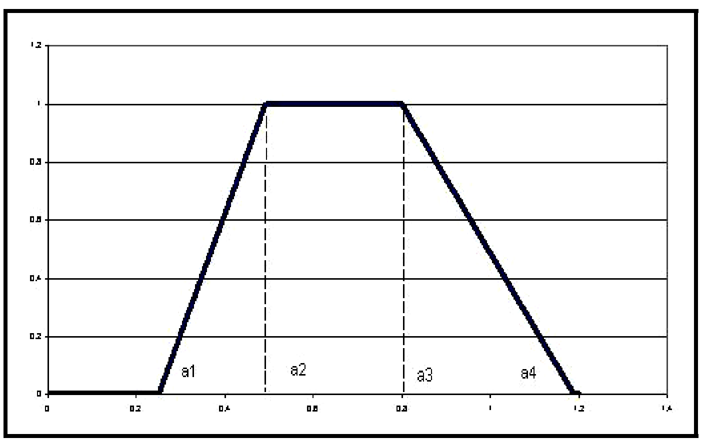

Figure 2.2 Function of belonging of a trapezoid number

As the borders of the interval are set imprecisely, it is reasonable to introduce the x-axis values of the tops of the trapeze as follows:

$$\boldsymbol{a = (a_1 + a_2)/2,\ в = (в_1 + в_2)/2} \qquad (2.5)$$

The distance between the tops $\boldsymbol{a_1, a_2}$ and $\boldsymbol{в_1, в_2}$ correspondingly is determined by the semantic meaning of the word *"approximately"*: the less disperse the quasi-statistics, the steeper the sides of the trapeze. At the limit the concept "approximately" degenerates into the concept "anywhere."

If we estimate the parameter qualitatively as in "This parameter value is average", for example, it is necessary to introduce a specifying statement which is a subject of expert evaluation (fuzzy classification) such as “An average value is approximately between a and b,” and then the trapezoid numbers can be used for modeling fuzzy classifications. Actually, it is the most natural way of an uncertain classification.

2.6.2 Triangular fuzzy numbers

Now for the same linguistic variable let’s the term-set T_1 = {U is approximately equal to a}.

It is clear, that $a \pm \delta \approx a$, and δ approaches zero, the extent of confidence of the evaluation approaches 1. From the point of view of the function of belonging, it gives to the latter a triangular shape (Fig. 2.3), and the extent of approximation is characterized by an expert.

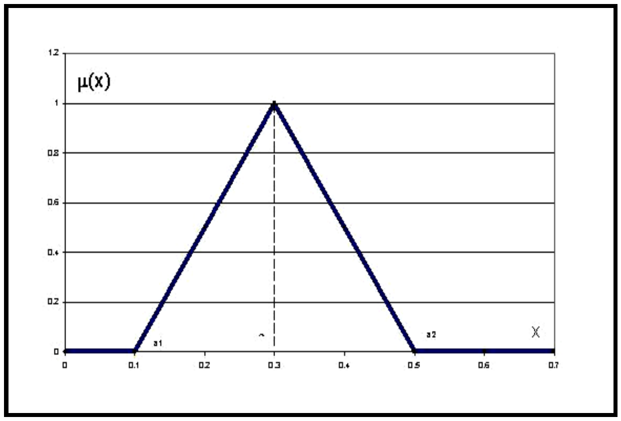

Figure 2.3 Function of belonging of triangular fuzzy number

Triangular numbers is the most often used in the practice type of fuzzy numbers, especially as the forecast values of a parameter.

2.6.3 Operations on fuzzy numbers

The whole section of the theory of fuzzy sets – soft calculations (imprecise arithmetic) – introduces a set of operations on fuzzy numbers. These operations are introduced through operations with the functions of belonging on the basis of so-called **segment principle**.

Let's define the *level of belonging* α as the ordinate of the function of belonging of a fuzzy number. Then, the intersection of the function of belonging with a fuzzy number generates a pair of values, which is commonly referred to as the *boundaries of the interval of reliability*.

Let’s set the fixed level of belonging α and define the corresponding intervals of reliability for two fuzzy numbers $\underline{\boldsymbol{A}}$ and $\underline{\boldsymbol{B}}$: $\mathbf{[a_1, a_2]}$ and $\mathbf{[b_1, b_2]}$, accordingly. Then, the basic operations with fuzzy numbers are reduced to operations with their intervals of reliability. The operations with intervals are, in turn, expressed through operations with real numbers – boundaries of the intervals:

“addition”:

$$\mathbf{[a_1, a_2]\ (+)\ [b_1, b_2] = [a_1 + b_1, a_2 + b_2],} \qquad \mathbf{(2.6)}$$

“subtraction”:

$$\mathbf{[a_1, a_2]\ (-)\ [b_1, b_2] = [a_1 - b_2, a_2 - b_1],} \qquad \mathbf{(2.7)}$$

“multiplication”:

$$\mathbf{[a_1, a_2]\ (*)\ [b_1, b_2] = [a_1 \times b_1, a_2 \times b_2],} \qquad \mathbf{(2.8)}$$

“division”:

$$\mathbf{[a_1, a_2]\ (/)\ [b_1, b_2] = [a_1 / b_2, a_2 / b_1],} \qquad \mathbf{(2.9)}$$

“exponentiation”:

$$\mathbf{[a_1, a_2]\ (\hat{}\,)\ i = [a_1^{\,i}, a_2^{\,i}].} \qquad \mathbf{(2.10)}$$

From the essence of the operations with trapezoid numbers it is possible to make a number of important statements (without proofs):

- the real numbers are a special case of the triangular fuzzy numbers;
- the sum of triangular numbers is a triangular number;
- a triangular (trapezoid) number multiplied by a real number, is a triangular (trapezoid) number;
- the sum of trapezoid numbers is a trapezoid number;

- the sum of a triangular and a trapezoid numbers is a trapezoid number.

Analyzing the features of nonlinear operations with fuzzy numbers (for example, the division) the researchers concluded that the form of functions of belonging of the resulting fuzzy numbers is often close to the triangular form. It enables us to approximate the result by reducing it to the triangular shape. Moreover, if the reduction is possible then *the operations with triangular numbers are reduced to operations with the abscissas of the peaks of their functions of belonging.*

That is, if we introduce the description of the triangular number as a set of abscissas of its peaks (a, b, c) we can write:

$$(a_1, b_1, c_1) + (a_2, b_2, c_2) \equiv (a_1 + a_2, b_1 + b_2, c_1 + c_2) \qquad (2.11)$$

It is the most common rule of the soft calculations.

2.7 Fuzzy sequences, fuzzy rectangular matrixes, fuzzy functions and operations with them

The fuzzy sequence is the numbered denumerable set of fuzzy numbers.

The fuzzy rectangular matrix is a finite set of fuzzy numbers indexed twice, with the first index M of lines, and the second index N of columns. Also, just like in the case of matrices of real numbers, the operations with fuzzy rectangular matrices are reduced to operations with fuzzy components of these matrixes. For example,

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} \otimes b_{11} \oplus a_{12} \otimes b_{21} & a_{11} \otimes b_{12} \oplus a_{12} \otimes b_{22} \\ a_{21} \otimes b_{11} \oplus a_{22} \otimes b_{21} & a_{21} \otimes b_{12} \oplus a_{22} \otimes b_{22} \end{pmatrix} \qquad \mathit{(2.12)}$$

where all operations with fuzzy numbers are described in the previous paragraph.

Fuzzy numbers field is an innumerable set of fuzzy numbers.

Fuzzy function is a one-for-one correspondence of two fields of fuzzy numbers. In our application the domain of fuzzy function is the real numbers axis, i.e. the degenerated case of the field of fuzzy numbers, when their triangular functions of belonging degenerate to the point with coordinates ***(a, 1)***.

It is appropriate to name a fuzzy function after the types of the numbers that characterize its range. If the range of the function is a field of triangular numbers it is fitting to call the function triangular.

For example, let's set the forecast of sales of a company (as a cumulative result) by the three functions of the variable: $f_1(T)$ is the optimistic forecast, $f_2(T)$ is the pessimistic one, and $f_3(T)$ is the mean expected value of sales, where T is the time of the forecast. Then, the linguistic variable **"The forecast of sales at the moment T"** is a triangular number ($f_1(T), f_2(T), f_3(T)$), and the whole forecast field is a triangular fuzzy function (Fig. 2.4), looking like a curvilinear strip.

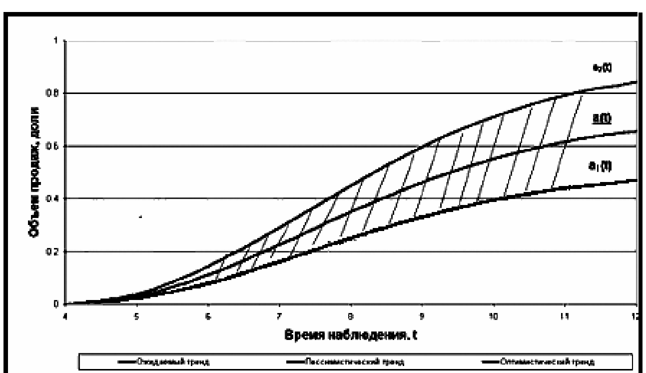

Figure 2.4 Fuzzy forecast of sales

Let's consider a number of operations with triangular fuzzy functions (statements without proof):

addition: the sum (or the difference) of triangular functions is a triangular function;

multiplication by a number transfers a triangular function into a triangular function;

differentiation (integration) of triangular fuzzy function is carried out by the rules of real differentiation (integration):

$$\frac{d}{dT}\,(f_1(T), f_2(T), f_3(T)) =$$

$$= \left(\frac{d}{dT} f_1(T),\ \frac{d}{dT} f_2(T),\ \frac{d}{dT} f_3(T)\right), \qquad (2.13)$$

$$\int (f_1(T), f_2(T), f_3(T))\, dT =$$

$$= \left(\int f_1(T) dT,\ \int f_2(T)\, dT,\ \int f_3(T)\, dT\right) \qquad (2.14)$$

a function dependent on a fuzzy parameter is fuzzy.

2.8 Probabilistic distribution with fuzzy parameters

Let us present a quasi-statistics and its histogram and let one of possible densities of probabilistic function of the distribution approximating the quasi-statistics be designated as $p\,(u, \aleph)$, where u is the value of the carrier, $u \in U$, $\aleph = (x_1,\ldots, x_N)$ is an **N**-dimensional vector of parameters of distribution.

Let's make hypothetical experiment. We will evaluate a type of the function of distribution **p(•)** varying all the parameters of vector $\aleph$. We will set a unimodal smooth function without discontinuities (for example, a quadratic multivariate parabola) as a criterion of plausibility of our distribution and normalize the value of the criterion. For example if the maximum of plausibility is L, the vector of parameters $\aleph$ gets the value, which we shall refer to as a *control point* or a *point of expectation* with coordinates $(x_{1L},\ldots, x_{NL})$. We can normalize the plausibility by presetting some percent of the maximum of plausibility, below which our probabilistic hypotheses are rejected. Then, all plausible probabilistic hypotheses are correlated with the set of vectors $\aleph'$, which is represented in N-dimensional phase space by a convex area with nonlinear borders.

Let's inscribe an N-dimensional parallelepiped of the maximal volume in this area, so that its sides are parallel to phase axes. Then, this parallelepiped represents the truncation of $\aleph'$ and can be described with the set of interval ranges of each component.

$$\aleph'' = (x_{11}, x_{12}; x_{21}, x_{22}; \ldots x_{N1}, x_{N2}) \in \aleph' \qquad (2.15)$$

Let's name $\aleph''$ a *zone of limiting plausibility*. Certainly, the control point gets into this zone, that is, the following is correct

$$x_{11} \le x_{1L} \le x_{12}, \ldots, x_{N1} \le x_{NL} \le x_{N2}, \qquad (2.16)$$

which follows from the unimodality and smoothness of the criterion of plausibility.

Then, we can consider the numbers (x_{i1}, x_{iL}, x_{i2}) as triangular fuzzy parameters of density of distribution, which in this case is also a fuzzy function. And the zone of limiting plausibility, then, is nothing else but a *fuzzy vector*.

We can see that the obtained probabilistic distribution has not only the frequency sense but also a subjective one because the zone of limiting plausibility depends on how we reject the probabilistic hypotheses. It seems this description completely corresponds to the nature of quasi-statistics as it was introduced here. The worse are the conditions for submission of plausible probabilistic hypotheses and the more difficult is to prove such plausibility – the greater is the significance of the factor of expert assessment. The resulting probabilistic description is a hybrid, which promises to be fruitful.

As an example, let us consider the normal law of distribution with fuzzy mean square deviation (Fig. 2.5). This fuzzy function is not in a shape of a strip. And it is just the time to notice, that the function with triangular fuzzy parameters is not generally triangular itself and cannot be reduced to a triangular shape.

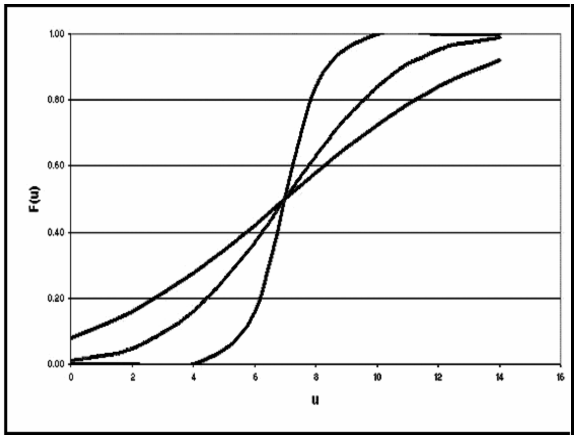

Figure 2.5 The normal law of distribution with fuzzy mean square deviation

The normalizing condition, however, is correct:

$$\int_{-\infty}^{+\infty} p(u,\aleph'')du = 1, \qquad (2.17)$$

where the right side represents the fuzzy number with function of belonging degenerated to a point. The integral undefined for fuzzy functions for general case, here represents the limit of the sums

$$\int_{-\infty}^{+\infty} p(u,\aleph'')du =$$

$$= \lim_{\Delta u \to 0} \sum_{(\Delta u)} (p(u,\aleph'') + p(u+\Delta u,\aleph''))\frac{\Delta u}{2} \qquad (2.18)$$

Let's apply all of the above to fuzzy assessment of parameters of profitability and risk of a funds index. Let's assume we have quasi-statistics of profitability $(r_1, \ldots r_N)$ of power N and the corresponding histogram $(v_1,\ldots, v_M)$ of the power M. Guided by the criterion of plausibility, we select a two-parameter normal distribution $\varphi(\bullet)$ with the expectation μ and the dispersion σ for these quasi-statistics.

$$F(\mu,\sigma) = -\sum_{i=1}^{M}\left(\frac{v_i}{\Delta r} - \varphi(r_i,\mu,\sigma)\right)^2 \to max, \qquad (2.19)$$

where r_i is the calculated value of profitability corresponding to the i column of the histogram, and Δr is the level of sampling of the histogram.

The problem (2.19) is a problem of non-linear optimization with the following solution

$$F_0 = max_{(\mu,\sigma)} F(\mu,\sigma), \qquad (2.20)$$

and μ_0, σ_0 – are the arguments of the maximum $F(\mu,\sigma)$, representing the *control point*.

Let's select the cutting off level $F_1 < F_0$ and recognize all probabilistic hypotheses plausible provided the corresponding criterion of plausibility lies in the interval between F_1 and F_0. Then, all plausible probabilistic hypotheses correspond to the set of vectors $\aleph'$, which in two-dimensional phase space is represented by a convex area with non-linear boundaries.

Let's inscribe in this area a rectangle of the maximal area, which sides are parallel to the phase axes. Then this rectangle is the zone of limiting plausibility, and it represents the truncation of $\aleph'$ which can also be described with the set of interval ranges of each component

$$\aleph'' = (\mu min, \mu max; \sigma min, \sigma max) \in \aleph'. \qquad (2.21)$$

Certainly, the control point is in this zone. That is the following is correct

$$\mu_{min} < \mu_0 < \mu_{max}, \sigma_{min} < \sigma_0 < \sigma_{max} \qquad (2.22)$$

which follows from unimodality and smoothness of the function of plausibility.

Then, we can consider the following numbers

$\mu = (\mu_{min}, \mu_0, \mu_{max})$, $\sigma = (\sigma_{min}, \sigma_0, \sigma_{max})$

as triangular fuzzy parameters of density of distribution $\varphi(\bullet)$, which in this case is also a fuzzy function.

2.9 Fuzzy knowledge

Let's refer to a natural language statement with the following structure as a **formal knowledge**:

$$IF\ (A_1 \Psi_1 A_2 \Psi_2 ... A_{N-1} \Psi_{N-1} A_N),\ THEN\ B \qquad (2.23)$$

where $\{A_i\}$, **B** are *atomic statements (predicates)*, Ψ_i are *logic connections* of the **AND/OR** type, and **N** is a *dimension of the condition*, and the atomic statements are as follows

$$a \Theta X \qquad (2.24)$$

where **a** is *a defined object (argument)*, Θ is a *logic connection of belonging* of the IS/IS NOT type, and **X** is a *generalization (the class of objects)*.

To understand the phrase the following precedence is observed: first all of the ***AND*** connections are applied to every two adjacent predicates and then all of the ***OR*** connections are applied to the results of the previous operations.

For example, it is possible to transform the classical conclusion "If Socrates is a person, and a person is mortal, then Socrates is mortal" to the structure of formal knowledge by the following rules:

we introduce two classes of objects X_1 = "Person (People)" and X_2 = "Mortal";

we then consider two arguments: a_1 = "Socrates", a_2 = "Person" = X_1.

Then, our knowledge has the formula

$$IF\ a_1\ IS\ X_1\ AND\ (a_2 = X_1)\ IS\ X_2$$

$$THAN\ a_1\ IS\ X_2 \qquad (2.25)$$

Classes of objects in the structure of knowledge are very often the fuzzy concepts. People can also draw conclusions containing the elements of uncertainty, appraisal. It forces us to switch from the knowledge in classical sense to indistinct knowledge.

Let's introduce the following set of linguistic variables with the term-set of values:

Θ = The relation of belonging = {It belongs, It most likely belongs, It probably belongs...., It probably does not belong, It most likely does not belong, It does not belong}. ***(2.26)***

Δ = The relation of following = {It follows, It most likely follows, It probably follows...., It probably does not follow, It the most likely does not follow, It does not follow}. ***(2.27)***

AND/OR = The relation of connection = {AND/OR, Most likely AND/OR, Probably AND/OR...} ***(2.28)***

Introducing these variables, we assume that they contain any number of gradation values arranged by their strengths (weaknesses) in the certain order. The unit interval can act as a carrier for these variables.

Then, the following formalism can be called **fuzzy knowledge**:

$$IF\ (a_1\Theta_1X_1\ \Psi_1\ a_2\Theta_2X_2\ \Psi_2 \ldots\ a_N\Theta_NX_N)\ \Delta\ a_{N+1}\Theta_{N+1}X_{N+1} \qquad (2.29)$$

where a_i and X_i are the values of linguistic variables, Θ_i is the value of variable belonging from the Θ set, Ψ_1 is the value of variable connection from the **AND/OR** set, and Δ is the term-value of variable of following from the Δ set.

The typical example of fuzzy knowledge is the following statement: "If the *expected in immediate future* ratio of the price of a share to its revenue is *about* 10, and (*though it is not necessary*) the capitalization of this company is at the 10 billion dollars *level*, then *most likely* these shares should be bought." Italics designate all assessments that make this knowledge fuzzy.

As the fuzzy knowledge is defined through linguistic variables, the operations of fuzzy logic conclusion can be also determined quantitatively on the basis of operations with the corresponding functions of belonging. However, we omit the detailed consideration of this question.

Some time ago fuzzy knowledge started to be actively applied to the development of broker recommendations on purchase (sale) of securities. For example, the monograph [Peray] considers the question on expediency of investment in share assets depending on character of the economic environment, and the parameters of this environment are fuzzy values. On his web page [Peray homepage] the author of the above mentioned monograph maintains the bulletin of macro-economic indicators and the corresponding investment conditions in various markets.

Specialized expert systems realizing the mechanism of fuzzy logic conclusion can be based on fuzzy knowledge. The simplest example of such system can be found on the web site [Option Advisor], where the elaboration of optional strategy is accompanied by fuzzy tentative estimation of the character of the market. Also of interest in this sense is the work [Trippi].

2.10 Fuzzy classifiers and matrix scenarios of data aggregation

Let us define an interval of real axis [0, 1] as a carrier of a linguistic variable. Any finite intervals of real axis can be reduced to the interval [0, 1] by a simple linear transformation therefore the selected unit interval is of universal nature and deserves a separate term. We shall refer to the carrier of the [0, 1] type as **01-carrier**.

Now let us introduce the linguistic variable **"The Level of the Factor"** with the term-set of values "Very Low, Low, Average, High, Very High". For the description of sub-sets of the term-set we shall introduce the system of five corresponding functions of belonging of the trapezoid kind:

$$\mu_1(x) = \begin{cases} 1, 0 \le x < 0.15; \\ 10(0.25 - x), 0.15 \le x < 0.25; \\ 0,\ 0.25 \le x \le 1; \end{cases} \qquad (2.30.1)$$

$$\mu_2(x) = \begin{cases} 0, 0 \le x < 0.15; \\ 10(x - 0.25), 0.15 \le x < 0.25; \\ 1,\ 0.25 \le x < 0.35; \\ 10(0.45 - x), 0.35 \le x < 0.45; \\ 0, 0.45 \le x <= 1; \end{cases}. \qquad (2.30.2)$$

$$\mu_3(x) = \begin{cases} 0, 0 \le x < 0.35; \\ 10(x - 0.35), 0.35 \le x < 0.45; \\ 1,\ 0.45 \le x < 0.55; \\ 10(0.65 - x), 0.55 \le x < 0.65; \\ 0, 0.65 \le x <= 1; \end{cases}. \qquad (2.30.3)$$

$$\mu_4(x) = \begin{cases} 0, 0 \le x < 0.55; \\ 10(x - 0.55), 0.55 \le x < 0.65; \\ 1,\ 0.65 \le x < 0.75; \\ 10(0.85 - x), 0.75 \le x < 0.85; \\ 0, 0.85 \le x <= 1; \end{cases}. \qquad (2.30.4)$$

$$\mu_5(x) = \begin{cases} 0, 0 \le x < 0.75; \\ 10(x - 0.75\ ), 0.75 \le x < 0.85;. \\ 1,\ 0.85 \le x \le 1; \end{cases} \qquad (2.30.5)$$

x is a 01-carrier everywhere in (2.30). The formed functions of belonging are shown on Fig. 2.6.

Let us also introduce the set of so called **nodes** α_j = (0.1, 0.3, 0.5, 0.7, 0.9) which, on the one hand, are the abscissas of maximums of the corresponding functions of belonging on 01-carrier, and, on the other hand, are equidistant from each other on the 01-carrier and symmetric about 0.5 node.

Then, we shall hereinafter refer to the introduced linguistic variable **"Level of the Factor"** defined on 01-carrier together with the set of the nodes as **standard five-level fuzzy 01-classifier**.

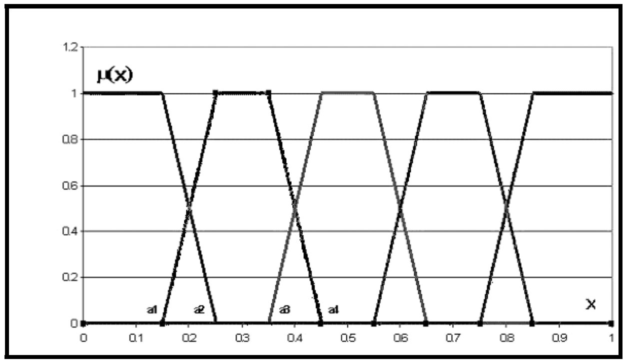

Figure 2.6 System of trapezoid functions of belonging on 01-carrier

The constructed fuzzy classifier is of great importance for further discussion. Its essence is that if nothing is known about the factor besides that it can accept any values within the boundaries of 01-carrier (the principle of equal preferences), and it is necessary associate qualitative and quantitative estimations of the factor, the offered classifier does it with the maximal reliability. Also, the sum of all the functions of belonging for any x is equal to one which points to the consistency of the classifier.

If an expert has additional information on the behavior of a factor (for example, its histogram) while identifying the level of the factor, the classification of the factor generally will not be standard because the nodes of the classification and the corresponding functions of belonging will lie asymmetrically on the carrier of the corresponding factor.

Also, if there is a set of N separate factors with current values x_i ($i = 1...N$), and each factor is assigned a five-level classifier (not necessarily standard or defined on 01-carrier) it is possible to pass from the set of separate factors to a single aggregated factor ***A_N***, and to identify its value subsequently with the help of the standard classifier. The quantitative value of the aggregated factor is determined according to the formula of double convolution:

$$A\_N = \sum_{i=1}^{N} p_i \sum_{j=1}^{5} \alpha_j \mu_{ij}(x_i), \qquad (2.31)$$

where α_j are the nodes of standard classifier, p_i is the weight of the i^th^ factor in the convolution, $\mu_{ij}(x_i)$ is the value of the function of belonging of the j^th^ qualitative level relative to the current value of the i^th^ factor. The factor ***A_N*** can be subjected to further recognition on the basis of standard fuzzy classifier, using functions of belonging of the type (2.30).

Formula (2.31) clarifies the purpose of nodes in the fuzzy classifier. These points serve as weights for aggregation of the system of factors at the level of their qualitative states. Thus, the nodes reduce the set of non-standard classifiers (with their asymmetrically located nodes) to the single standard classifier, simultaneously transitioning from the set of non-standard carriers of separate factors to the standard 01-carrier.

It is possible to construct a matrix, with the rows containing the factors, and the columns containing their qualitative levels. The values of functions of belonging of the corresponding qualitative levels lie on the intersections of rows and columns. Let us supplement the matrix with one more column of the weights of factors in the convolution $\mathbf{p_i}$ and with one more row of the nodes α_j. Then the obtained matrix contains all the necessary initial data for calculating the aggregated factor ***A_N*** according to (2.31). Therefore, it is appropriate to call the offered scenario of the data aggregation the **matrix scenario**.

For a long time now, matrix scenarios based on the five-level classifiers have been rather successfully applied to complex evaluation of the level of functioning of multifactor systems, including the financial ones (for example, the corporate finances). It will be discussed in chapters 3, 5 and 8 of this book.

Our discussion in this paragraph is based on a five-level classifier. In reality, there may be any number of levels in the classifier and it is determined only by the convenience of the modeling. The simplest qualifier is **binary** (good - bad, high - low), but it is too rough since it does not fix the characteristic of the average position around which the majority of quantitative states is grouped in real life. Here is an abstract analogy: the extremes of life are observed especially vividly from the position of mediocrity (see «The Steppe Wolf» by H. Hesse). Therefore, it is expedient to consider the **standard tri-level fuzzy 01-classifier** (with the *Low, Average*, and *High* states) with functions of belonging shown on Fig. 2.7.

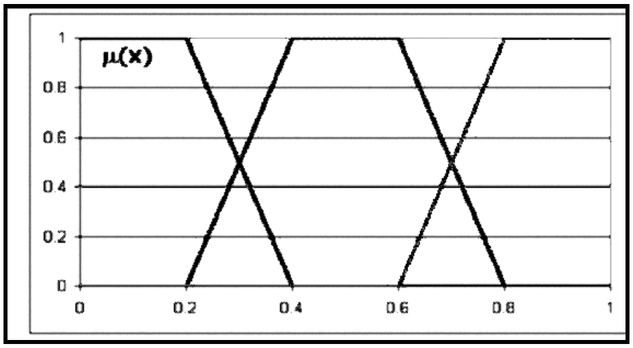

Figure 2.7 Tri-level 01-classification

$$\mu_1(x) = \begin{cases} 1, 0 \le x < 0.2; \\ 5(0.4 - x), 0.2 \le x < 0.4;. \\ 1, \; 0.4 \le x \le 1; \end{cases} \qquad (2.32.1)$$

$$\mu_2(x) = \begin{cases} 0, 0 \le x < 0.2; \\ 5(x - 0.2), 0.2 \le x < 0.4; \\ 1, \; 0.4 \le x < 0.6; \qquad . \\ 5(0.8 - x), 0.6 \le x < 0.8; \\ 0, 0.8 \le x <= 1; \end{cases} \qquad (2.32.2)$$

$$\mu_3(x) = \begin{cases} 0, 0 \le x < 0.6; \\ 5(x - 0.6), 0.6 \le x < 0.8;. \\ 1, \; 0.8 \le x \le 1. \end{cases} \qquad (2.32.3)$$

Similarly, the tri-level classifier matrix scenario of data aggregation is based on the following formula:

$$A\_N = \sum_{i=1}^{N} p_i \sum_{j=1}^{3} \alpha_j \mu_{ij}(x_i)\,. \qquad (2.33)$$

This completes the statement of the basic formalisms of the theory of fuzzy sets. Let us see now how they help.

II. FINANCIAL MANAGEMENT UNDER FUZZY CONDITIONS

3. FINANCIAL ANALYSIS AND BANKRUPTCY RISK EVALUATION

The existing approaches

The main attention of an investor in the securities arena should be focused on the financial health of the issuer. An investor (or proprietor) expects to get income both in the form of dividends or interest on bonds, and as a rate growth of the corresponding investment tools. Worsening of financial health of the issuer, accompanied by the growth of its liabilities, causes the risk of failure of payments on liabilities, termination of any payments and curtailment of the activity of the unfortunate market subject. In other words, there is a risk of bankruptcy. To minimize the risk of bankruptcy and maximally improve the financial well-being of a corporation is the task of its financial management.

The task of determining the extent of risk of bankruptcy is vital for both proprietors of the enterprise and its creditors. That is why any scientifically proved techniques of the risk of bankruptcy evaluation are of interest.

The extent of bankruptcy risk is a complex factor describing both the financial position of an enterprise, and the quality of its management, which in the final analysis is expressed in the financial equivalent, but is not exhausted by the financial consequences only.

So, careless borrowing will sooner or later lead to the situation when the borrowed amount will exceed the real capabilities of an enterprise to pay its creditors. It means the loss of financial stability, which can be easily measured on the balance sheet of the firm. But the root of the problem is not in the finance itself, but rather in its inadequate management. The finance is only the mirror of the problem, which should frequently be solved by not even the financial means (for example – to dismiss an incompetent manager).

In practice of the financial analysis, a number of factors describing individual sides of the current financial position of an enterprise are well-known. These are the factors of liquidity, profitability, stability, turnover of capital, profitableness, etc. For a number of factors the certain specifications describing their positive or negative value are known. For example, when own means of an enterprise exceed half of its total liabilities, the factor of the autonomy corresponding to this proportion exceeds 0.5, and this value is considered to be "good" (accordingly, when it is less than 0.5, it is considered to be "bad"). But in most cases the factors estimated by the analysis cannot be normalized unequivocally. It is related to the specificity of industries of the economy, current features of existing enterprises, and the state of the economic environment.

Nevertheless, any person interested in the position of an enterprise (we refer to them as decision makers), is not content with just quantitative evaluation of factors. It is important for a decision maker to know, whether the received values are acceptable, whether they are good, and to what extent. Also, the decision maker aspires to establish a logic connection between the quantitative values of factors of the selected group and the risk of bankruptcy. That is, decision makers cannot be satisfied with the binary evaluation "good – bad", they are interested in the gradations of a situation and economic interpretation of these gradation values. The problem is complicated by the existence of many factors, they change frequently in different directions and consequently, a decision maker tries to "reduce" the set of all the researched individual financial factors to a one complex factor, the value of which shows the extent of well-being ("vitality") of a firm and how far the enterprise is from bankruptcy or how close to it.

A successful analysis of the bankruptcy risk of an enterprise is possible only on the basis of the following fundamental preconditions.

The analysis is based on the results of observation of an enterprise for as long period of time as possible.

The balance sheet used in the analysis, should authentically display the real financial state of an enterprise.

Only the most critical factors germane to a potential bankruptcy of the given enterprise are used for the analysis. It is only possible when a decision maker estimates not only the financial status of an enterprise, but also its industry position.

An analyst must have representative statistics of bankruptcies, which should be also verified for pertinence to a potential bankruptcy of the given enterprise – from the point of view of the economy industry, the country and the period of time for which the analysis is carried out.

All this tells us that an expert-analyst should form a clear picture about what is "good" or "bad" on the given enterprise's industry scale.

So, for example, an investor in securities should watch how the key P/E ratio of an enterprise share correlates to the P/E ratios of the other enterprises in the same sector of the economy. Such information can be found on practically all large American financial companies' web sites, for example, [Quick Stock Evaluation] contains the comparison of two levels of factors and the conclusion to what qualitative extent these levels are far from each other.

In the developed countries, the problem of supplying the interested people with full and updated economic statistics is successfully solved. So [MGFS Industry Groups.], 9000 American corporations, whose shares are quoted at leading stock exchanges of the country, are classified and related to 9 industries, 31 industrial economic groups and 215 sectors. For each of these groups there is accessible information with the broad spectrum of financial factors of the group's activity, obtained as an average of all enterprises included in the group. Such a large base for comparative analysis allows a decision maker to make reliable decisions. In Russia similar work had only just begun, hence it is necessary to base the classification of factors not only on statistics, but also on opinions of experts having long-term actual experience of financial analysis of enterprises.

In the USA the most common approach to the analysis of an enterprise bankruptcy risk is the **Altman's approach** [Altman] described below.

A set of separate financial factors of an enterprise which, based on the preliminary analysis, are the most pertinent to a potential bankruptcy is formed with the reference to the given country and to the interval of time. Let us say we have N such factors.

The hyper-plane is drawn in an N-dimensional space formed by the selected factors. It separates the successful enterprises from the bankrupt ones in the best way, based on the examined statistics. The equation of this hyper-plane is

$$Z = \sum_{(i)} \alpha_i \times K_i \, , \qquad \textbf{(3.1)}$$

where K_i are the functions of the balance sheet factors, α_i are the weights obtained as a result of the analysis.

Moving the plane (3.1) parallel to its original position it is possible to observe how the number of successful and unsuccessful enterprises in a particular sub-area, cut off by the given plane, is redistributed. Accordingly, it is possible to establish threshold values Z_1 and Z_2: when $Z < Z_1$, the bankruptcy risk of an enterprise is high, when $Z > Z_2$ the bankruptcy risk is low, and for $Z_1 < Z < Z_2$ the status of an enterprise cannot be determined.

Edward Altman who developed this approach in 1968 applied it in the same year to the US economy. It resulted in a well known formula:

$$Z = 1.2K_1 + 1.4K_2 + 3.3K_3 + 0.6K_4 + 1.0K_5 \, , \qquad \textit{(3.2)}$$

where:

K_1 = own current capital / total assets;

K_2 = retained earnings / total assets;

K_3 = profit before interests payment / total assets;

K_4 = market value of own capital / borrowed capital;

K_5 = sales volume / total assets.

Altman's interval evaluation: at $Z < 1.81$ there is a high probability of bankruptcy, at $Z > 2.67$ the bankruptcy probability is low.

Later (1983) Altman applied his approach to the companies, whose shares are not quoted on the market. The relation (3.2) in this case looks like

$$Z = 0.717K_1 + 0.847K_2 + 3.107K_3 + 0.42K_4 + 0.995K_5 \, . \qquad \textit{(3.3)}$$

Here K_4 is the book value of own capital / borrowed capital. Altman diagnosed high probability of bankruptcy at $Z < 1.23$.

Altman's approach also known as the method of discriminant analysis was subsequently applied by Altman and his followers in a number of countries (England, France, Brazil, etc.). So, for example Toffler and Tisshaw [Toffler] obtained the following formula for the Great Britain:

$$Z = 0.53K_1 + 0.13K_2 + 0.18K_3 + 0.16K_4, \tag{3.4}$$

where
K_1 = sales profit / current liabilities;

K_2 = current capital / total liabilities;

K_3 = current liabilities / total assets;

K_4 = sales volume / total assets.

The researchers declare the probability of bankruptcy to be low at Z > 0.3.
Let's list more similar models.

The Lees' Model:

$$Z = 0.063K_1 + 0.092K_2 + 0.057K_3 + 0.001K_4, \tag{3.5}$$

where
K_1 = current capital / total assets;

K_2 = sales profit / total assets;

K_3 = retained earnings / total assets;

K_4 = market value of own capital / borrowed capital.

The probability of bankruptcy is high at Z < 0.037.

The Chesser's Model [Chesser]:

$$P = \frac{1}{1 + e^Y}, \tag{3.6}$$

where

$$Y = -2.0434 - 5.24K_1 + 0.0053K_2 - 6.6507K_3 + \\ + 4.4009K_4 - 0.07915K_5 - 0.102K_6, \tag{3.7}$$

K_1 = liquid assets / total assets;

K_2 = sales volume / liquid assets;

K_3 = gross income / total assets;

K_4 = borrowed capital / total assets;

K_5 = fixed capital / net wealth;

K_6 = current capital / sales volume.

There is a high probability of bankruptcy at P > 0.5.

The comparison of data received for a number of countries, shows, that the weights in Z convolution and the threshold interval [Z1, Z2] vary significantly not only between the countries but also for different years within the framework of one country (it is possible to compare Altman's conclusions about the US enterprises' conditions for 10 years of analysis). It turns out that the Altman's approach **is not stable when the initial data vary**. Even though the statistics on which Altman and his followers rely upon might be representative, it does not possess an important property of statistical uniformity of sampling of events. It is one thing when the statistics is applied to sampling of radio components of the same batch, and another when it is applied to companies with various technical-organizational specific characters, unique market niches, strategies and purposes, phases of the life cycle, etc. Here it is impossible to speak of statistical uniformity of events, hence, the permissibility of the application of probabilistic methods and the term "probability of bankruptcy" itself is doubtful.

Certainly, we have a right to expect that the higher level of financial autonomy of an enterprise corresponds to the less likelihood of its bankruptcy. This correspondence is expressed by all the monotonous dependences obtained on the basis of the Altmnan's approach. How large this distance is in a specific case of a particular enterprise, however, is a question which most likely cannot be answered by the Altman's approach because this conclusion cannot be based on the enterprise data, but on the statistics (quasi-statistics) of all probable bankruptcies. There ripens a desire not to apply the general to the particular, but to study in detail these particularities, in relation to the general, i.e. to change the direction of the research.

The Altman's approach makes sense when there is (or is proved in modeling) uniformity and representation of events of survival/bankruptcy. But the problem of qualitative statistics is not even the key restriction of this method. The point is that the classical probability is a characteristic of a *general set of events* and not a separate object or event. Considering a particular entity we probabilistically describe its relation to the whole group. But the uniqueness of any enterprise is that it can survive even with very weak chances and, of course, vise versa. The singularity of an enterprise fate nudges a researcher to look at it narrowly, to decipher its uniqueness, its specificity, instead of treating all entities alike, not to search for similarities, but, on the contrary, to diagnose and describe differences. There is no place for statistical probability with such approach. A researcher feels it intuitively and stresses the recognition of the current situation and finding how far is the enterprise from bankruptcy rather than bankruptcy *forecasting* (which with the absence of full statistics turns into divination on the coffee ground).

A researcher analyzing the similar in market sense enterprises proves in modeling their quasi-homogeneity within the limits of the set sample. A researcher collects the quasi-statistics in the sense of the section 1.4 of this monograph. Then the comparative analysis of the sampled enterprises and their fuzzy classification by the level of separate financial factors become scientifically proven.

3.2 Matrix method of evaluation of a corporation bankruptcy risk

Uncertainty of an expert that arises in the course of various sorts of classifications causes fuzzy sets to appear in the structure of the risk analysis method. The good examples are when an expert cannot precisely differentiate the concepts of “high” and “maximum” probability or when it is necessary to draw a border between average and low levels of a parameter value. Then the application of fuzzy descriptions means the following.

An expert creates a linguistic variable with its term-set of values. For example, a variable “**Level of Management**” can have a term-set of values “Very Low, Low, Average, High, Very High”.

To describe a linguistic variable structurally, the expert chooses its corresponding quantitative attribute – for example, the factor of the level of management constructed in a certain way, which takes values from zero to one.

Next, the expert assigns a **function of belonging** of the level of management to some fuzzy sub-set to each value of linguistic variable (which, by construction, is a **fuzzy sub-set** of values on the interval (0, 1) – range of the factor of the level of management). The common functions in this case are the **trapezoid** functions of belonging (see Fig. 3.1). The top base of the trapezoid corresponds to the complete confidence of the expert in the accuracy of the classification, and the bottom one corresponds to the confidence that any other values of the interval (0, 1) are not included in the chosen fuzzy sub-set.

To describe the trapezoid functions of belonging $\mu(x)$ compactly it is convenient to use the trapezoid numbers of type

$$\beta(a_1, a_2, a_3, a_4), \qquad (3.8)$$

where a_1 and a_4 are the abscissas of the bottom base, and a_2 and a_3 are the abscissas of the top base of the trapezoid (Fig. 3.1), specifying $\mu(x)$ in an area with a non-zero belonging of the **carrier** x to the corresponding fuzzy sub-set.

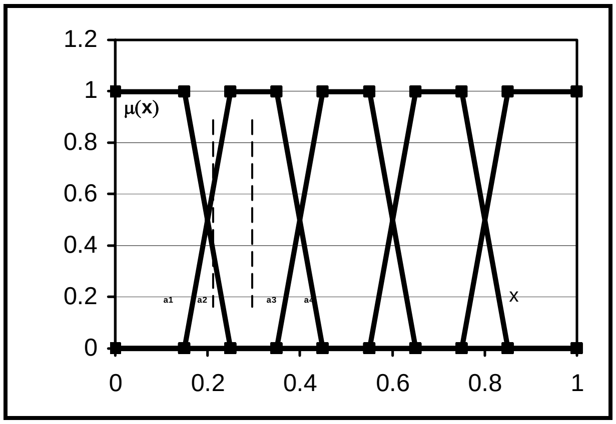

Figure 3.1 Trapezoid functions of belonging

Now the description of the linguistic variable is complete, and an analyst can use it as a mathematical object in corresponding operations and methods. We shall demonstrate it by the example of our own method first stated in [Nedosekin][1].

Stage 1. Linguistic variables and fuzzy sub-sets.

a. Linguistic variable **E "The State of an Enterprise"** has five values:
$\mathbf{E_1}$ is a fuzzy sub-set of the states of the "maximum trouble";
$\mathbf{E_2}$ is a fuzzy sub-set of the states of the "trouble";
$\mathbf{E_3}$ is a fuzzy sub-set of the states of the "average quality";
$\mathbf{E_4}$ is a fuzzy sub-set of the states of the "relative well-being";
$\mathbf{E_5}$ is a fuzzy sub-set of the states of the "maximum well-being".

b. Corresponding to variable **E** linguistic variable **G "Risk of Bankruptcy"** also has 5 values:
$\mathbf{G_1}$ is a fuzzy sub-set "maximum risk of bankruptcy",
$\mathbf{G_2}$ is a fuzzy sub-set "high risk of bankruptcy",
$\mathbf{G_3}$ is a fuzzy sub-set "average risk of bankruptcy",
$\mathbf{G_4}$ is a fuzzy sub-set "low risk of bankruptcy",
$\mathbf{G_5}$ is a fuzzy sub-set "insignificant risk of bankruptcy".
The carrier of the set G – index of the extent of bankruptcy risk g – can take values from 0 to 1 by definition.

c. Let us define linguistic variable $\mathbf{B_i}$ **"Level of factor $\mathbf{X_i}$"** for a single arbitrary financial or managerial factor $\mathbf{X_i}$ with the following term-set:
$\mathbf{B_{i1}}$ is a sub-set "very low level of factor X_i",
$\mathbf{B_{i2}}$ is a sub-set "low level of factor X_i",
$\mathbf{B_{i3}}$ is a sub-set "average level of factor X_i",
$\mathbf{B_{i4}}$ is a sub-set "high level of factor X_i",
$\mathbf{B_{i5}}$ is a sub-set "very high level of factor X_i".

Stage 2. Indices.

Let us form a set of **N** individual indices $\mathbf{X = \{X_i\}}$**,** which in the opinion of an expert-analyst influence the evaluation of an enterprise bankruptcy risk, on the one hand, and assess the various sides of business and financial life of an enterprise of different nature, on the other (to avoid duplication of indices from the point of view of their importance for the analysis). For example, a choice of a system of indices might be:

$\mathbf{X_1}$ is a coefficient of autonomy (ratio of own capital to balance currency);

$\mathbf{X_2}$ is a coefficient of provision of current assets with own means (ratio of net current capital to current assets);

$\mathbf{X_3}$ is a coefficient of intermediate liquidity (ratio of the sum of cash and receivables to short-term liabilities);

$\mathbf{X_4}$ is a coefficient of the absolute liquidity (ratio of total cash to short-term liabilities);

[1] I developed this method in cooperation with O.Maksimov

X_5 is an annual turnover of total assets in (ratio of sales revenues to average for the period cost of assets);

X_6 is a profitability of capital (ratio of net profit to average for the period cost of assets).

Stage 3. Importance.

Let us assign each index **X_i** with the level of its **importance** for the analysis **r_i**. To estimate this level we must arrange all indices in the decreasing order of their importance, so that

$$r_1 \geq r_2 \geq \ldots r_N . \qquad (3.9)$$

If the system of indices is arranged in the order of decreasing of their importance, then the importance r_i of i[th] index should be determined by the Fishburn's rule [Fishburn]:

$$r_i = \frac{2(N-i+1)}{(N+1)N} . \qquad (3.10)$$

The Fishburn's rule reflects the fact that only information we have about level of indices' importance is (3.9). Then, the estimation (3.10) corresponds to the maximum of entropy of available informational uncertainty about the object of the research.

If, however, all factors are of the equal importance (they are equally preferable or there is no system of preferences), then

$$r_i = 1/N. \qquad (3.11)$$

Stage 4. Classification of the extent of risk.

Let us form a **classification** of the current value g of the extent of risk as a criterion of splitting of present set into fuzzy sub-sets (Table 3.1).

Table 3.1 Classification of extent of bankruptcy risk

Interval of values *g*	Classification of the level of parameter	Extent of the trust (the function of belonging)
$0 \leq g \leq 0.15$	G_5	1
$0.15 < g < 0.25$	G_5	$\mu_5 = 10 \times (0.25 - g)$
	G_4	$1 - \mu_5 = \mu_4$
$0.25 \leq g \leq 0.35$	G_4	1
$0.35 < g < 0.45$	G_4	$\mu_4 = 10 \times (0.45 - g)$
	G_3	$1 - \mu_4 = \mu_3$
$0.45 \leq g \leq 0.55$	G_3	1
$0.55 < g < 0.65$	G_3	$\mu_3 = 10 \times (0.65 - g)$
	G_2	$1 - \mu_3 = \mu_2$
$0.65 \leq g \leq 0.75$	G_2	1
$0.75 < g < 0.85$	G_2	$\mu_2 = 10 \times (0.85 - g)$
	G_1	$1 - \mu_2 = \mu_1$
$0.85 \leq g \leq 1.0$	G_1	1

Stage 5. Classification of values of indices.

Let us construct the classification of current values **x** of indices **X** as a criterion of splitting of full set of their values into fuzzy sub-sets of the **B** type. To avoid cluttering our description, let us look at the example of such classification (specific to Russia) for the case of the stage 2 above with 6 indices (Table 3.2). Here, the cells of the table contain trapezoid numbers describing the corresponding functions of belonging. For example, classifying the level of the parameter X_1, and having hard time differentiating the level between "low" and "average", an expert defined the range of uncertainty as the interval (0.25, 0.3).

Table 3.2 Classification of individual financial factors

The index Code	T-numbers {γ} for values of linguistic variable "Size of the parameter"				
	"Very Low"	"Low"	"Average"	"High"	"Very High"
X_1	(0,0,0.1,0.2)	(0.1,0.2,0.25,0.3)	(0.25,0.3,0.45,0.5)	(0.45,0.5,0.6,0.7)	(0.6,0.7,1,1)
X_2	(-1,-1,-0.005, 0)	(0.005,0,0.09,0.11)	(0.09,0.11,0.3,0.35)	(0.3,0.35,0.45,0.5)	(0.45,0.5,1,1)
X_3	(0,0,0.5,0.6)	(0.5,0.6,0.7,0.8)	(0.7,0.8,0.9,1)	(0.9,1,1.3,1.5)	(1.3,1.5,∞, ∞)
X_4	(0,0,0.02,0.03)	(0.02,0.03,0.08,0.1)	(0.08,0.1,0.3,0.35)	(0.3,0.35,0.5,0.6)	(0.5,0.6,∞, ∞)
X_5	(0,0,0.12,0.14)	(0.12,0.14,0.18,0.2)	(0.18,0.2,0.3,0.4)	(0.3,0.4,0.5,0.8)	(0.5,0.8,∞, ∞)
X_6	(-∞,-∞, 0,0)	(0,0,0.006,0.01)	(0.006,0.01,0.06, 0.1)	(0.06,0.1,0.225, 0.4)	(0.225,0.4,∞, ∞)

Stage 6. Evaluation of the level of indices.

Let us make the evaluation of current level of indices and present the obtained results in Table 3.3.

Table 3.3 Current level of indices

Index	Current Value
X_1	x_1
…	…
X_i	x_i
…	…
X_N	x_N

Stage 7. Classification of the level of indices.

Let us classify the current values ***x*** by the criterion of the table of the type of Table 3.2. The result of the classification is the Table 3.4, where λ_{ij} is the level of belonging of the carrier x_i to the fuzzy sub-set $\mathbf{B_j}$.

Table 3.4 Levels of belonging of carriers to fuzzy sub-sets

Index	Result of classification by sub-sets				
	B_{i1}	B_{i2}	B_{i3}	B_{i4}	B_{i5}
X_1	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}
…	…	…	…	…	…
X_i	λ_{i1}	λ_{i2}	λ_{i3}	λ_{i4}	λ_{i5}
…	…	…	…	…	…
X_N	λ_{N1}	λ_{N2}	λ_{N3}	λ_{N4}	λ_{N5}

Stage 8. Evaluation of the extent of risk.

Now let us execute the formal arithmetic operations to evaluate the extent of the bankruptcy risk ***g***:

$$g = \sum_{j=1}^{5} g_j \sum_{i=1}^{N} r_i \lambda_{ij} , \qquad (3.12)$$

where

$$g_j = 0.9 - 0.2 * (j - 1) , \qquad (3.13)$$

λ_{ij} is taken from Table 3.4, and r_i is found according to the formulae (3.10) or (3.11).

The essence of formulae (3.12) and (3.13) is as follows. First of all we estimate the weights of some sub-set of ***B*** in the evaluation of the state of corporation ***E*** and in evaluation of the extent of risk ***G*** (internal summation in (3.12)). These weights are then used in external summation to find the mean value of the factor ***g*** where $\boldsymbol{g_j}$ is none other than the estimated average of ***g*** from the corresponding range of the table 3.1 of the stage 4 of this method.

Stage 9. Linguistic recognition.

Let us classify the obtained value of the extent of risk based on the data of Table 3.1. The classification results in the linguistic description of the extent of bankruptcy risk. In addition it provides the **level of confidence** of an expert in the correctness of the classification. Thus our conclusion of an enterprise's risk extent not only takes the linguistic form, but also acquires the characteristic of the quality of our statements.

The full description of the method is complete. Now let us consider an example.

The problem definition.

Let us consider the corporation "**CD**" (real corporation functioning in Russia), which is analyzed for two periods – the fourth quarter of 1998 and the first quarter of 1999. We have to make a complex evaluation of its financial status for the specified period of time.

The solution.

(The numbering corresponds to the numbers of stages of the method).

1. Let us define the sets ***E, G*** and ***B*** the way it was done in the 1st stage of the method.
2. The system ***X*** consisting of 6 indices defined in the 2nd stage remains unchanged.
3. We also accept, that all indices are equally important for the analysis ($\boldsymbol{r_i}$ = 1/6).
4. The extent of the risk is classified by the rule of Table 3.3 of the 4th stage of the method.
5. The chosen indices are classified in Table 3.2 based on the preliminary expert analysis.
6. The financial state of the enterprise "CD" is characterized by the following financial indices (Table 3.5).

Table 3.5 Current level of indices

Code of the index X_i	Value of X_i for the period $I(x_{I,i})$	Value X_i for the period $II(x_{II,i})$
X_1	0.619	0.566
X_2	0.294	0.262
X_3	0.670	0.622
X_4	0.112	0.048
X_5	2.876	3.460
X_6	0.113	0.008

7. Let us carry classify the current values x by the criterion of Table 3.2. The result is given in Table 3.6.

Table 3.6 Classification of the levels of indices

Index X_i	Value {λ} for the period I					Value {λ} for the period II				
	$\lambda_1(x_{I,i})$	$\lambda_2(x_{I,i})$	$\lambda_3(x_{I,i})$	$\lambda_4(x_{I,i})$	$\lambda_5(x_{I,i})$	$\lambda_1(x_{I,i})$	$\lambda_2(x_{I,i})$	$\lambda_3(x_{I,i})$	$\lambda_4(x_{I,i})$	$\lambda_5(x_{I,i})$
X_1	0	0	0	0.81	0.19	0	0	0	1	0
X_2	0	0	1	0	0	0	0	1	0	0
X_3	0	1	0	0	0	0	1	0	0	0
X_4	0	0	1	0	0	0	1	0	0	0
X_5	0	0	0	0	1	0	0	0	0	1
X_6	0	0	0	1	0	0	0.5	0.5	0	0

8. The analysis of Table 3.6 shows us that after the second period there was a qualitative drop of coverage simultaneously with the qualitative growth of the turnover of assets.

9. The evaluation of the extent of bankruptcy risk according to the formula (3.12) produces $\mathbf{g_I}$ = 0.389, $\mathbf{g_{II}}$ = 0.420, which brings us to the conclusion, that there was a **serious worsening** of the state of the enterprise (the sharp quantitative growth of the turnover was not accompanied with the qualitative growth, on the other hand, we observe qualitative decline of the autonomy, absolute liquidity, and the profitability).

10. The linguistic recognition of the extent of risk according to Table 2.2 gives us the extent of bankruptcy risk as a borderline between the low and the average, and the expert's level of confidence that the extent is average, grows from period to period.

4. THE INVESTMENT PROJECT: EFFICIENCY AND RISK

4.1 Limited nature of the existing approaches to evaluation of the investment project

Let's begin this section with the following three basic definitions.

Investment (in a broad sense) is a temporary refusal of the economic subjects from consumption of resources (capital) at their disposal and use of these resources to increase their well-being in the future.

Investment project is a plan or a program of actions related to the capital investments with the purpose of their subsequent compensation and getting a profit.

Investment process is the temporally unfolded fulfillment of the investment project. The beginning of the investment process is making the investments decision, and its ending is either the achievement of all the project objectives, or its forced termination.

The investment project assumes temporal planning of the three basic monetary flows: the flow of investments, the flow of current (operating) payments and the flow of receipts. Neither the flow of current payments, nor the inflows of receipts can be planned quite precisely, because there can be no full certainty about the future of the market. The factual future prices and sales volumes of the products sold, the prices of raw materials, and other monetary and cost parameters of the market environment can strongly differ with assumed planned values which are estimated from the standpoint of the current positions.

Unavoidable informational uncertainty implies equally unavoidable investment decisions **risk**. There is always a possibility that a project considered to be valid, de-facto turns out unprofitable, because the parameters' values achieved in the course of the investment process deviated from the planned ones, or some factors were completely overlooked. An investor will never have a comprehensive evaluation of risk, because a number of variances of the environment always exceeds managerial capabilities of a decision maker [Ashby], and there always will be an unexpected scenario (any disaster, for example) that will not be taken into account in the project, but can happen, nevertheless, and disrupt the investment process. At the same time an investor must make efforts to increase the level of his or her awareness and to try to measure the degree of risk of the investment decisions both at the development stage of the of the project, and in course of the investment process. If the extent of risk rises to the intolerable levels, and an investor does not know about it, he or she is doomed to operate blindly.

The method of the investment risk evaluation is directly connected with the way of the description of a project's initial data informational uncertainty. If the initial parameters have the **probabilistic** description, then the investments' efficiency indices are also random variables with the implicative probabilistic distribution. However, the less the extent of statistical nature of some parameters, the informational context of evidence of the described market environment status, and the level of the intuitive activity of experts, the less grounded is the application of any types of probabilities in the investment analysis.

An alternative way of accounting for uncertainty is the so-called **mini-max** approach. A certain class of the expected scenarios of the investment process is formed and from this class the minimum and the maximum process efficiency scenarios are chosen. Then, the expected effect is estimated according to the Gurvitz formula with the consent parameter λ . At $\lambda = 0$ (the Wald's point) the most pessimistic assessment of the project's efficiency, when everything is done to lower the expected losses in case of the most adverse scenario, is chosen as the decision-making basis. This approach, certainly, minimizes the risk for an investor. However, using such an approach will scrap the majority of the projects even those having rather decent chances of success. There is a danger of paralysis of business activity, and degradation of the investor as a decision maker.

There is a vivid example from the practice of gambling. Any player in preference[2] knows that a player must repeatedly bid one or two tricks more than he has at hand, hoping for a good widow. Otherwise, by the end of the game a player will lose or at best come out even, because the other players are inclined to **reasonable aggression**, i.e. to justified risk. Looking at the investments as a kind of business game we say by analogy: an investor must risk, but risk rationally, assigning to each of the investment process potential scenarios its extent of expectancy. Otherwise there is a risk to lose from indecision – **the loss of being excessively overcautious**. In gambling, a decent hand of cards and a good widow don't happen very often. A preference player, who bids six tricks and wins eight, causes a general discontent. You feel embarrassed for your partner, for his inaptitude, his inability to grab the opportunity afforded to him by a decent hand of cards that comes so seldom.

The tool that allows us to evaluate the opportunities (the expectations) is the theory of fuzzy sets. We find its first application to the investment analysis in the works of Professors A.Kaufmann and J.Gil Aluja [Gil Aluja]. Using the approach offered in these works, we shall form the method of the investment risk evaluation, both at the project, and in the course of the investment process.

4.2 Method of evaluation of the investment project with fuzzy sets

In the literature on the investment analysis the formula of Net Present Value (NPV) of investments is well-known. Let us further consider one important special case of the NPV evaluation:

All investment receipts come at the beginning of the investment process.

The evaluation of liquidating cost of the project is made *postfactum*, after the expiration of the project's life tem.

Then, the formula for NPV is as following:

$$NPV = -I + \sum_{i=1}^{N} \frac{\Delta V_i}{(1+r_i)^i} + \frac{C}{(1+r_{N+1})^{N+1}}, \qquad (4.1)$$

where, **I** is a starting volume of investments, **N** is a number of the planned intervals (periods) of the investment process corresponding to the project's life tem, ΔV_i is a circulating balance of receipts and payments in the ith period, r_i is the rate of discounting chosen for the ith period that takes into account the estimations of the expected cost of capital used in the project (for example, the expected rate on the long-term liabilities), **C** is the liquidating cost of net assets, developed in course of the investment process (including the residual cost of fixed assets on the balance of an enterprise).

The investment project is recognized to be **effective**, when **NPV** evaluated according to (4.1) exceeds some projected level **G** (in the most common case **G** = 0).

Notes:

NPV is estimated according to the formula (4.1) in terms of the constant (real) prices.

The rate of discounting is planned such, that the period of charging interests on a debt capital coincides with the corresponding period of the investment process.

The interval **(N+1)** is not related to the life term of the project, and it is used in the model to fix the moment of termination of monetary interline accounting by all parties in the investment process (the investors, the creditors and the debtors) on credits, deposits, dividends, etc. when the final financial result of the project will become unequivocal.

If all the parameters in (4.1) possess a "fuzzy" nature, i.e. their exact planned values are unknown, then it is appropriate to use the triangular fuzzy numbers with the function of belonging shown on Fig. 4.1 as the initial data. These numbers model the statement "The parameter A is approximately equal to $\bar{a}$ and is unequivocally in the range $[a_{min}, a_{max}]$".

[2] A card game popular in Russia (*Translator's note*)

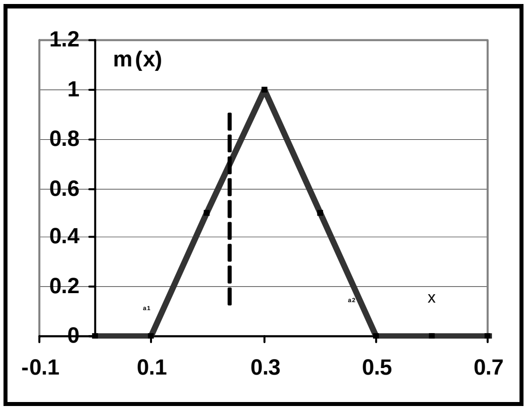

Figure 4.1 A triangular number

The obtained description allows a developer of an investment project to take the interval $[\mathbf{a}_{\min}, \mathbf{a}_{\max}]$ of the parameter and its most expected value $\bar{\mathbf{a}}$ as the initial information, and then the corresponding triangular number $\underline{A}$ = $(\mathbf{a}_{\min}, \bar{a}, \mathbf{a}_{\max})$ is formed. We shall hereafter refer to the parameters $(\mathbf{a}_{\min}, \bar{a}, \mathbf{a}_{\max})$ as *significant points* of the triangular fuzzy number $\underline{A}$. Generally speaking, the allocation of the three significant points of initial data is rather common for the investment analysis (see, for example, [Behrens]). These points are often compared with the subjective probabilities of realization of the corresponding scenarios of initial data ("pessimistic", "normal" and "optimistic"). However, we do not believe we have the right to operate the probabilities which values we can neither determine, nor assign (we have touched this subject in chapter 1 of this book, in particular, speaking about the principle of the maximum entropy). Therefore, in the investment analysis we replace the concept of *randomness* with the concepts of the *expectancy* and the *opportunity*.

Now we can define the following set of fuzzy numbers for the analysis of the efficiency of the project:

$\underline{I}$ = $(\mathbf{I}_{\min}, \bar{I}, \mathbf{I}_{\max})$ – an investor cannot precisely estimate the volume of investment resources available at the moment of the decision-making;

$\underline{r_i}$ = $(\mathbf{r}_{i\ \min}, \overline{r_i}, \mathbf{r}_{i\ \max})$ – an investor cannot precisely estimate the cost of capital used in a project (for example, the ratio of own capital to the borrowed funds, and also the interest on long-term liabilities);

$\underline{\Delta V_i}$ = $(\mathbf{V}_{\min}, \overline{\Delta V_i}, \mathbf{V}_{\max})$ – an investor forecasts the range of monetary results of project realization taking into account the possible fluctuations of the prices for the sold products, the costs of spent resources, the conditions of taxation, and the influences of other factors;

$\underline{C}$ = $(\mathbf{C}_{\min}, \bar{C}, \mathbf{C}_{\max})$ – an investor indistinctly depicts the potential conditions of the future sale of the current business or its liquidation;

$\underline{G}$ = $(\mathbf{G}_{\min}, \bar{G}, \mathbf{G}_{\max})$ – an investor indistinctly depicts the criterion on which the project can be recognized effective, or does not fully realize the possible meaning of the "efficiency" at the end of investment process.

Notes:

In case any of the parameters $\underline{A}$ is known quite precisely or set unequivocally, the fuzzy number $\underline{A}$ degenerates into real number **A** with $\mathbf{a}_{\min} = \bar{a} = \mathbf{a}_{\max}$. The essence of the method remains unchanged.

Let us examine $\underline{G}$. An investor, choosing an expected estimation $\bar{G}$, is probably guided not only by tactical, but also by strategic reasons. For example, a project can be allowed to be somewhat unprofitable if it diversifies the investor's activity and improves his or her business's reliability. As a variant, an investor realizes a dumping project, the capture of the market and the super-profit will compensate the temporary loss of profitability, but the investor wants to cut off the excessive losses when the market will be already redistributed in his or her favor. Conversely, to increase the average profitability of his or her business an investor bears the increased risk.

Thus, the problem of the investment choice in the above mentioned situation is a process of the decision-making under **uncertainty**, when the decision is reached by merging the objectives and the restrictions [Bellmann-Zadeh].

To transform the formula (4.1) to be suitable for using fuzzy initial data, we shall apply the **segment method**, described in chapter 2 of this book.

Let us set a fixed level of belonging α and define the corresponding intervals of reliability for two fuzzy numbers $\underset{\sim}{A}$ and $\underset{\sim}{B}$: **[a_1, a_2]** and **[b_1, b_2]**, accordingly. Then, the basic operations with fuzzy numbers are reduced to operations with their intervals of reliability. And the operations with intervals, in turn, are expressed through operations with real numbers – the boundaries of the intervals:

"addition":

$$[a_1, a_2]\ (+)\ [b_1, b_2] = [a_1 + b_1, a_2 + b_2], \qquad (4.2)$$

"subtraction":

$$[a_1, a_2]\ (-)\ [b_1, b_2] = [a_1 - b_2, a_2 - b_1], \qquad (4.3)$$

"multiplication":

$$[a_1, a_2]\ (*)\ [b_1, b_2] = [a_1 \times b_1, a_2 \times b_2], \qquad (4.4)$$

"division":

$$[a_1, a_2]\ (/)\ [b_1, b_2] = [a_1 / b_2, a_2 / b_1], \qquad (4.5)$$

"exponentiation":

$$[a_1, a_2]\ (\wedge)\ i = [a_1^{\,i}, a_2^{\,i}]. \qquad (4.6)$$

Let us get the intervals of reliability **[I_1, I_2], [r_{i1}, r_{i2}], [ΔV_{i1}, ΔV_{i2}],** and **[C_1, C_2]** for each fuzzy number in structure of the initial data. Then, substituting the corresponding borders of intervals in (4.1) by the rules (4.2) – (4.6), we get the following for the set level α:

$$[NPV_1, NPV_2] =$$

$$(-)\ [I_1, I_2]\ \ (+)\left(\sum_{i=1}^{N}\right)\left[\frac{\Delta V_{i1}}{(1+r_{i2})^i}, \frac{\Delta V_{i2}}{(1+r_{i1})^i}\right]$$

$$(+)\ \left[\frac{C_1}{(1+r_{N+1,2})^{N+1}}, \frac{C_2}{(1+r_{N+1,1})^{N+1}}\right] =$$

$$= \left[-I_2 + \sum_{i=1}^{N}\frac{\Delta V_{i1}}{(1+r_{i2})^i} + \frac{C_1}{(1+r_{N+1,2})^{N+1}}\right.,$$

$$\left. -I_1 + \sum_{i=1}^{N}\frac{\Delta V_{i2}}{(1+r_{i1})^i} + \frac{C_2}{(1+r_{N+1,1})^{N+1}}\right] \qquad (4.7)$$

Having set an acceptable level of sampling on α on the interval of belonging [0, 1], we can reconstruct the resulting fuzzy number $\underset{\sim}{NPV}$ by its approximating its function of belonging μ_{NPV} with a kinked curve through the intervals' end points.

It is often possible to **reduce** $\underset{\sim}{NPV}$ to a **triangular shape**, limiting the calculations to the significant points of initial data's fuzzy numbers. It allows calculation of all the key parameters in the evaluation of the risk extent analytically, rather than approximately, as we will show below.

4.3 Evaluation of risk of a project inefficiency on the basis of fuzzy sets

Let us pass to the evaluation of risk of investments. The functions of belonging $\underset{\sim}{NPV}$ and the criterion value $\underset{\sim}{G}$ are shown on Fig. 4.2.

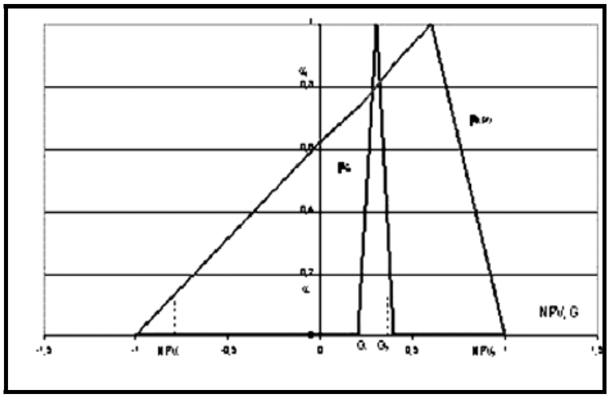

Figure 4.2 The correlation of NPV and the efficiency criterion

These two functions of belonging intersect at the point with the ordinate α_1. Let us choose arbitrary level of belonging α and let us also determine the corresponding intervals $[\mathbf{NPV_1}, \mathbf{NPV_2}]$ and $[\mathbf{G_1}, \mathbf{G_2}]$. When $\alpha > \alpha_1$ and $\mathbf{NPV_1} > \mathbf{G_2}$, the intervals do not intersect, and the confidence in the effectiveness of a project is absolute, therefore, the extent of the risk of investments' inefficiency is zero. It is appropriate to label level α_1 **the upper boundary of the risk zone**. When $0 \le \alpha \le \alpha_1$, the intervals intersect.

The shaded zone of inefficient investments is limited by the straight lines $\mathbf{G = G_1}$, $\mathbf{G = G_2}$, $\mathbf{NPV = NPV_1}$, $\mathbf{NPV = NPV_2}$, and the bisector of the quadrantal angle $\mathbf{G = NPV}$ as shown on Fig. 4.3.

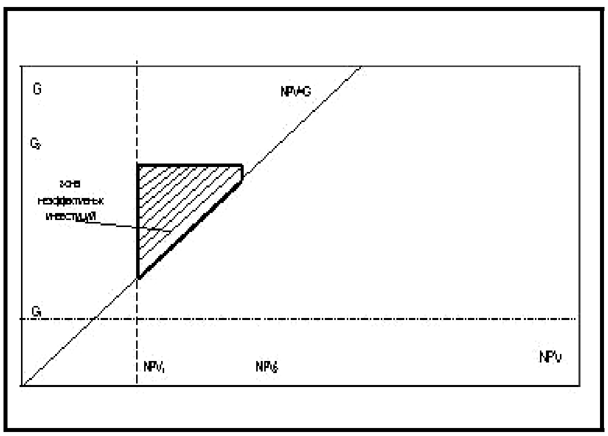

Figure 4.3 Zone of inefficient investments

The mutual correlation of parameters $\mathbf{G_{1,2}}$ and $\mathbf{NPV_{1,2}}$ result in the following calculation for the area of the shaded plane figure:

$$S_\alpha = 0 \text{ at } NPV_1 \ge G_2;$$

$$S_\alpha = \frac{(G_2 - NPV_1)^2}{2} \text{ at } G_2 > NPV_1 \ge G_1, NPV_2 \ge G_2;$$

$$S_\alpha = \frac{(G_1 - NPV_1) + (G_2 - NPV_1)}{2} \times (G_2 - G_1) \text{ at } NPV_1 < G_1, NPV_2 \ge G_2;$$

$$S_\alpha = (G_2 - G_1)(NPV_2 - NPV_1) - \frac{(NPV_2 - G_1)^2}{2} \text{ at } NPV_1 < G_1 \le NPV_2, NPV_2 < G_2;$$

$$S_\alpha = (G_2 - G_1)(NPV_2 - NPV_1) \text{ at } NPV_2 \ge G_1. \qquad (4.8)$$

Since all variants **(NPV, G)** are equally possible at the set level of belonging α, the extent of the risk of project inefficiency $\varphi(\alpha)$ is a geometrical probability of finding a point **(NPV, G)** in the zone of inefficient investments:

$$\varphi(\alpha) = \frac{S_\alpha}{(G_2 - G_1)(NPV_2 - NPV_1)}, \quad (4.9)$$

where S_α is estimated according to (4.8).

Then, the final value of the extent of risk of project inefficiency is:

$$V\,\&\,M = \int_0^{\alpha_1} \varphi(\alpha)\,d\alpha \quad (4.10)$$

In the important special case (see Fig. 4.4), when the restriction $\underline{G}$ is precisely determined by the level **G**, the passage to the limit in (4.9) as $G_2 \rightarrow G_1 = G$ produces:

$$\varphi(\alpha) = \begin{cases} 0, & at \quad G < NPV_1; \\ \dfrac{G - NPV_1}{NPV_2 - NPV_1}, & at \; NPV_1 \leq G \leq NPV_2; \\ 1, & at \quad G > NPV_2; \end{cases}$$

$$\alpha = [0, 1]. \quad (4.11)$$

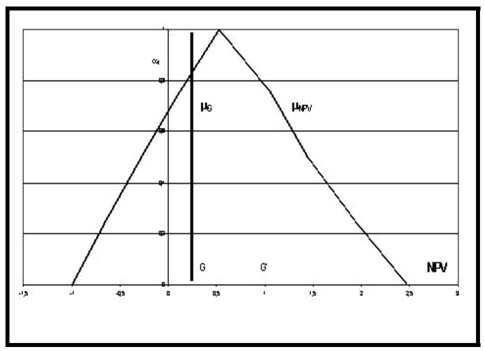

Figure 4.4 Punctual lower boundary of efficiency

To collect all the necessary initial data for risk evaluation we need the two values of the inverse function $\mu_{NPV}^{-1}(\alpha_1)$. The first value is **G** (by the definition of the upper boundary of the risk zone α_1), and the second value we shall designate **G'**. Let us similarly designate the two values of the inverse function $\mu_{NPV}^{-1}(0)$ NPV_{min} and NPV_{max}. Let us also introduce the designation $\overline{NPV}$ as the most expected value of $\underline{NPV}$. Then, the expression for the extent of the investment risk V&M, taking into account (4.11) and a long chain of transformations, looks like:

$$V\,\&\,M = \begin{cases} 0, \quad G < NPV_{min}; \\ R \times \left(1 + \dfrac{1-\alpha_1}{\alpha_1} \times \ln(1-\alpha_1)\right), \\ NPV_{min} \leq G < \overline{NPV}; \\ 1 - (1-R) \times \left(1 + \dfrac{1-\alpha_1}{\alpha_1} \times \ln(1-\alpha_1)\right), \\ \overline{NPV} \leq G < NPV_{max}; \\ 1, \quad G \geq NPV_{max}, \end{cases} \quad (4.12)$$

where

$$R = \begin{cases} \dfrac{G - NPV_{min}}{NPV_{max} - NPV_{min}}, & G < NPV_{max}, \\ 1, \; G \geq NPV_{max} \end{cases} \quad (4.13)$$

$$\alpha_1 = \begin{cases} 0, G < NPV_{min}; \\ \dfrac{G - NPV_{min}}{\overline{NPV} - NPV_{min}}, \\ NPV_{min} \le G < \overline{NPV}; \\ 1, \quad G = \overline{NPV} \\ \dfrac{NPV_{max} - G}{NPV_{max} - \overline{NPV}}, \\ \overline{NPV} < G < NPV_{max}; \\ 0, \; G \ge NPV_{max}. \end{cases} \qquad (4.14)$$

Let us study the expression (4.12) for the following three special cases:

When $\mathbf{G = NPV_{min}}$ (extremely low risk), $\mathbf{R = 0, \alpha_1 = 0, G' = NPV_{max}}$, the passage to the limit in (4.12) gives us **V&M = 0**.

When $\mathbf{G = G' = \overline{NPV}}$ (average risk), $\mathbf{\alpha_1 = 1, R = (NPV_{max} - \overline{NPV})/(NPV_{max} - NPV_{min})}$ = 1-P, the passage to the limit in (4.12) gives us $\mathbf{V\&M = (NPV_{max} - \overline{NPV})/(NPV_{max} - NPV_{min})}$.

When $\mathbf{G = NPV_{max}}$ (extremely high risk), $\mathbf{P = 0, \alpha_1 = 0, G' = 0}$, and the passage to the limit in (4.12) produces **V&M = 1**.

Thus, the extent of risk **V&M** takes the values from 0 to 1. Every investor, having allocated the interval of unacceptable values of risk, can classify the values **V&M** according to his or her investment preferences. A more detailed gradation of the extents of risk is also possible. For example, after introducing a linguistic variable "**the extent of risk**" with the term-set of values {*Insignificant, Low, Average, Relatively High, Unacceptable*}, every investor can make an independent description of the corresponding fuzzy sub-sets, by setting five functions of belonging μ_***(V&M)**.

The description of the fuzzy method of the investments' efficiency analysis with the evaluation of the extent of risk of the investment decision mistake is completed. Let us consider a simple explanatory example.

The initial data of the project: $N = 2$, $\underline{I} = (1, 1, 1)$ is a precisely known size of investments, $\underline{r_1} = \underline{r_2} = \underline{r} = (0.1, 0.2, 0.3)$, $\underline{\Delta V_1} = \underline{\Delta V_2} = \underline{\Delta V} = (0, 1, 2)$, $\underline{C} = (0, 0, 0)$ – the residual cost of the project is zero, $\underline{G} = (0, 0, 0)$ – the criterion of the efficiency is a non-negative value of NPV.

The results of calculations according to the formula (4.1) for the levels of belonging α = [0, 1] with the step 0.25 are shown in Table 4.1.

Table 4.1 The results of the project efficiency calculations

α	Intervals of reliability according to the level of belonging α for:		
	r	**ΔV**	**NPV**
1	[0.2, 0.2]	[1, 1]	[0.527, 0.527]
0.75	[0.175, 0.225]	[0.75, 1.25]	[0.112, 1.068]
0.5	[0.15, 0.25]	[0.5, 1.5]	[-0.280, 1.438]
0.25	[0.125, 0.275]	[0.25, 1.75]	[-0.650, 1.944]
0	[0.1, 0.3]	[0, 2]	[-1, 2.470]

The approximation of the function μ_{NPV} (Fig. 4.5) shows that it is close to the triangular shape

$$\mu_{NPV}(x) = \begin{cases} 0, & at \quad x < -1; \\ \dfrac{x+1}{0.527+1}, & at \quad -1 \le x < 0.527; \\ \dfrac{2.47 - x}{2.47 - 0.527}, & at \quad 0.527 < x \le 2.47; \\ 0, & at \quad x > 2.47; \end{cases} \qquad (4.15)$$

and we shall use this shape in calculations.

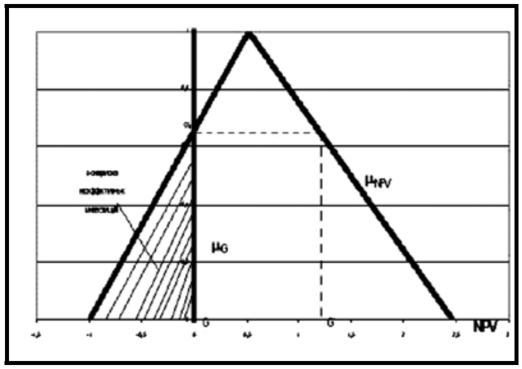

Figure 4.5 Reduction of the function of belonging to the triangular shape

Let us make the positive decision on investment of capital $\underline{I}$. Then, $\alpha_1 = \mu_{NPV}(0) = 0.655$, $G' = \mu_{NPV}^{-1}(\alpha_1) = 1.197$, and, according to (4.11) – (4.15), **R** = 0.288, **V&M** = 0.127.

We'll go on with the example. Let's say the decision to begin the investment process was made, and by the results of the first period the turnover balance was $\Delta V_1 = 1$ with the actually measured rate of discounting $r_1 = 0.2$. Then, the re-calculation of interval evaluation of **NPV** according to (4.1) produces:

$$[NPV_1, NPV_2] =$$

$$= [-0.167 + \frac{\Delta V_{21}}{(1+r_{22})^2} \; , \; -0.167 + \frac{\Delta V_{22}}{(1+r_{21})^2}]. \qquad (4.16)$$

The results of calculations according to the formula (4.16) are shown in Table 4.2.

Table 4.2 The results of the calculations of the project efficiency

α	Intervals of reliability according to the level of belonging *α* for:		
	r	ΔV	NPV
1	[0.2, 0.2]	[1, 1]	[0.527, 0.527]
0.75	[0.175, 0.225]	[0.75, 1.25]	[0.333, 0.738]
0.5	[0.15, 0.25]	[0.5, 1.5]	[0.153, 0.967]
0.25	[0.125, 0.275]	[0.25, 1.75]	[-0.012, 1.227]
0	[0.1, 0.3]	[0, 2]	[-0.167, 1.489]

The reduction of $\underline{NPV}$ to a triangular form produces:

$$\mu_{NPV}(x) = \begin{cases} 0, & at \quad x < -0.167 \\ \dfrac{x+0.167}{0.527+0.167}, & at \quad -0.167 \le x < 0.527 \\ \dfrac{1.489-x}{1.489-0.527}, & at \quad 0.527 < x \le 1.489 \\ 0, & at \quad x > 1.489 \end{cases} , \qquad (4.17)$$

which gives us $\alpha_1 = \mu_{NPV}(0) = 0.241$, $G' = \mu_{NPV}^{-1}(\alpha_1) = 1.257$, and according to (4.11) – (4.14), **R** = 0.101, **V&M** = 0.013.

We can see that due to the reduction of the level of uncertainty the extent of risk decreased by almost an order of magnitude. Thus, an investor has an effective tool of control of the efficiency of the investment process.

Calculations also show us that the more significant the uncertainty of the initial data, the higher the risk. Therefore, in some cases an investor just **must refuse to make a decision** and should take additional steps to fight the uncertainty. To know when it is justified to refuse to make a decision, an investor must have a gauge of uncertainty of the current informational situation (of instability of a project). It is logical to use factor α_1 to take such measurements. In case of a full certainty, $\alpha_1 = 0$. With reference to $\mu_{NPV}(x)$ of (4.16), the calculations produce $\alpha_{11} = 0.655$, and for $\mu_{NPV}(x)$ of (4.17), $\alpha_{12} = 0.241 < \alpha_{11}$. An investor can also interpret α_1 linguistically just as in the case of linguistic evaluation of the extent of risk, and thus, he or she can establish a boundary of α_1 beyond which the uncertainty becomes unacceptable.

4.4 The simplest method to evaluate the investments' risk

Let's consider the process of business planning under uncertainty, when the uncertainty of initial data is such that allows to generate **interval-symmetric estimations** (for example: the minimum of sales is $5 million, the maximum of sales is $10 million, the average value is (5 + 10)/2 = $7.5 million). This situation is especially typical for draft business projects, when the initial data contains the maximum of uncertainty.

The interval-symmetric fuzzy parameters can be characterized with the two, rather than three, real numbers: the mean value of the parameter and the scatter.

If all the parameters of a business plan are interval-symmetric, then it is possible to reduce the resulting index of business plan efficiency – **the net present value of a project** (***NPV***) – to the interval-symmetric form, neglecting the error introduced by the asymmetry of the fuzzy factor of discounting. Let's designate **NPVav –** a mean expected value of NPV, **Δ –** a disperse of NPV from the mean, i.e. $\Delta = NPV_{av} - NPV_{min} = NPV_{max} - NPV_{av}$, $NPV = NPV_{av} \pm \Delta$.

Let's introduce the coefficient of stability of a business plan:

$$\lambda = NPV_{av}/\Delta. \tag{4.18}$$

It is clear, that the higher the coefficient of stability of a business plan, the more reliable the investment decision. At $\lambda \to \infty$, there is no data scatter, and the investment project can be accepted or rejected without any risk of wrong decision. However, in reality there are always adverse scenarios, when $NPV_{min} = NPV_{av} - \Delta < 0$, i.e. $\lambda < 1$.

Thus, the rational investment projects assume a positive mean expected outcome of the project, i.e. $\lambda > 0$ is carried out.

Hence, we study the risk of an investment project assuming its stability within the limits of $0 < \lambda < 1$.

Let's use the results from [Nedosekin, 2000] to reproduce the derivation of the formula for evaluation of risk of a project in the simplest case. If the **NPV** of a project is a triangular fuzzy number **(NPV_{min}, NPV_{av}, NPV_{max})**, **the risk of the project RE** (Risk Estimation is an expectation of **NPV** < 0) is estimated as follows:

$$RE = \int_0^{\alpha_1} \varphi(\alpha) d\alpha, \tag{4.19}$$

where

$$\varphi(\alpha) = \begin{cases} 0 & , \ at \ \ 0 < NPV_1; \\ \dfrac{-NPV_1}{NPV_2 - NPV_1} & , \ at \ NPV_1 \le 0 \le NPV_2; \\ 1 & , \ at \ \ 0 > NPV_2; \end{cases},$$

$$\alpha = [0, 1]. \tag{4.20}$$

$$NPV_1 = NPV_{min} + \alpha \times (NPV_{av} - NPV_{min}), \tag{4.21}$$

$$NPV_2 = NPV_{max} - \alpha \times (NPV_{max} - NPV_{av}), \tag{4.22}$$

$$\alpha_1 = - NPV_{min} / (NPV_{av} - NPV_{min}). \tag{4.23}$$

Let's designate

$$l = - NPV_{min}, \ \ m = NPV_{av} - NPV_{min}, \ q = NPV_{max} - NPV_{min}. \tag{4.24}$$

Then (4.19) becomes:

$$RE = \int_0^{\alpha_1} \varphi(\alpha) d\alpha =$$

$$= \int_0^{\alpha_1} \frac{l - m\alpha}{q(1-\alpha)} d\alpha =$$

$$= \frac{m}{q}\alpha_1 - \frac{l-m}{q} ln(1-\alpha_1) =$$

$$= \frac{-NPV_{min}}{NPV_{max} - NPV_{min}} +$$

$$+ \frac{NPV_{av}}{NPV_{max} - NPV_{min}} ln \frac{NPV_{av}}{NPV_{av} - NPV_{min}}. \qquad (4.25)$$

Taking into account the symmetry of evaluations, we have:

$$RE = \frac{\Delta - NPV_{av}}{2\Delta} +$$

$$+ \frac{NPV_{av}}{2\Delta} ln \frac{NPV_{av}}{\Delta} =$$

$$= \frac{1}{2} + \frac{\lambda}{2}(ln\lambda - 1). \qquad (4.26)$$

This is the elementary formula for the risk evaluation. Fig. 4.6 shows the dependence of the extent of risk of a project on the index of stability of a business plan (we shall refer it to as a risk function).

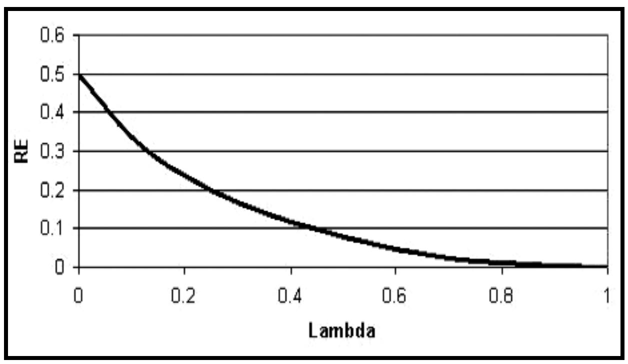

Figure 4.6 The risk function

From Fig. 4.6 we can see that the acceptable risk of a project is up to 10% (the risk function grows slowly, almost linearly). At the risk from 10% to 20% we have a boundary situation, and when risk is over 20% the function of risk grows excessively, and the risk becomes unacceptable. Such subjective evaluations of the acceptable risk result in specifications of the type shown in Table 4.3 (for the evaluations of the first column of Table 4.3 the equation (4.26) was solved for **λ: RE** = 10%... 20%).

Table 4.3 Risk level and the status of risk of the project

Value λ	Risk level of the project	The status of risk of the project
0.44-1	<10%	Acceptable risk
0.25-0.44	10% – 20%	Threshold risk
0-0.25	> 20%	Unacceptable risk

Now it is possible and very simple to determine the status of risk of an investment project in one step.

Calculation example. By the results of the financial analysis of a business plan we've obtained the triangular interval-symmetric evaluation **NPV** = (-40, 40, 120) thousand euro, or in other notation, **NPV** = 40 ± 80 thousand euro. Let's determine the status of risk of the project.

Solution.

$\lambda = 40 / (120 - 40) = 0.5 > 0.44$. The risk of the project is acceptable (7.7%).

5. THE EVALUATION OF INVESTMENT ATTRACTIVENESS OF AMERICAN STOCK

Let's use the matrix method described in chapter 3 of this book to evaluate the investment attractiveness of the American stock. The similar analysis was done two years ago in the article [Nedosekin 2001], but since then the market has changed, and some revision of reference points and classifiers as well as the revision of the expert model are in order.

We still claim that after two years the P/E ratio (the profitability of investments) and the capitalization of assets Cap (the indirect factor of reliability) remain to be the key factors for evaluation of investment attractiveness of stock. Analyzing the investment attractiveness of the Software and Programming industry of the Technology sector in 2001, we could not find any under-valued companies to invest in. The course of events had proved our fears: all companies in the chosen industry have depreciated greatly, and some of them had gone bankrupt.

At the same time in the last two years the importance of certain factors in the complex evaluation of the stock quality has changed. The factors directly connected with own capital of the company, such as the return of equities (ROE), the burden of owns capital with long-term liabilities (Debt/Equity Ratio), and market reassessment of own capital (Price/Book Ratio), attracted the additional attention of investors.

So, we modified the system of complex evaluation factors, adding the factors connected with own capital and the efficiency of its use by corporation. The analysis of quasi-statistics of the Technology sector conducted in January, 2003, enabled us to construct the corresponding factors' histograms (see example in Fig. 5.1) and to adjust the fuzzy classifier of factors of the type of table 5.1. Please note that we characterize the factors' levels as "low" or "high" based on their attribution to complex evaluation of the investment quality of stock, rather than from quantitative point of view. For example, the value P/B = 5 is quantitatively high, but this evaluation is "unhealthy" and causes the risk of low reassessment, and from this point of view the investment attractiveness of the stock is low.

Table 5.1 Fuzzy classifier of factors

Level of factor	**Range of values for factors:**					
	***P/E* for Cap**		**Cap,**	**ROE%**	**D/Eq**	**P/B**
	<1 billion	**> 1 billion**	**Million $**			
Very low (VL)	30 - ∞	45 - ∞	0-50	<0	> 1	> 4.5
VL – L	25-30	40-45	50-100	0-5%	0.7-1	4-4.5
Low (L)	20-25	30-40	100-300	5-10%	0.4-0.7	3.5-4
Very Low (VL)	15-20	25-30	300-500	10-15%	0.3-0.4	3-3.5
Average (Av)	10-15	20-25	500-1 000	15-25%	0.2-0.3	2.5-3
Average-High	7-10	15-20	1 000-3 000	25-30%	0.15-0.2	2-2.5
High (H)	5-7	10-15	3 000-5 000	30-35%	0.1-0.15	1.5-2
H – VH	5-5	10-10	5 000-10 000	35-40%	0.05-0.1	1-1.5
Very High (VH)	2-5	5-10	Over 10 000	> 40%	0-0.05	<1

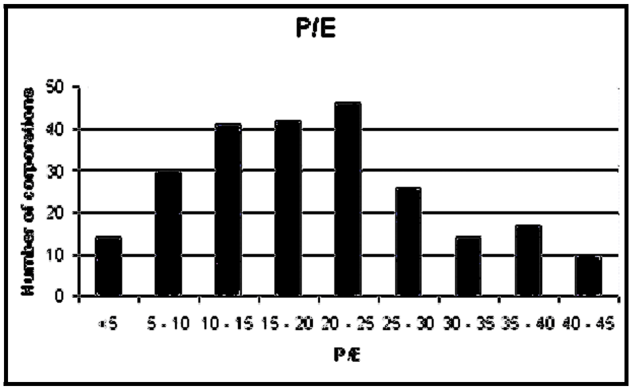

Figure 5.1 Histogram for the P/E ratio

Ranging the chosen five factors by their importance for complex evaluation, we arrive to the following system of preferences:

$$\mathbf{P/E \succ Cap \succ ROE = D/E = P/B} \qquad \textbf{(5.1)}$$

That results in the choice of factors' weights in the complex evaluation [Nedosekin 2001]:

$$p_1 = \frac{1}{3},\ p_2 = \frac{4}{15},\ p_3 = p_4 = p_5 = \frac{2}{15},\ \sum_{i=1}^{5} p_i = 1 \qquad \textbf{(5.2)}$$

Then by analogy to the technique described in section 3, we get a complex factor A_N for each stock by method of double convolution:

$$A\_N = \sum_{j=1}^{M} \alpha_j \sum_{i=1}^{N} p_i \lambda_{ij}, \qquad \textbf{(5.3)}$$

where i is the index of separate factor out of their total number **N** = 5, j is the index of the level of the factor out of total number of levels **M** = 5, λ_{ij} is the rank of the ith factor of the jth level, defined with the functions of belonging of the corresponding trapezoid fuzzy numbers (abscissas of their tops are defined in Table 5.1),

$$\alpha_j = 0.2 * j - 0.1 \qquad \textbf{(5.4)}$$

these are the abscissas of maximums of functions of belonging of the term-set of linguistic variable "**The evaluation of stock**". Then, the mean expected rank of jth level, weighed on all **N** factors, is estimated by the formula

$$y_j = \sum_{i=1}^{N} p_i \lambda_{ij}, \qquad \textbf{(5.5)}$$

and it is justified.

$$A\_N = \sum_{j=1}^{M} \alpha_j y_j\ . \qquad \textbf{(5.6)}$$

Having obtained the evaluation **A_N**, we can recognize it according to the Table 5.2.

Table 5.2 Function of belonging for integrated factor "The evaluation of stock"

Value of A_N	Values of functions of belonging for sub-sets of the variable "The evaluation of stock":				
	VL	L	Av	H	VH
0-0.15	1	0	0	0	0
0.15-0.25	*(0.25-A_N)* *10*	*(A_N- -0.15)* *10*	0	0	0
0.25-0.35	0	1	0	0	0
0.35-0.45	0	*(0.45--A_N)* *10*	*(A_N- 0.35)**10*	0	0

Value of A_N	Values of functions of belonging for sub-sets of the variable "The evaluation of stock":				
	VL	L	Av	H	VH
0.45-0.55	0	0	1	0	0
0.55-0.65	0	0	*(0.65-A_N)**10*	*(A_N-0.55)**10*	0
0.65-0.75	0	0	0	1	0
0.75-0.85	0	0	0	*(0.85-A_N)**10*	*(A_N-0.75)**10*
0.85-1.0	0	0	0	0	1

Let's define the linguistic variable "Stock trade recommendation" with the term-set of values "Strong Buy (SB – definitely buy), Moderate Buy (MB – maybe buy), Hold (H – hold), Moderate Sell (MS – maybe sell), Strong Sell (SS – definitely sell)". This very system of trading recommendations is offered on-line by [Zaks.com].

Let's establish a bi-unique conformity of the linguistic variable at the level of sub-sets introduced by us: VL – SS, L – MS, Av – H, H – MB, VH – SB. Thus, we have connected the quality of a stock with its investment attractiveness. Then, the variable **A_N** is also the carrier for the term-set of linguistic variable «**Trade recommendation**», with the same functions of belonging of the carrier to sub-sets of values.

So, we have made all necessary calculations by the formulae (5.1) – (5.6), having studied the quasi-statistics of the Technology sector in January, 2003 (we had all the necessary initial information on 493 corporations). The results of our scoring are summarized in the Table 5.3.

Table 5.3 The results of the scoring of the Technology sector

Factors	Distribution of factors by levels				
	VL	L	Av	H	VH
Cap	33%	24%	25%	12%	6%
P/E	37%	22%	26%	12%	3%
ROE	13%	48%	27%	4%	7%
D/E	9%	16%	12%	10%	52%
P/B	16%	8%	19%	30%	27%
Summary	26%	23%	23%	13%	14%

The histogram of the resulting factor on the selected corporations is shown on Fig. 5.2.

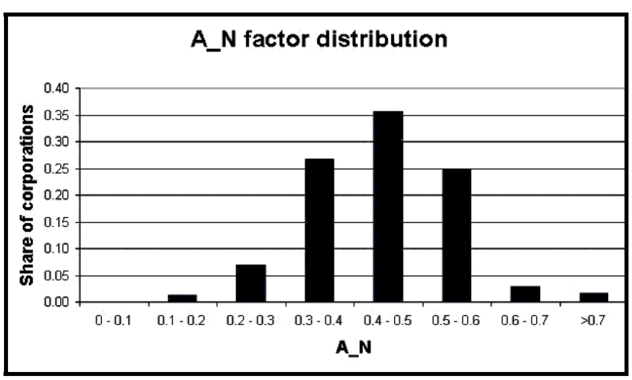

Figure 5.2 Histogram of the factor A_N

It follows from the obtained data that the lion's share of enterprises have the A_N factor within the limits of 0.3 – 0.5. This characterizes the investment attractiveness of the Technology sector as something intermediate between **low and average**. To be more exact, we have obtained the factor **A_N = 0.431** for the sector. According to the table 5.2, the odds that the value of the factor is, nevertheless, **average** are 80 to 20. This fact, however, does not bring any good news for the sector. The conclusions are unfavorable.

The trading recommendation for the sector is "Hold", with dynamics towards "Moderate Sell".

Only 6 companies out of the 493 studied issuers of the sector have their score higher than 0.65 (when the trade order is "Moderate Buy"). Adding further restriction of capitalization of at least 5 billion dollars, we get a winner with the score A_N = 0.706: EDS – Electronic Data Systems. We will watch this stock. We do not believe it will fall sharply moreover, it has growth potential. The corporation had big problems in September, 2002 (they even mentioned bankruptcy), but after transition into the range of realistic prices (from $12 to $72) and after the Pentagon awarded EDS a contract to upgrade electronic systems of the US Air Force and Navy, the business picked up. The war with Iraq seems to be coming (we write these words on 01/29/2003), and someone can profit from trouble, why not EDS? Today (01/29/2003) the company's price per share is $16.85. Let's remember this number.

Unfortunately, the Technology sector market as a whole is most likely headed for an unpleasant future. We expect a profound correction of NASDAQ down to the level of, let's say, 1000 – 1100. It shall be the second fundamental fall of the market since July, 2002, but there is no other way. There are only two ways to return the investment attractiveness to this market: by greatly increasing the profits or by reducing the stock prices. The first is hardly realistic, so we are left with the second.

Table 5.3 represents something like a phase portrait of the sector, like a map. It shows the key problems for the sector. First of all, it is low profitability of the sector corporations which places both the P/E and ROE into the "red corner". Secondly, over 50% of the sector corporations have low capitalization (less than 500 million dollars), therefore, there is an increased risk of investments in these issuers' stock (against the background of the continuing US economic recession). Of course, the peculiarity of wartime may change everything. When the companies working for war revive, it traditionally affects the whole market. However, in the times of peace the existing picture can provoke only one reaction – withdrawal of capital.

6. FUZZY OPTIMIZATION OF A STOCK PORTFOLIO

Historically, the first method of optimization of a stock portfolio was offered by Markowitz [Markowitz.] Its essence is as follows.

Let a portfolio contain N types of securities, each characterized by the following five parameters:

- The initial price W_{i0} of one security before putting it into the portfolio;
- The number of securities n_i in the portfolio;
- The initial investments S_{i0} into the given portfolio segment, where

$$\mathbf{S_{i0} = W_{i0} * n_i}; \qquad \mathbf{(6.1)}$$

- The mean expected value of the return of security r_i;
- Its standard deviation σ_i from r_i.

It is clear from the listed conditions, that the random variable of the security's return has normal distribution with the first initial moment r_i and at the second central moment σ_i. This distribution does not necessary have to be normal, but the normality follows automatically from the conditions of the Wiener's stochastic process.

The portfolio itself is characterized by the following:

- Total volume of portfolio investments S;
- Share price distribution of portfolio securities $\{x_i\}$, where the following is true for the initial portfolio

$$x_i = \frac{S_{i0}}{S}, \qquad \sum_{i=1}^{N} x_i = 1, \qquad i = 1,\dots,N; \qquad \mathbf{(6.2)}$$

- The correlation matrix $\{\rho_{ij}\}$. Its indices characterize the connection between the returns of i^{th} and j^{th} securities. ρ_{ij} = -1 means a fully negative correlation, and ρ_{ij} = 1 means a fully positive one. ρ_{ii} = 1 is always true as a security fully positively correlates with itself.

Thus, the portfolio is described with the system of statistically connected random variables with normal distributions. Then, according to theory of random numbers, the expected profitability r of a portfolio is

$$r = \sum_{i=1}^{N} x_i * r_i \ , \qquad (6.3)$$

and its standard deviation σ is

$$\sigma = (\sum_{i=1}^{N}\sum_{j=1}^{N} x_i * x_j * \rho_{ij} * \sigma_i * \sigma_j)^{\frac{1}{2}} . \qquad (6.4)$$

We can describe the task of managing such portfolio has as follows: to determine the vector $\{x_i\}$ that maximizes the criterion function r of (6.3) with the set limit on the risk level σ, estimated in (6.4):

$$\{x_{opt}\} = \{x\} \mid r \to max, \sigma = \text{const} \le \sigma_M, \qquad (6.5)$$

where σ_M is the risk of a security with the maximum mean expected profitability.

Formula (6.5) is none other than the classical problem of square-law optimization solved by any known computational approach.

Note. In the Markovitz approach to portfolio selection the risk is understood as an extent of fluctuation of the expected portfolio income either towards its decrease or increase, rather than a risk of inefficiency of investments. We can effortlessly change the problem (6.5) to the one limited by the probability of a portfolio profitability ending up below a pre-conditioned level, rather than by the fixed standard deviation.

Setting various levels of restrictions for σ to solve the problem (6.5), it is possible to obtain the dependence of maximal profitability on σ

$$r_{max} = r_{max}(\sigma). \qquad (6.6)$$

The expression (6.6) referred to as **effective boundary** of the portfolio set in coordinates "risk – profitability" is a piecewise-parabolic concave continuous function. The right boundary limit is the point corresponding to the case of portfolio containing one security with the maximum mean expected profitability.

The very widespread in practice of portfolio management Markovitz' approach has, however, a number of modeling assumptions poorly reflecting the reality of the described object – the stock market. First of all, there is no time invariance of price processes. That disallows describing the profitability of a security by a random variable with known parameters. The same also applies to correlation.

Considering a portfolio from modeling classes, and the price background of indices of modeling classes as quasi-statistics, we should model this quasi-statistics with multi-variant fuzzy-probabilistic distribution with parameters in the form of fuzzy numbers. Then, the conditions (6.3) – (6.4) are **formulated in a fuzzy set form**, and the problem of square-law optimization is also solved in this form. The solution of the problem is an effective boundary in a shape of a strip fuzzy function. It should be reduced to the triangular shape by the usual rules.

A fuzzy vector of the optimum portfolio shares corresponds to each piece of the effective boundary corresponding to the abscissa of portfolio risk.

And, finally, given the control specifications of profitability and risk (the benchmark of the modeling portfolio) which the results of portfolio management should meet, if the benchmark falls into the effective boundary zone, there is a risk that the modeling portfolio will underperform the benchmark as far as the factor of profitability is concerned. Because the expected profitability of a portfolio is a triangular fuzzy number, the risk of inefficiency of portfolio can be evaluated by the same formula, as the risk of inefficiency of investments (the method of evaluating the risk of investments is considered in chapter 5 of this book).

Thus, the statement of the modified Markovitz' approach is finished. From this point on the text of the monograph assumes that the method deals with quasi-statistics of modeling indices in portfolio. This quasi-statistics is modeled by means of N-dimensional fuzzy-probabilistic distribution. Having evaluated the parameters of this distribution as fuzzy numbers, we solve the problem of square-law optimization in fuzzy terms, obtaining an effective boundary in the form of curvilinear strip.

Let's consider an elementary example of an American modeling portfolio consisting of two modeling classes: the governmental long-term bonds (**Class 1** described with the LB Government Bond index) and highly capitalized stocks (**Class 2** described with the S&P500 index). The summary data of both indices is shown in Table 6.1.

We could also evaluate the correlation of the two indices. But, as it will be shown later, it won't be necessary in this case. For now though, we shall designate the factor of correlation ρ_{12} for generality.

We must notice before we begin that the case of two components portfolio is **degenerated** from the optimization standpoint. In this situation the full set of portfolio decisions corresponds to a piece of a curve line on the plane in general case, and it is also an effective boundary. So, in this example we are mainly looking for an analytical formula of the effective boundary in coordinates "risk-profitability," rather than trying to solve the optimization problem.

Table 6.1 Initial data of modeling classes

Number of the modeling class	Expected profitability $r_{1,2}$ % annually			Expected volatility $\sigma_{1,2}$, % annually		
	min	average	max	min	average	max
1 Bonds	6.0	6.1	6.2	0.6	0.7	0.8
2 Shares	10	12.5	15	20	25	30

Let's write down (6.3) – (6.4) in a particular case

$$r = x_1 r_1 + x_2 r_2 ; \tag{6.7}$$

$$\sigma^2 = x_1^2 \sigma_1^2 + 2x_1 x_2 \sigma_1 \sigma_2 \rho_{12} + x_2^2 \sigma_2^2 ; \tag{6.8}$$

$$x_2 = 1 - x_1. \tag{6.9}$$

All "constant" coefficients in (6.7) - (6.9) are the triangular fuzzy numbers, and the operations of addition, multiplication, and deduction are defined in the space of triangular fuzzy numbers.

Since in our case $\sigma_2 >> \sigma_1$, there exists approximate equality:

$$\sigma = x_2 \times \sigma_2 , \tag{6.10}$$

which gives us

$$r = \frac{r_2 - r_1}{\sigma_2} \times \sigma + r_1 ; \tag{6.11}$$

the equation of the effective boundary in the shape of a strip with rectilinear borders (see Fig. 6.1).

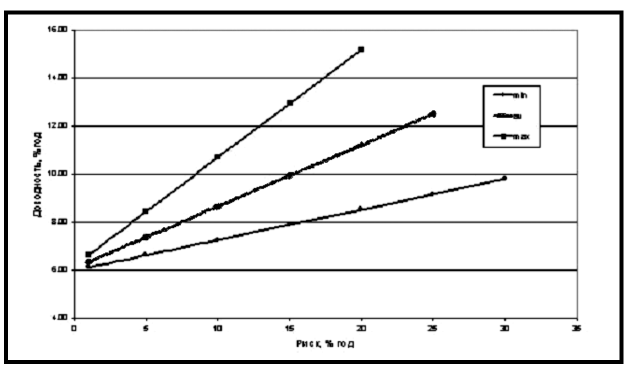

Figure 6.1 Effective boundary as a strip with linear borders

The coefficient of proportionality in (6.11) is none other than the well-known in portfolio management Sharpe's factor [Sharpe] – the relation of the index profitability (minus risk-free component of profitability) to its volatility. In our case it is fuzzy and can be reduced to the triangular form by the following rule:

$$\left(\frac{r_{2min} - r_{1max}}{\sigma_{2max}}, \frac{r_{2av} - r_{1av}}{\sigma_{2av}}, \frac{r_{2max} - r_{1min}}{\sigma_{2min}}\right) \tag{6.12}$$

Table 6.2 contains the boundaries of modeling class of bonds in the structure of modeling portfolio for various risk levels.

Table 6.2 Optimum share of bonds in a portfolio

Risk of portfolio, % annually:		1	5	10	15	20	25	30
Share of bonds in portfolio	Max	0.967	0.833	0.667	0.500	0.333	0.167	0.000
	Av	0.960	0.800	0.600	0.400	0.200	0.000	0
	Min	0.950	0.750	0.500	0.250	0.000	0	0
Disperse		0.067	0.083	0.167	0.250	0.333	0.167	0

The dispersion of the portfolio boundaries at the edges of the strip is lower than in the middle. That is because at the edges of the strip of the effective boundary the portfolio possesses quite a certain style: the modeling class of stocks corresponds to the greater profitability, and the modeling class of bonds corresponds to smaller risk.

It is also necessary to mention that the dispersion of the profitability and risk parameters influences the solution of the optimization of funds portfolio problem much more appreciably than the dispersion of the parameters of correlation matrix (as proved in the [Chopra V.K., Ziemba W.T]). Therefore, when preparing the initial data for the portfolio analysis one should mainly concentrate on minimization of the dispersion of profitability and risk parameters of the portfolio assets.

Thus, by the results of fuzzy-set optimization, we obtained the optimal distribution of modeling assets with fuzzy rather than fixed boundaries. It is the maximum of what we can achieve under the substantial uncertainty.

7. FORECASTING OF FUNDS INDICES

The optimization of model funds portfolio is based on the initial data of indices obtained as a result of scientific forecasting. When it is required that the theory of forecasting must predict quite exact values of some parameters in the future, the forecasting of the stock indices is no longer a scientific problem. The modern theory of stock indices forecasting is based on a different subject of forecasting. It is not the indices themselves, but rather **their rational tendencies** stipulated by rational behavior of a collective investor in the funds assets.

There exists a whole class of theories of forecasting based on historical analysis of data. None of these theories control the consistency of the input data to the corresponding methods. However, in the case when there is an epistemological paradigm break between historical data and the future, the corresponding past history of indices essentially devalues, and the methods based on the use of these statistics begin to produce erroneous and unverifiable forecasts. The last crisis of the stock market was an excellent test for all the existing methods of forecasting, all of which have failed this test.

Hence, the science of forecasting of the stock market tendencies has to change its bases. The theory of rational investment choice might just be the new possible basis for the modern theory of forecasting. The materials of the US stock market can supply the demonstrative base of this theory.

The American market was euphoric about its economic capability for a long time. At the moment, it is trying to overcome the recession and panic mood of investors, and searches for new economic reference points. I believe several more years of upheaval still await us, but we can already see the light at the end of the tunnel. It is an **increasing rationalization of the investment choice**, and for at least the next five years the world stock market will sail under this flag. The shock from bursting of the bubble of the "new economy" must still be overcome and thought over.

As a consequence, the optimal management of both personal and institutional funds portfolios gradually acquires the traits of **active, efficient and alert management**. *Active* management assumes the abandonment of passive strategy of portfolio management (for example, following the market indices, the way the balanced funds do). *Efficient* management is carried out in real time, with continuous reassessment of the level of portfolio optimality (even within one trading day, modern computer programs facilitate it). *Alert* management assumes the presence of the established warning signals in the system triggered by a change in the preset macro-economic, financial, political, and other parameters. The triggered alert signal causes automatic execution of some chain of the pre-established crucial rules of re-balancing of funds portfolio.

The optimal portfolio management based on the fuzzy evaluations of profitability and risk factors of assets must take into account valid prognostic models (see [Nedosekin] for brief description of generating such a model). It seems apparent that those groups of market subjects that forecast the financial flows and operate them more successfully, under conditions of the new world will have exclusive advantages that can be hardly overstated. To own the information is to own the world.

The primary factor of success here is to understand the **rational investment behavior**, and to have qualitative and quantitative mathematical model of such behavior. It took a lot of scientific effort to describe the rational investment choice (for example, with the function of investment utility, even using fuzzy sets [Mathie-Nikot]). However, a research of the aspects of rational investment behavior not based on the detailed analysis of the stock market and macro-economic conditions in the country where the investments are made is **useless**, and unfortunately practically everybody is looking at the problem from the standpoint of such analysis. The approach used by the **Lattice Financial** [Lattice Financial] is a pleasant exception. This approach traces detailed modeling connection between macro-economic factors and quantitative assessments of the tendencies of the stock market. Here, however, we can see another extreme: there is excessive measure of mechanistic understanding of connections on macro- and micro-levels in the [Lattice Financial] models. In this case there is a direct temptation of a "recursive forecasting," where the future is determined by the present within the precision of probabilistically distributed stochastic signal. The factor of rationalization of the choice is completely excluded of such models.

It is necessary to fill this gap in the theory of funds investments, and simultaneously to develop the mathematical tools of rational investment choice models by introducing the formalisms of the theory of fuzzy sets. T he fuzzy sets are a natural choice as a number of model parameters cannot be determined quite precisely because we are dealing with subjective human preferences. These preferences are blurry not only because we cannot collect plausible statistics, but also because investors themselves sometimes do not fully understand what they want, and how they distinguish "good" stocks from the "bad" ones. The purpose of this study is to understand what is "good" and what is "bad" for an investor.

7.1 Theoretical background for a rational investment choice

The simplest and most constructive definition of a rational investment choice is this: it is a choice that is profitable in the medium-term prospect (with possible intermediate losses). For example, if the rationally expected profitability of a stock is negative for the next 2-3 years, such choice cannot be rational. It means that an investor does not understand something in the nature of the market. The whole story of the previous two years is a story of how the investors in the US stocks lost the money – the story of irrational investments. Hereinafter, we study the rational investment choice, i.e. the choice of investments into various stock instruments with the scientific expectation in the increase of the investment capitalization.

We obtain a hypothetical model of the effective (equilibrium, rational) market, when several equally informed and abiding by the same rules agents, who do not form the coalitions, operate in the economic game. There is no rational market in reality because there are always unfair insiders who create a veil of information noise around their activity, and profit off the irrational acts of other investors. It is a dishonest activity, unfair competition, which in some cases is prosecuted by law. Some experts add oil to the fire. While clearly understanding the nature of macro-economic processes, they give advices that generate mass irrational investment choice resulting in losses. I, in particular, refer to the advices of one of the most respectable US consultants Abby G. Cohen who in 2001 advised investors "to sit tight", copying the principle of balanced index funds, buying or selling nothing (see [Pundit Watch: Abby Cohen] for details). This "advice" brought the losses of hundreds of billions of dollars.

The fact that the stock market bubble of the "new economy" bursted (though not completely), is itself the characteristic of the inefficient market that searches for a new balance, new efficiency, and rationality. And our task is to determine this hypothesis of a new efficiency, to formulate the paradigm of that rational market that America and the rest of the world are striving for.

So, we shall consider the behavior of a rational investor (private or institutional), who is forming his or her generalized modeling investment portfolio of securities of the three basic types issued in one country:

State bonds.
Corporate bonds.
Corporate stocks.

Remark 1. We do not consider interest-free cash bank accounts in the currency of the country because on the long run the money is the assets with negative profitability (due to the inflation). Therefore, such investment choice cannot be rational. The money in the framework of rational choice is not an investment resource, but the means of immediate payment for goods. It becomes the investment resource only when it brings profits, being invested somewhere and bringing revenues as a payment for their delayed demand in payments.

Remark 2. At this stage of modeling we do not consider separately the behavior of the investor related to hedging of the investment risks by means of derivative securities. It is the subject of a separate research.

We assume that at the initial moment of investment (t=0) an investor sinks into the generalized investment portfolio the monetary capital nominally equal to a **unit** in the currency of the country, where the investments are made.

Analyzing the rational investment choice, we take into consideration the macro-economic conditions in the chosen country at the moment of making investment decision. We will see later what these conditions are.

Our scientific task is to determine the **cause-and-effect relation of rational investment choice**, i.e. to answer to the following question: what external quantitative and qualitative macro-economic factors will force a rational investor to form the generalized investment portfolio (in some proportion). Understanding this causal relationship quantitatively and qualitatively, we can proceed to the construction of prognostic models. We do not expect that the real market behavior will follow our forecast absolutely precisely (we do not believe in the exact forecasts at all). We forecast the rational trend of market behavior, and not the behavior itself. At the same time we assume that the real market will asymptotically approach this trend during the next five years, and we ascribe the fluctuations of the market to irrational investment choice caused by erroneous (unscientific) assessment of news, rumors and market alerts, including the macro-economic ones.

The grouping of assets declared above is justified because the liabilities, regardless of their nature (securities or cash deposits), express investor's expectation of certain future fixed revenues. The criteria of clusterization are a profitability of assets investments, reliability of the assets issuer, and the character of the assets volatility (Table 7.1):

Table 7.1 Integrated classifications of stock investments

Type of real assets	Profitability of real assets	Reliability of real assets (risk 1)	Volatility of real assets (risk 2)
State bonds	Low	High	Low
Corporate bonds	Low and average	Average and low	Low and average
Corporate stocks	Average and high	Average and low	High

Reliability and volatility are the two sides of the risk related to the assets investments. Combining these two measures in one, we can assert that the risk of investments into state bonds is low, into corporate bonds it is average, and into corporate stocks it is high.

If we consider selected types of assets as **modeling classes** than each class can be compared with the stock index in the form of the index of cumulative final profitability in currency of the country as explained in the previous section of this book. We also believe, that the default risks of real assets in the structure of modeling assets are eliminated, and the main risks is the **synchronous volatility** of the rate price of real assets (due to almost complete correlation of real assets inside the modeling assets).

Clearly, studying the historical data and using experts' reasoning or prognostic models (table 7.2) it's possible to perform point estimate of profitability and risk based on these indices. For simplicity, at this stage of consideration we assume the obtained estimations to be **non-fuzzy**.

Table 7.2 Initial data for modeling assets

Type of assets	Asset profitability	Asset risk	Assets weight in portfolio
State bonds	r_1	σ_1	x_1
Corporate bonds	r_2	σ_2	x_2
Corporate stocks	r_3	σ_3	x_3

The sum of weights in the portfolio is equal to a unit. Depending on the type of the choice (conservative, intermediate, aggressive) the investor increases or reduces the share of stocks and bonds.

Remark 3. At the beginning of the research we don't know the point forecast estimate of profitability and the risk of assets (otherwise solving our problem is pointless). We do know, however, the relations of order of profitability and risks which we will include in the mathematical model later.

Remark 4. Let's reiterate that the rational investment assumes rational evaluations of the assets' profitability and risk. Hereinafter, we speak about rational evaluations for making rational investment decisions, unless indicated otherwise. We will discuss later how to obtain these rational evaluations.

Certainly, the constructed generalized investment portfolio is monotonous ([Nedosekin]). That is, we know that the monotonous decrease of profitability from asset to asset is accompanied in our model with the corresponding monotonous decrease of the risk of investments. The monotonicity of portfolio is what renders it balanced (equilibrium) and conforming to the golden rule of investing, and all modeling assets comprising a monotonous portfolio participate in forming of the effective boundary.

Therefore, we assert that the simultaneously investing into the three selected assets makes the investment choice rational, irrespective of the allotment of these assets in the portfolio. It follows also from the simple reasons that all the listed assets organically supplement each other, creating fully diversified set of investment instruments. None of the three modeling assets is unnecessary, because these assets fully overlap the space of rational values "risk-profitability." Of course, the real assets comprising modeling components of a portfolio can force each other out from the effective boundary and then the presence of "retarded" real assets makes the portfolio non-monotonic.

In the most general case the effective boundary of a modeling assets portfolio set is a concave continuous function in the "risk – profitability" coordinates. Let's draw on the diagram, alongside with the effective boundary, the isolines of two-dimensional function of utility of investment preference ([Sharpe], Fig. 7.1) that have common tangent with the effective boundary. Each isoline will correspond to the certain type of the investment behavior. The aggressive rational investors correspond to the isoline with the smaller inclination of the tangent, and the conservative rational investors correspond to the isoline with large inclination (they demand greater profitability as a payment for the increasing risk, than the aggressive investors).

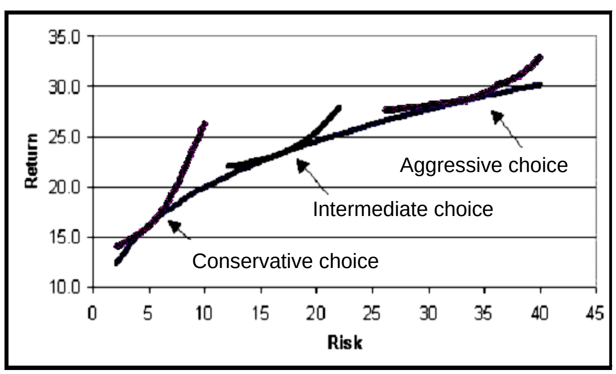

Figure 7.1 Effective boundary and isolines of the utility function

Naturally, the traditional or fuzzy classification of investment preferences by the shape of the effective boundary suggests itself. Here is the simplest method of such classification. Lets define σ_{min} to be the risk of the left end of the effective boundary, σ_{max} to be the risk of the right end of the effective boundary, and $\Delta = (\sigma_{max} - \sigma_{min})/3$. Then, the investment choice can be tied to the risk extent of a stock portfolio as follows:

- A conservative choice corresponds to the portfolio risk from $\boldsymbol{\sigma_{min}}$ to $\boldsymbol{\sigma_{min} + \Delta}$;
- An intermediate choice corresponds to the portfolio risk from $\boldsymbol{\sigma_{min} + \Delta}$ to $\boldsymbol{\sigma_{min} + 2\Delta}$;
- An aggressive choice corresponds to the portfolio risk from $\boldsymbol{\sigma_{min} + 2\Delta}$ to $\boldsymbol{\sigma_{max}}$.

Fig. 7.1 shows an effective boundary of a portfolio in the most general form. We shall see that for a generalized investment portfolio in our definition effective boundary degenerates to almost a straight line. Let's prove this statement using the theory of monotonous portfolio [Nedosekin]. Since our generalized investment portfolio is monotonous, there is a relation of the order of magnitude of portfolio assets' profitability and risks. The elementary market studies give us such relation:

$\mathbf{r_3 >> r_2 \approx r_1}$

$$\sigma_3 >> \sigma_2 \approx \sigma_1 \qquad (7.1)$$

The relation (4.29) is universal and it is correct for all generalized classes of investment instruments in all countries and at all times. It captures the essence of the major difference between the fixed and uncertain income issues: since the issue's income is not known beforehand (which is an essential risk), it should be paid for with an essential gain of profitability. At the same time, comparing to the risk and profitability of stocks the risk of state and corporate bonds is imperceptible. This is also correct for profitability of assets.

Let's mention one more time: we are studying the behavior of modeling issues, not the real ones. For example, it is well-known that the so-called "junk bonds" can make profit comparable to that of stocks. However, the portion of trade in such bonds is so small that its weight in the index of bonds is very low and does not violate the condition (4.29).

To show the correctness of (7.1) quantitatively let's build a Russian portfolio with the following fuzzy expert evaluations of parameters for 2002 (table 7.3).

Table 7.3 Data on Russian funds portfolio for 2002

Type of assets	Profitability of assets% annually in RuR	Risk of assets,% annually in RuR	Weight of assets in portfolio,%
State bonds	(16,17,18)	(1,2,3)	25
Corporate bonds	(20,21, 22)	(2,4,6)	25
Corporate stocks	(40,60,80)	(20,30,40)	50

The correlational matrix of assets formed as a point estimate for the previous two years of processing of the historical data is shown in Table 7.4.

Table 7.4 Correlational matrix of Russian funds assets

Type of assets	State liabilities	Corporate liabilities	Corporate shares
State bonds	1	0.96	0.26
Corporate bonds	0.96	1	0.02
Corporate stocks	0.26	0.02	1

Fig. 7.2 shows the result of modeling by SBS Portfolio Optimization System programs (we will discuss this program in chapter 10):

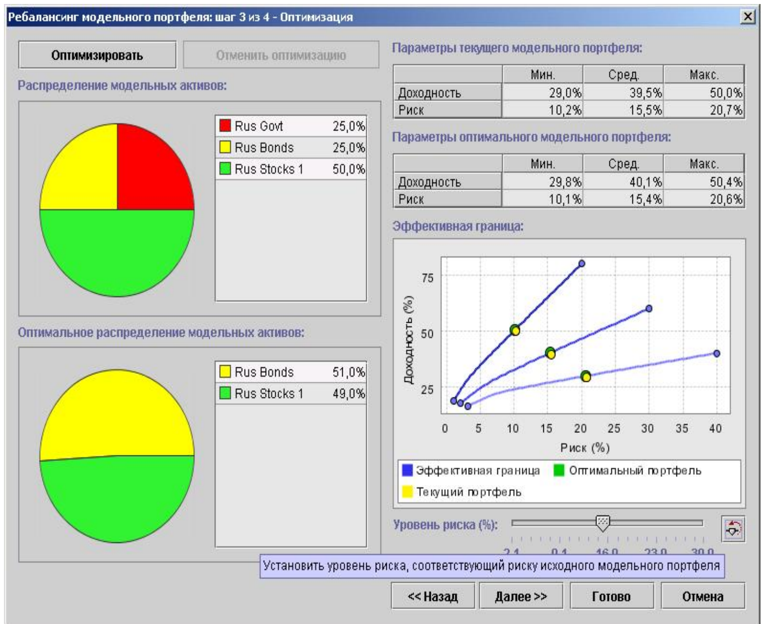

Figure 7.2 The result of modeling of generalized Russian investment portfolio

One can see that in this case the effective boundary is a strip with almost rectilinear borders which can be without essential error easily interpolated with a straight line. This feature of the strip has been shown in the example in chapter 5: for generalized portfolio consisting of two assets (stocks and bonds), by the virtue of (7.1), an effective boundary asymptotically transforms into a strip with straight lines for top and bottom, described by the formula:

$$r = \frac{r_A - r_B}{\sigma_A}\sigma + r_B\,; \qquad (7.2)$$

where r_A is a profitability on stocks, r_B is a profitability of bonds, σ_A is a risk of stocks, and σ_B is a risk of bonds, all are triangular fuzzy numbers.

Since the profitability and risk of the state and corporate bonds are close (in comparison with the same for stocks), and the correlation of these bonds is close to a unit (for understandable reasons, because all these bonds circulate on the internal market in uniform macro-economic environment) all bonds can be incorporated into one super class of assets. Then (7.2) is true and the statement that our generalized investment portfolio has an effective boundary in the shape of a strip with linear borders is proved.

Three very important conclusions follow from the above:

Conclusion 1. As the difference between the state and corporate bonds is hardly noticeable in generalized investment portfolio, the optimal decision is to make the shares of these components in the portfolio equal. This rational requirement will relieve us of the effect of "foolish optimization", when in the optimal portfolio corporate bonds supersede the state ones exactly because of the notorious lack of difference between them (see Fig. 7.2, where the bottom circular diagram corresponding to portion distribution in the optimum portfolio excludes the state bonds).

Conclusion 2. Let's reduce the equation of the straight line (7.2) to the canonical form:

$$\frac{r - r_B}{\sigma} = \frac{r_A - r_B}{\sigma_A} = const\,. \qquad (7.3)$$

The factor on the left hand side of (7.3) is about equal to the Sharp factor of a portfolio (should the numerator allow for the state bonds only). We see that on all intervals of the effective boundary the investment choice of an investor, irrespective of his or hers slant (conservative, intermediate, aggressive), has the same extent of economic efficiency (which can be approximately evaluated by the Sharpe factor for the stock index). In other words, the payment for the risk in the form of increasing profitability is compounded uniformly, and it is impossible to achieve the special conditions of investment with the maximum economic benefit. For example, for the boundary of Fig. 7.1 such maximum exists in the intermediate type of the investment choice range, accordingly, there is an economic preference of this choice. In our case it is not available.

Conclusion 3. *The choice of two modeling assets is always optimal and rational.* It follows from the monotonicity of the generalized portfolio, because the sub-set of assets of monotonous portfolio also forms a monotonous portfolio.

All the above-stated tells us that the problem of rational choice is reduced to the problem of defining the relation between stocks and bonds, on the one hand, and the stock and other markets, on the other hand. If stocks are “superheated”, it is necessary to gradually replace them with bonds. If the bonds are “superheated” (low revenues, high cost) it is necessary to get rid of bonds. There is also a situation, when it is necessary to leave the stock market fully or partly. The main question is the same: in what proportion and why it should be done? The answer to this question is given by the principle of investment balance.

7.2 Principle of investment balance

The investment balance is a basis of rational investment choice. This principle originates in the mathematical theory of games (in particular, an equilibrium game is a game with the zero sum [Neumann - Morgenstern]). The balance principle is the analogue of the law of conservation of energy and substance. If the capital is not placed well, it will flow to a better location. If there is no good place for it anywhere within the limits of its form, it will change the form.

For example, the current American stocks crisis of overvaluation is a search for a new equilibrium level. The capital is restless in super-heated stocks, and it outflows from there. It tries to settle in bonds, but it is not a good place for it, either. The conditions of government bonds are not interesting, and the conditions of corporate bonds are unreliable (all these conclusions are made within the limits of the current situation of the US stock market). So, what are the options? Capital either escapes abroad, where it changes currency and accumulates in European banks, or gradually settles in less liquid forms (precious metals, antiques, real estate, etc.).

The balance is an equal preference. From the point of view of the investment choice it is indifference. We have just shown that the effective boundary of the generalized investment portfolio is linear or is close to it. At no point of the boundary the economic advantage (additional gain) can be achieved by the Sharpe criterion. No economic advantage means nobody wins in the market game (the game sum is zero). People who invest into super-heated stocks lose. Those investing into under-valued shares win. But, when all players operate rationally nobody gets any additional gain, because all players equally effectively distribute the basic source of the income – the gross domestic product of the country – at the level of industries and corporations, where investments are made. Accordingly, a rational investor does not care where to invest on the rational market. And, with the absence of additional reasons, he or she just invests 50% in bonds, and 50% in stocks, positioning the investment choice as intermediate (under additional reasons here we understand, for example, the old age of an investor inclining one to be more conservative). We shall name the 50:50 choice the control portfolio point.

There are other important applications of the principle of balance. The monotonous portfolio is balanced, because it is formed according to the golden rule of investment, and this rule interprets the principle of balance, as a principle of reasonable diversification. Irrespective of the type of choice, a reasonable investor “never puts all eggs in one basket.” However wholeheartedly one likes to risk, he or she must save for a rainy day. On the other hand, by investing in bonds only one will never get rich or provide for retirement, so it is necessary to risk. And the fact of incomplete correlation of stocks and bonds indices testifies to the mutual elimination of risks of these indices in diversified portfolio.

Let's also notice that there is such a thing as irrational (unreasonable) diversification. The anti-scientific formula of “following the market,” unshakable belief that market is always right, generate the effect of erroneous balancing by the Abby Cohen scenario (discussed above) – when instead of immediately leaving stock market (because the collapse is unavoidable, and all macro-economic factors testify to that), and converting stocks in dollars, and in turn dollars in euro (it’s time to forget about diversification when market collapses), we are forced “to balance,” i.e. to bear the losses.

Let's construct the quantitative model of the principle of balance. For this purpose let’s correct the generalized investment portfolio as follows:

- Modeling class of stocks ($\mathbf{r_A}$ is a profitability of stocks, $\boldsymbol{\sigma_A}$ – stock risk, and $\mathbf{x_A(t=0) = x_{A0}}$ is a starting portion of stocks in the portfolio).

- Modeling class of bonds (r_B is a profitability of bonds, σ_B is a risk of bonds, and $x_B(t=0) = x_{B0}$ is a starting portion of bonds in the portfolio).
- The fictitious modeling class of non-fund assets described only by the size of the portion of the capital $x_N(t)$ withdrawn from the stocks (**A**) and bonds (**B**) assets. Initially $x_N(t=0) = 0$, i.e. conditions of modeling assume that an investor forms the funds portfolio first.

The essence of the correction is that we have decided to consolidate all bonds since they are hardly distinguishable against the background of stocks. We have also provided for the possibility that an investor might move capital from stock securities to other types of investments. For all cases, the equation of the balance of shares remains true:

$$x_A(t) + x_B(t) + x_N(t) = 1, \qquad (7.4)$$

And in the portfolio control point the following is true

$$x_A(t) = x_B(t) = (1 - x_N(t))/2. \qquad (7.5)$$

Let's introduce into the model three additional exogenous macroeconomic factors:

- The profitability r_I and risk σ_I of the index of inflation of the country's currency. Notice that the parameters of profitability and risk here are close to the ones for bonds. The state bonds can lag the inflation a little, and the corporate bonds can outpace it, but they are very close in comparison with the parameters of profitability and risk of stocks;
- The profitability r_{GDP} and risk σ_{GDP} of the index of growth of gross domestic product (GDP) of the region, where the investments are made;
- The profitability r_V and risk σ_V of the index of the currency exchange rate of the region where the investments are made, in relation to the RuR.

Also in course of forecasting of stock indices we shall continuously observe and forecast (on the basis of all the foregoing initial information) the P/E ratio (formed as follows: the numerator is a price index of stocks, the denominator is a net profit of corporations per averaged share, and the growth rate of this profit can be evaluated by the growth rate of GDP and the inflation rate).

With the reference to the USA, the index of inflation (estimated by the factor of current profitability, according to the data [USA CPI]) is shown on Fig. 7.3, and the P/E ratio index is shown on Fig. 7.4 (the data is from [Luskin]).

Before developing the models of the investment balance let's ask a qualitative question: is there a balance between the inflation and profitability of capital in general, and if not, with causes the upset?

In 1996 the Chairman of the US Federal Reserve System Alan Greenspan said [Greenspan]:

> "*Clearly, sustained low inflation implies less uncertainty about the future, and lower risk premiums imply higher prices of stocks and other earning assets. We can see that in the inverse relationship exhibited by price/earnings ratios and the rate of inflation in the past.*"

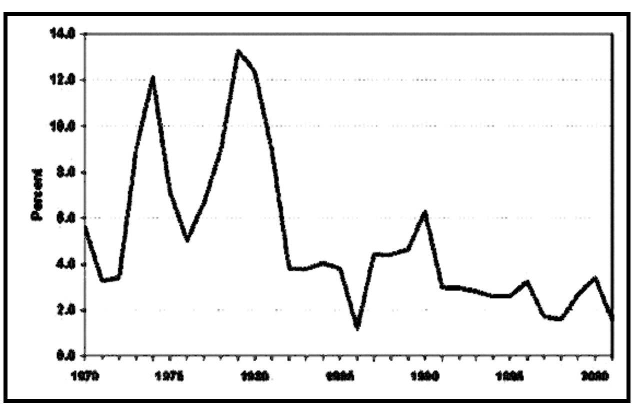

Figure 7.3 Inflation in the USA from 1971 to 2002

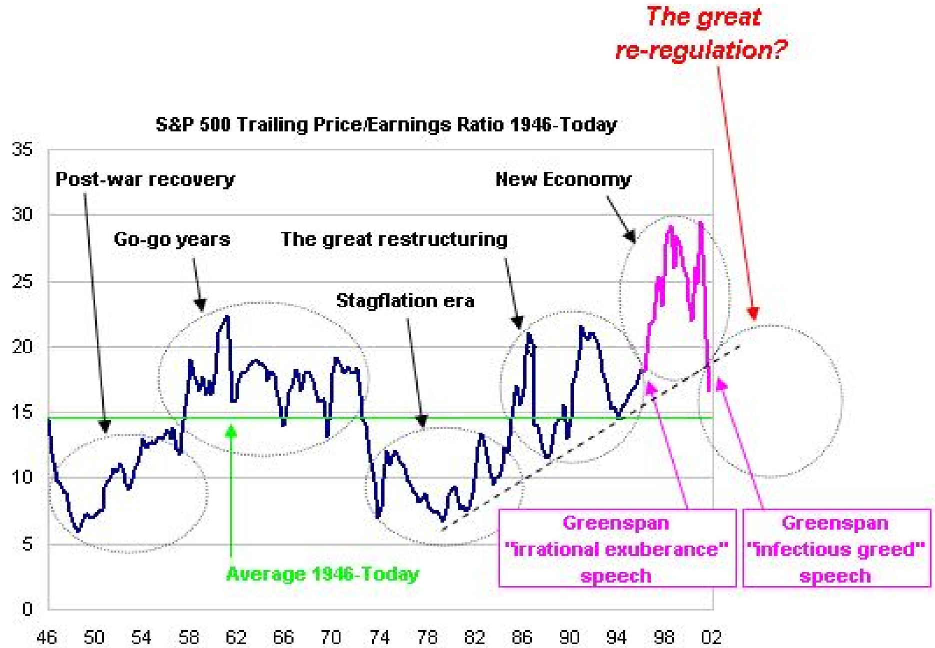

Figure 7.4 US capital profitability from 1946 to 2002 (by the P/E ratio index)

For stocks, the risk premium is the level of profitability of capital we study. Here, Greenspan is right. For example, during the era of stagflation (1975 – 1982) high rates of inflation provoked low values of the P/E ratio. It is explained by the fact that the government and corporate bonds have always been leveled by inflation, outstripping it a little, and that made them an attractive investment alternative to stocks (see the historical data on state bonds with annual maturity [the US Treasury]). And in this sense the market always looked for the investment balance.

But once (after 1995) the balance was lost, and Greenspan had predicted it in the same speech [Greenspan], continuing the foregoing:

> *"But how do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade? And how do we factor that assessment into the monetary policy? We as central bankers need not be concerned if a collapsing financial asset bubble does not threaten to impair the real economy, its production, jobs, and price stability."*

Many saw a prophecy in this Greenspan's statement, and, as a matter of fact, it was. Greenspan points out that there is a sea of "easy money" which does not want to reckon with macroeconomics, and this money, over-heating stock values, create an investment disparity.

The only thing Alan Greenspan does not want to take into account is the social consequences caused by the crisis of large scale over-valuation of stocks. The evaporation of pension assets makes people distinctly alarmed; they mistrust the stock market and want to leave it. The crack in the US pension system can bring far-reaching consequences, up to the partial curtailing of the voluntary component of this system. This undermines the corporate investment mechanism, and can lead to the essential slowdown of the rates of economic growth and cardinal worsening of financial health of corporations. It leads to the drop in profits and, as a consequence, to an even greater drop in quotations. This is how the spiral of the contraction of corporate financing that collapses the economy works.

Let's consider a simple estimation factor of the disparity of stock investments, obtained from:

A_N Score (t) = I (t) * PE Ratio (t), (7.6)

where I(t) is the rate of inflation in share units. We also suppose that the following is true

$r_B(t) = I(t) + \Delta(t)$, (7.7)

where $\Delta(t)$ is the level of risk premium (today in the USA this factor fluctuates about 1-5% annually, depending on the type of bonds).

The factor of the disparity is shown on Fig. 7.5.

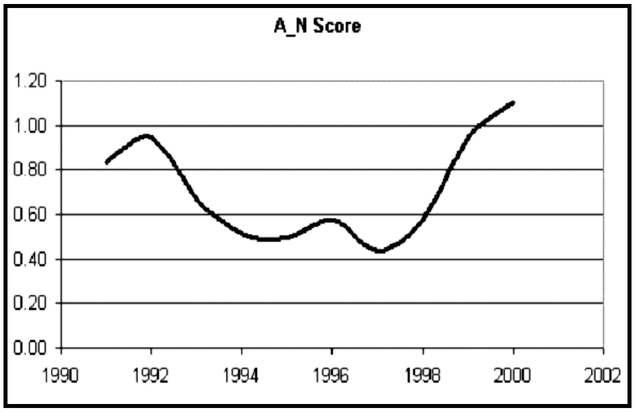

Figure 7.5 Factor of investment disparity (USA)

From the analysis of historical data on Fig. 7.3 – 7.5 it is obvious, that the positive disparity is reached, when A_N Score (t) < 0.5 (the situation of 1994 – 1997, when the P/E ratio varied in the range of 17 to 22 with inflation of 2.5-3% annually). Clearly, the bonds are uninteresting, and the return of capital of 5% annually (plus the expected rate of growth) can leave nobody indifferent. The inflow of capital and growth are expected, and the growth comes. At the same time, the "rally" (i.e. the steady "bullish" market) keeps the volatility of the stock index at the level it was "before the rise."

The balance is achieved at 0.6 < **A_N Score (t)** < 0.7 (the situation 1994 – 1997 and 1998 – 1999 when the P/E ratio varied in the range of 24 to 28 with inflation of 2.5-3.5% annually).

The negative disparity happens at **A_N Score (t)** > 0.7 (1991 – 1992, 2000 – 2001, when the P/E ratio achieved and exceeded 30, and the inflation exceeded 5-6% annually). The stocks cease to be interesting, the bonds start to come into play; however, the inflation itself raises the system risk of the stock market, its unreliability. The outflow of capitals and recession are expected, and the recession comes (at the same time steady "bearish" market returns the volatility of indices to the level it was "before the rise"). Fig. 7.6 shows, how the rate volatility of the stock index grows as the negative disparity tends to increase **[Luskin].**

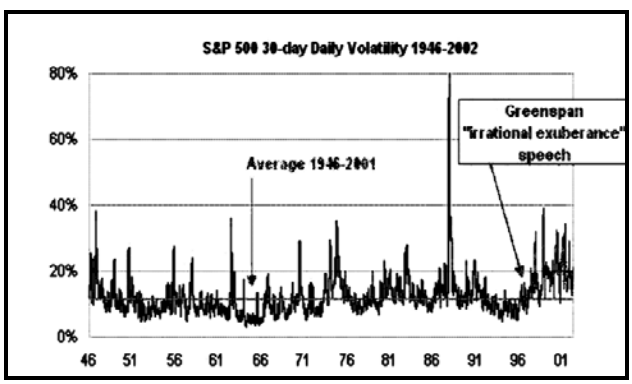

Figure 7.6 The growth of the rate volatility of the index of shares

The problem is that we cannot directly transfer obtained boundaries of the parity equilibrium choice, without considering a number of remarks which essentially correct our evaluations.

First of all, the boom of corporate scandals in the USA shows that the assessments of profitability of the enterprises are over-valued. It leads to the correction of the equilibrium range of the P/E ratio from 24-28 (historically) to 18-22 (for the period from 2003 to 2008-2010). An investor demands additional risk premium because the new developments in accounting manipulation were revealed. Secondly, a long-term investor accounts for the potential growth of inflation tendency from 2 to 3-4% annually with restoration of the investment picture of the beginning of the 90's. Applying this to the factor of investment disparity, the balance is reached at the level of 0.65 – 0.75. Provided during the foreseeable period the inflation does not increase, the P/E ratio of 18-22 is the level of positive disparity when it is possible to return to the purchase of stocks.

7.3 The model of rational dynamics of stock investments

So, modeling rational investment choice, we establish that it is controlled by the principle of the investment balance. When disequilibrium occurs due to the internal conditions of the stock market or by the virtue of changed macro-economic conditions, a disparity arises, and the system seeks to return the lost balance through the flow of capitals from one kind of assets into another.

We shall construct our model of the investment balance as a description of a dynamic system (a finite state machine where the investment tendencies are the states, we will discuss it later) where the initial allocation of stock assets and the subsequent flows between assets on the interval of discrete forecast time $\mathbf{t_{start}, t_{start}+1, ..., t, \dots, t_{end}}$ is modeled**.** By default, we choose the unit interval of forecasting $\mathbf{\Delta T}$ = 0.25 year (one quarter).

To begin with, let's classify the tendencies arising in course of the investment choice.

From the point of view of flow of capital it is possible to single out the following tendencies:

- The *attraction* tendency (when the capital is withdrawn from other forms and is invested in stocks);
- The *waiting* tendency (when the inflow of capital stops, but there is still no outflow from stocks);
- The *withdrawing* tendency (when the capital flows from the stock market into other forms).

From the point of view of portfolio choice, it is possible to single out the following:

- The *aggressive* tendency (when the capital prefers stocks to bonds and other forms);
- The *intermediate* tendency (when the capital searches for investment balance between stocks and bonds);
- The *conservative* tendency (when the capital prefers bonds and other forms to stocks).

On the Cartesian product of the above-stated classifications the combined tendencies are formed: the waiting-aggressive, the attractive-conservative and so on.

Initial rational allocation of assets is modeled in Table 7.5. The parameters a_i and b_{ij}, found in Table 7.5, they are particular for each country and for each period of the forecasting. Within the limits of five-year term of the forecasting we assume these parameters to be constant, provided the expert model does not postulate the opposite.

Table 7.5 Initial distribution of capital

Number of input situation	Rate of inflation	Level of P/E	Rational share distribution of investments			Tendency
			$x_A(t_{start})$	$x_B(t_{start})$	$x_N(t_{start})$	
1	Low inflation, deflation: $0 - a_{1,\%}$	Till b_{11}	1	0	0	Attractive-aggressive
2		b_{11} - b_{12}	0	0	1	Withdrawing
3		Over b_{12}	0	0	1	Withdrawing
4	Moderate inflation: $a_1 - a_{2,\%}$	Till b_{21}	0.5	0.5	0	Attractive-intermediate
5		$b_{21} - b_{22}$	0	1	0	Attractive-conservative
6		Over b_{22}	0	0.5	0.5	Withdrawing-conservative
7	High inflation, hyper-inflation, stagflation: higher than $a_{2,\%}$	Till b_{31}	0	1	0	Attractive-conservative
8		$b_{31} - b_{32}$	0	0	1	Withdrawing
9		Over b_{32}	0	0	1	Withdrawing

Table 7.6 Scenario of investment transitions

Number of input situation from Table 7.5	Rational flows of capital: + inflow, - outflow, 0 – no flow			Tendency
	A	B	N	
1	+	-	0	Waiting-aggressive
2	0	0	0	Waiting
3	-	0	+	Withdrawing
4	+	+	-	Attraction
5	0	+	-	Attractive-conservative
6	-	+	0	Waiting-conservative
7	0	+	-	Attractive-conservative
8	-	0	+	Withdrawing-conservative
9	-	-	+	Withdrawing

From Tables 7.5 and 7.6 it is obvious that with the increase of the investment risk (with the growth of inflation or with the drop in the return of capital) the capital in hands of a rational investor seeks to change its form, and that is immediately fixed by the corresponding change of the tendency towards withdrawing.

7.4 The phases of the forecasting

All the necessary theoretical qualitative prerequisites for the construction of a forecasting model are stated. A general scheme of modeling constructed on the basis of the principle of investment balance and the corresponding rational investment choice, seems to be as follows (provided that the securities investments are made on the territory of the third country, for example, in Russia):

- **Phase 1.** Initial modeling of capital allocation is carried out according to Table 7.5. All initial values of the forecasted stock indices are fixed (these values are either known or formed by the researcher on the basis of additional reasons).
- **Phase 2.** Exogenous macro-economic tendencies, such as gross domestic product, inflation, the relation of national currency to Russian RuR are analyzed on the whole interval of the forecasting.
- **Phase 3.** Current rational tendencies of the flow of capital are quantitatively determined according to Table 7.6.
- **Phase 4.** The calculated corridor of profitability on cumulative indices is predicted based on the following specialized models:
  - risk premiums for bonds
  - elasticity of profitability on the factor of the return of capital for stocks and mutual fund shares
  - reducibility of parameters – for the second echelon stock (low capitalization).
- **Phase 5.** The profitability and the risk of index assets are evaluated.
- **Phase 6.** The forecast of share relation in the generalized investment portfolio (A, B, N) is modeled on the basis of the specialized models of rebalancing.
- **Phase 7.** The values of the index and the level of profitability of the investment capital are forecasted.
- **Phase 8.** The forecasting discrete time is incremented, and the forecasting process restarts from the stage of quantitative analysis of tendencies according to Table 7.6 (phase 3). If the forecast is completed we pass to the next phase.
- **Phase 9.** The translation of indices in national currency into indices in RuR (the standard index) is carried out.
- **Phase 10**. The calculated corridor of final profitability for standard indices is estimated.
- **Phase 11**. The expert evaluation of final profitability and risk on standard indices is constructed.

The foregoing procedure is based on the application of the specialized models and techniques which are considered hereafter.

7.5 Models and methods for stock indices forecasting

We shall construct fuzzy macro-economic model based on the forgoing presuppositions. We shall then use this model to describe the method of stock indices forecasting. The detailed description of the model and the method can be found in the Appendix 1 and in [Nedosekin].

7.6 Example of the forecast (USA)

The initial conditions for modeling are shown in Table 7.7.

Table 7.7 Initial conditions of forecast modeling

Forecasting factor	Code	Initial data (January, 01 2002)
Starting value of indices on the basis of national currency	Stocks (S&P500)	1,154
	Bonds (TYX cumulative)	1.0
	The P/E ratio	37
	GDP rate (GDP)	1.1%
	Inflation rate (I)	2.1%
	Currency exchange (J)	30.1
Starting returns and risks		
Stocks, % annually	r	-16%
	Sigma	24%
Bonds, % annually	r	5.5%
	Sigma	0.2%
The modified Sharpe factor	Sh (t_{start})	-0.896
The investment tendency for capital redistribution	Number	3
The comment (tendency)		Withdrawing

The result of modeling, according to the mathematical calculations from the Appendix 1, is shown on Fig. 7.7 (the relation of the forecasted and actual tendencies of the American stock market).

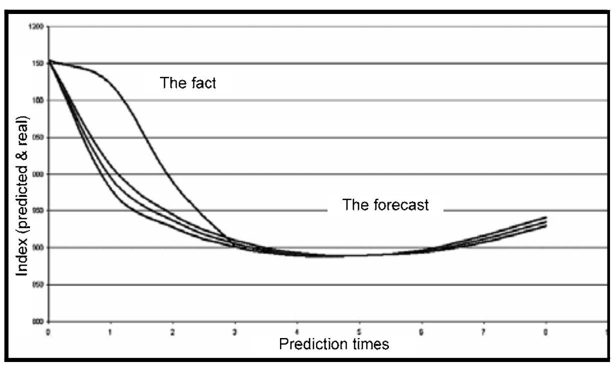

Figure 7.7 The forecasted and the actual index of American stocks

The qualitative assumptions about over-valuation of the US stock market made by the author in [Nedosekin] (the approximate bottom of the S&P500 index for 2nd quarter of 2002 is also found there), had gotten its quantitative confirmation. Back-testing the models for the first two quarters of 2002 had shown that due to panic fear of losses the American investors habitually support the market certainly doomed to fall (indicated by the concavity of the curve of actual values of the index), instead of hastily getting rid of falling stocks and bonds. Thus, the divergence of the forecast and the reality is caused by extremely irrational behavior of investors in their hopeless struggle.

The optimum management of our investment portfolio is shown on Fig. 7.8.

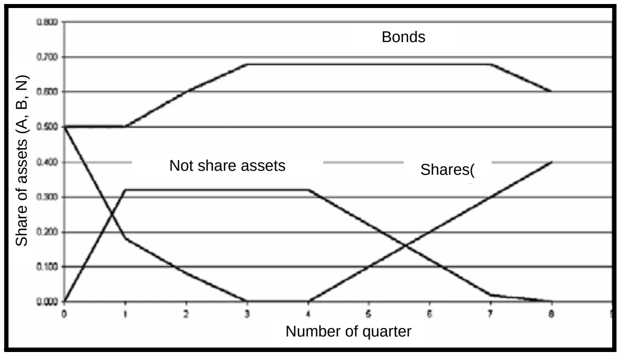

Figure 7.8 The trajectory of rational management of stock portfolio

Should we operate according to the Abby Cohen scenario (the balancing in the control point) we would lose (Fig. 7.9) up to the third of the capital.

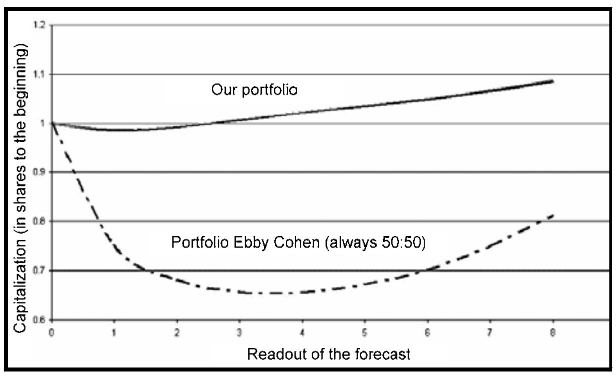

Figure 7.9 Comparative capitalizations of the two portfolios (ours and the Abby Cohen's)

But because we had withdrawn a third of the capital from the market for half a year, thus reducing the share of stocks to zero, we saved the assets from the depreciation and now we can return on the market when it reaches the investment balance (according to the plan – in 2003 - 2004). Actually, the whole 2002 was a year to stay away from the American stock market.

Thus, we have shown that the scientifically grounded forecast of stock indices based on the hypothesis of rational investment choice is a panacea from long-term losses and the precondition for competent optimization of modeling funds portfolio.

8. STRATEGIC PLANNING

The large versatile companies (hereafter referred to as Corporations), conducting the worldwide coordinated business, quite frequently apply the matrix structure in the strategic planning. The rows of such a matrix contain the countries, where the business is run, and the columns contain the main lines of the Corporation's business. An intersection of a row and a column represents a business-unit with double subordination: to regional management, on the one hand, and to the management of businesses of the Corporation, on the other.

The specificity of strategic planning in such complex economic systems as Corporations is in the simultaneous optimization of two business-portfolios: the regional portfolio and the portfolio of businesses. Consider the following:

- not only the classical factors of economic efficiency (sales revenues, profit, economic value added, etc.) are used as the criteria of the optimization of business-portfolios, but also the factors of the prospects of the business, considered from the point of view of its life cycle;
- strategic planning is multi-level and it is conducted at the levels of the regional communities of a Corporation, on the one hand, and at the levels of the businesses of a Corporation, on the other hand;

- planning is developed under the maximum of uncertainty of the market factors. It includes two types of uncertainty:
  a) uncertainty of qualitative recognition of the current quantitative level of factors;
  b) uncertainty of forecasted values of parameters of a strategic plan.

This chapter offers a number of ways to apply the formalisms of the theory of fuzzy sets to account for the uncertainty in strategic planning. Let's consider, for example, the simplest strategic plan of a regional community of a Corporation for the current fiscal year, with the assumption, that the regional community itself (hereafter referred to as Company) is a three-level hierarchical system: the Company contains several departments (business-units in the strategic plan of the Corporation), and the departments themselves include some local businesses. The strategic planning in the Company is carried out on all three levels: for the local businesses, for the departments and for the Company as a whole. It is expedient for the painless aggregation of information that the plans' structures on all the selected levels of hierarchy are of the same type.

The structure of a strategic plan usually includes the following main blocks:

- the macroeconomic block that describes the external environment of the business;
- the marketing block that describes the market of the businesses and the competition on that market;
- the financial block that contains all the financial factors of the planned object;
- the decisions block that records the actions on the business enhancement, the schedules and the responsible persons.

In the course of further discussion we will consider the typical and quite pertinent options for the application of fuzzy descriptions for each selected block (except for the decisions block where the mathematics does not take part).

8.1 Macroeconomic block. The PETS-analysis

In the course of the primary analysis of a business's macro-economic environment the four components PETS-model is often applied (**P** – Political and Legal, **E** – Economic, **T** – Technological, **S** – Social).

The model considers the expectancy of the events, which are considered an opportunity or a risk for the given business. Often managers of the top echelons push the business executives responsible for the development of a strategic plan to determine the probabilities of certain events quantitatively. Certainly, for such quantitative evaluation there are no bases. The term "probability" itself does not hold water in such application, because individual events of non-uniform origins are not statistical, and it is impossible to speak of the frequency of their occurrence.

The following two ways of introducing fuzzy descriptions in the PETS-model immediately arise:

- replacement of the "probability" with the **expectancy** expressed in such qualitative terms as "very low expectancy," "low expectancy," "average expectancy," "high expectancy," "very high expectancy." Here, the expectancy cannot have the quantitative carrier;
- replacement of the binary scale "business opportunity/risk" with the quinary scale: "more likely opportunity," "presumably opportunity," "uncertainty," "presumable risk," "likelier risk."

The field of events (and their evaluation) can be formed based on the polling of experts.

8.2 Marketing block. Analysis of strengths and weaknesses of a business

For the evaluation of strengths and weaknesses of a business (the SWOT-analysis, S – Strength, W – Weakness, O – Opportunities, T – Threats) it is possible to use both quantitative, and qualitative scales.

Let's introduce a two-level scale containing a number of **basic factors**, which, in turn, are characterized with the sets of their **component factors**. We can choose the following basic factors to characterize the strength /weakness of a business: Technology, Quality, Costs, Sales, Prices, Service, and Logistics. The component factors, for example, of the "Sales" factor, are the access to the developed channels of sales, regional presence, access to the key consumers, advertising, the qualification of personnel, etc.

Then, the aggregation of the component factors to comprise the basic factors can be carried out on the basis of the matrix scheme, considered in sections 2.10 and 3.2 of this book and as applied to the complex financial analysis of the businesses.

Let's consider an example. Let the basic factor be defined by two component factors with weights 0.6 and 0.4, and let the level of the first component factor be determined by an expert as 0.8, and the level of the second component factor – as 0.5. We have to determine the level of the basic factor qualitatively.

Solution. Let's take as a basis the set of functions of belonging of the standard quinary classification on the 01-carrier. The function of belonging of the sub-set "High level of the factor", defined on 01-carrier x, has the following analytical form:

$$\mu_4(x) = \begin{cases} 0, 0 \le x < 0.55; \\ 10(x - 0.55), 0.55 \le x < 0.65; \\ 1, \ 0.65 \le x < 0.75; \\ 10(0.85 - x), 0.75 \le x < 0.85; \\ 0, 0.85 \le x <= 1. \end{cases} \quad (8.1)$$

In turn, the function of belonging of the sub-set "Average level of the factor" has the following analytical form:

$$\mu_3(x) = \begin{cases} 0, 0 \le x < 0.35; \\ 10(x - 0.35), 0.35 \le x < 0.45; \\ 1, \ 0.45 \le x < 0.55; \\ 10(0.65 - x), 0.55 \le x < 0.65; \\ 0, 0.65 \le x <= 1. \end{cases} \quad (8.2)$$

Accordingly, the identification of the first component factor's level tells us that it is high with the extent of confidence of 0.5, and it is very high with the same extent confidence. The identification of the second component factor's level results in an unequivocal recognition of this level as average.

To evaluate the strength /weakness of the business by the basic factor, let's make a table for calculation of SW by the matrix scheme (Table 8.1).

Table 8.1 Matrix for evaluation of SW

Factors	Values	Functions of belonging for levels of component factors				
		Very low (μ_1)	Low (μ_2)	Average (μ_3)	High (μ_4)	Very high (μ_5)
1	0.6	0	0	0	0.5	0.5
2	0.4	0	0	1	0	0
Nodes		0.1	0.3	0.5	0.7	0.9

Then, the calculation by the matrix of Table 8.1 produces:

$$SW = 0.6 * (0.5 * 0.7 + 0.5 * 0.9) + 0.4 * 1 * 0.5 = 0.68 \quad (8.3)$$

Identifying this level of SW according to the formula (8.1), positions it as 100% high.

The example is finished. It is possible to carry out the matrix convolution in a similar fashion while transitioning from the individual figures of strength/weakness of a business by the basic factors to the integral figure of strength/weakness of the business. You only need to determine the weights of the basic factors in the integral evaluation.

8.3 Marketing block. Two-dimensional analysis "competitiveness – prospects"

Let us have two integral measures: business competitiveness and its prospects. Then, we can carry out the analysis within the framework of the model Shell/DPM 3x3 which has a high practical significance for strategic planning [Hoichens-Robinson]. The main conclusion which can be made on the basis of the model is to position the business and by that to determine its place and the role in cumulative portfolio of the businesses of a Company.

It is possible to estimate the **competitiveness (A)** on the basis of the following main factors:

The ratio of the business's share and that of the basic competitor (RCP – Relative Competitive Position) – $\mathbf{a_1}$;

The Company name recognition – $\mathbf{a_2}$;
The strength of the business/Company brand – $\mathbf{a_3}$;
The development of distribution network – $\mathbf{a_4}$;
The technological position of the business – $\mathbf{a_5}$.

It is possible to estimate the **prospects of the business (B)** on the basis of the following main factors:

The share of the business in the structure of the department of the Company – $\mathbf{b_1}$;
The rate of growth of the business – $\mathbf{b_2}$;
The intensity of the business's competition on the open market – $\mathbf{b_3}$;
The profitability of the business – $\mathbf{b_4}$;
The sensitivity of the business to business-cycles – $\mathbf{b_5}$.

All the listed basic factors $\mathbf{a_i}$, $\mathbf{b_j}$ can be assigned a 01-carrier. If historically these factors are measured on the basis of a different quantitative scale (for example, from 1 to 5) then we can transform from the existing scale to 01-carrier with a simple linear conversion.

We can evaluate the integrated factors **A** and **B** quantitatively by the formula (1) (the standard matrix scheme of evaluation), but to recognize the levels of these factors it is necessary to apply **three-level 01-classification** (Fig. 8.1) with sub-sets "Low level, Average level, High level" of the linguistic variable "**Level of factor**" rather than the standard five-level 01-classification. The Shell/DDM is a 3x3 dimensional model (there are only 9 positions of the business), causing the transition from five levels to three.

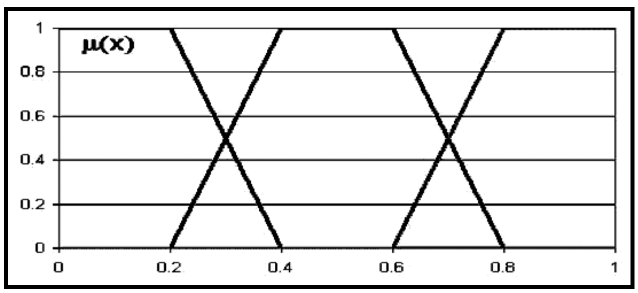

Figure 8.1 Three-level 01-classification

The weights of the basic factors in integral evaluation are selected on the basis of additional reasons. One possible reason is the Fishburn's principle.

Let's consider an example. Let the integral factor A of the business be defined by the five basic factors with the system of weights and quantitative levels established by table 8.2, and let the integral factor B of the same business be defined by the five basic factors with the system of weights and quantitative levels established by table 8.3. We must qualitatively determine the levels of integral factors ***A*** and ***B*** on the basis of three-level 01-classification.

Table 8.2 Matrix for evaluation of integral factor A

Factor	Value	Function of belonging for the levels of component factors:				
		Very low (μ_1)	Low (μ_2)	Average (μ_3)	High (μ_4)	Very high (μ_5)
a_1	0.3	0	1	0	0	0
a_2	0.15	0	0	0	0	1
a_3	0.15	0	0	0	1	0
a_4	0.2	0	0	0	1	0
a_5	0.2	0	0	0	0	1
Nodes		0.1	0.3	0.5	0.7	0.9

Table 8.3 Matrix for evaluation of the integral factor B

Factor	Value	Function of belonging to levels of component factors:				
		Very low (μ_1)	Low (μ_2)	Average (μ_3)	High (μ_4)	Very high (μ_5)
b_1	0.15	0	0	0	0	1
b_2	0.3	0	1	0	0	0
b_3	0.15	0	0	1	0	0
b_4	0.25	0	0	0	1	0
b_5	0.15	0	0	0	1	0
Nodes		0.1	0.3	0.5	0.7	0.9

Solution. We have based the recognition on the set of functions of belonging of the type shown on Fig. 8.1. The function of belonging of the sub-set "High level of the factor", defined on 01-carrier x, has the following analytical form:

$$\mu_3(x) = \begin{cases} 0, 0 \le x < 0.6; \\ 5(x-0.6), 0.6 \le x < 0.8; \\ 1, \; 0.8 \le x \le 1. \end{cases} \qquad (8.4)$$

In turn, the function of belonging of the sub-set "Average level of the factor" has the following analytical form:

$$\mu_2(x) = \begin{cases} 0, 0 \le x < 0.2; \\ 5(x-0.2), 0.2 \le x < 0.4; \\ 1, \; 0.4 \le x < 0.6; \\ 5(0.8 - x), 0.6 \le x < 0.8; \\ 0, 0.8 \le x <= 1. \end{cases} \qquad (8.5)$$

Accordingly, the following is true:

$$\mu_1 (x) = 1- \mu_2 (x)- \mu_3 (x) \qquad (8.6)$$

The calculation with the reference to the Tables 2.11 and 2.12 produces:

$$A = 0.\,3*0.3 + 0.15*0.9 + 0.15*0.7 + 0.2*0.7 + 0.2*0.9 = 0.65, \qquad (8.7)$$

$$B = 0.15*0.9 + 0.3*0.3 + 0.15*0.5 + 0.25*0.7 + 0.15*0.7 = 0.58, \qquad (8.8)$$

According to formulae (8.4) and (8.5) these results position the level A as 25% high and 75% average, and the level B as 100% average.

This completes the example. Now, having recognized the levels A and B, it is possible to position the business according to the Shell/DDM model. Table 8.4 contains the positions of the model [Hoichens-Robinson] and the possible strategies for the business.

Table 8.4 Business positions according to the Shell/DDM model

№	Levels of factors		The position and its brief characteristic	Possible strategy of the business
	A	B		
1	H	H	**The business leader** The industry is attractive and the enterprise is strongly positioned, being the leader. The potential market is large, the rates of the market growth are high, and neither the enterprise's weaknesses nor the obvious threats from competitors are noticeable.	Continue investing into the business to protect the leading positions, while the industry continues to grow. Big capital investments (exceeding those which can be provided from own assets) are required. Continue to invest, foregoing momentary gain for future profits.
2	H	Av	**Strategy of growth** The industry is moderately attractive, but the enterprise occupies strong positions. Such enterprise is one of the leaders, mature in the life cycle of the business. The market is moderately growing or stable with good rate of return and with no strong competitors.	Try to keep the occupied positions; the position can provide the necessary financial assets for self-financing and have enough surpluses to invest into other promising areas of business.
3	H	L	**Strategy of the cash generator** The enterprise occupies rather strong positions in an unattractive industry. It is one of the leaders here, if not the leader. The market is stable, but reducing, and the industry's rate of return is declining. Competitors also present a certain threat though the efficiency of the enterprise is high, and the costs are low.	Such business is the basic source of the income of the enterprise. As no further development of this business is required, the strategy is to make insignificant investments, extracting the maximum revenues.
4	Av	H	**Strategy of strengthening of competitive advantages** The enterprise occupies the average position in an attractive industry. As the market share, the quality of products, and the reputation of the enterprise are pretty high (almost as high as that of the industry leader) the enterprise can become the leader if it properly allocates its resources. Before incurring any expenses in this case, it is necessary to analyze carefully the dependence of economic benefit on capital investments in the given industry.	Invest, if the business-area is worth it, conducting the necessary detailed analysis of investments; large investments are required to move into the position of the leader; the business-area is considered very suitable for the investment, if it can strengthen the competitive advantages. The necessary investments will exceed the expected revenues therefore additional capital investments might be required for the further struggle for your market share.
5	Av	Av	**Proceed with caution** The enterprise occupies average positions in an industry with average attractiveness. There exist no special strengths or opportunities for additional development of the enterprise; the market grows slowly; the rate of return slowly decreases.	Invest cautiously and by small portions, making sure of the fast return and constantly conduct thorough analysis of the economic situation.

6	Av	B	**Strategy of partial curtailing** The enterprise occupies average positions in an unattractive industry. There are no strengths and practically no opportunities for the development of the enterprise; the market is unattractive (low rate of return, potential excess of capacities, high density of capital in the industry)	As it is improbable that in this position the enterprise will continue to earn substantial revenues, the offered strategy is not to develop the given type of business, but to try to transform the physical assets and the position on the market into money supply, and then to use your own resources to develop a more perspective business
7	L	H	**Double the volume of the production or curtail the business** The enterprise occupies weak positions in an attractive industry.	Invest or leave this business. Since an attempt to improve competitive positions of such an enterprise by means of wide front attack would require very large and risky investments, it might only be undertaken after the detailed analysis. If it is established, that the enterprise is able to struggle for the leading positions in the industry, then the strategy should be focused on "doubling." Otherwise, the strategic decision should be to abandon the business.
8	L	Av	**Proceed with caution or partially curtail production** The enterprise occupies weak positions in a moderately attractive industry.	Stop investments, all management should be focused on the balance of cash flows; try to hold in the given position until it's profitable; gradually curtail the business.
9	L	L	**Strategy of business curtailing** The enterprise occupies weak positions in an unattractive industry,	As the company in this position loses money, it is necessary to make all efforts to get rid of such business, and the quicker, the better.

Schematically the positions of the model are shown on Fig. 8.2. We see that under conditions of the example the business being evaluated is found on the line 5 of table 8.4 "Proceed with caution." At the same time, some displacement towards the area of high competitiveness (A^+) testifies to the business's increasing competitive advantages which combined with the cautious investments, might allow it to own a larger market share, i.e. to increase the amount of profit.

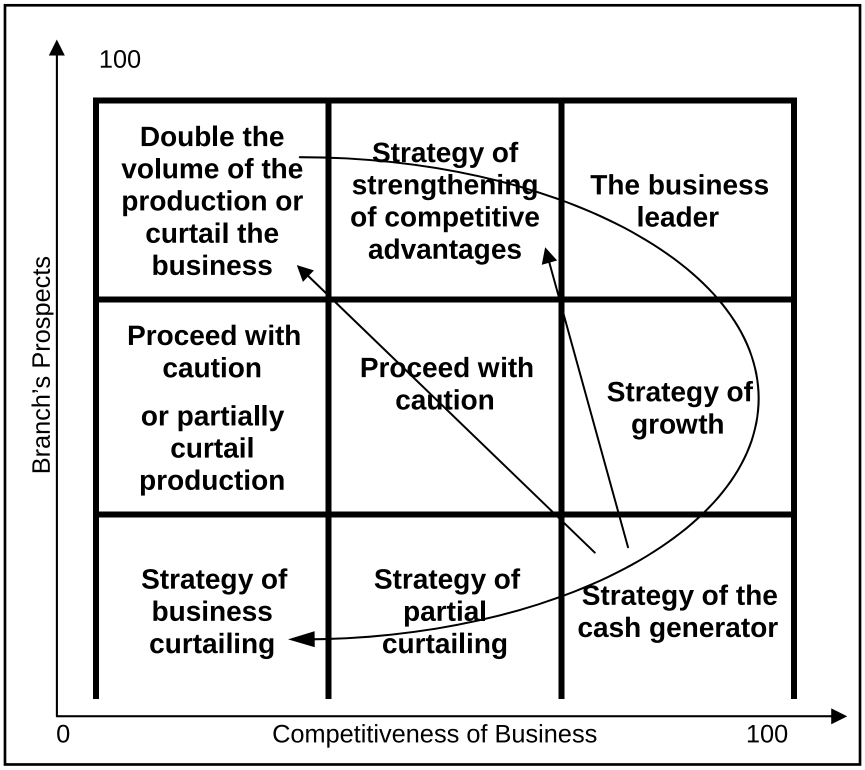

Figure 8.2 3x3 position matrix

8.4 Financial block. Business - plan

As we have shown in chapter 4, it is pertinent to present all financial factors in a business-plan for a number of years as triangular-fuzzy sets describing the optimistic, pessimistic and the most expected financial scenarios. The resulting factors of the business-plan summed over several years (NPV, EVA with the increasing total, IRR, etc.) take a triangular-fuzzy form. Accordingly, it allows **evaluating the risks** (of investment activity, of default on financial liabilities, etc.) by the method of the evaluation of investment risk considered in section 4.3 of this monograph. For example, if the resulting triangular factor $\mathbf{Z = \{Z_{min}, Z_{av}, Z_{max}\}}$ at the moment t must exceed the preset value $\mathbf{P(t)}$, than the risk of the opposite event (the failure of the plan) is calculated according to the formula:

$$Risk(t) = \begin{cases} 0, \quad P(t) < Z_{min}(t) \\ R \times \left(1 + \dfrac{1 - \alpha_1}{\alpha_1} \times \ln(1 - \alpha_1)\right), \\ Z_{min}(t) \le P(t) < Z_{av}(t)\,; \\ 1 - (1 - R) \times \left(1 + \dfrac{1 - \alpha_1}{\alpha_1} \times \ln(1 - \alpha_1)\right), \\ Z_{av}(t) \le P(t) < Z_{max}(t)\,; \\ 1, \quad P(t) \ge Z_{max}(t), \end{cases} \tag{8.9}$$

where

$$R = \begin{cases} \dfrac{P(t) - Z_{min}}{Z_{max} - Z_{min}}, \\ P(t) < Z_{max}(t); \\ 1, \; P(t) \ge Z_{max}(t); \end{cases} \tag{8.10}$$

$$\alpha_1 = \begin{cases} 0, P(t) < Z_{min}(t); \\ \dfrac{P(t) - Z_{min}(t)}{Z_{av}(t) - Z_{min}(t)}, \\ Z_{min}(t) \le P(t) < Z_{av}(t); \\ 1, \ P(t) = Z_{av}(t); \\ \dfrac{Z_{max}(t) - P(t)}{Z_{max}(t) - Z_{av}(t)}, \\ Z_{av}(t) < P(t) < Z_{max}(t); \\ 0, \ P(t) \ge Z_{max}(t). \end{cases} \qquad (8.11)$$

In the simplest case, for the triangular-symmetric resulting factors, it is possible to use the formula for the risk evaluation from section 4.4. Let it be as follows:

$$\begin{aligned} &Z_{av,} = (Z_{max} + Z_{min,})/2; \\ &\Delta = Z_{av} - Z_{min} = Z_{max} - Z_{av}; \\ &Z = Z_{av} \pm \Delta; \\ &\lambda = Z_{av}/\Delta. \end{aligned} \qquad (8.12)$$

Then (8.8) – (8.10) becomes compact:

$$Risk = \frac{1}{2} + \frac{\lambda}{2}(\ln \lambda - 1)\,. \qquad (8.13)$$

9. ACTUARIAL PENSION FUND MODELING

A large number of actuarial models for evaluation of pension systems has been developed in the world. However, the developed models do not have a satisfactory solution to many problems. We are talking about the evaluation of the efficiency of accumulation investments on the stock market.

The problem is that the stock market is the research object possessing fundamentally different level of uncertainty than the sources of earnings and recipients of payments in pension systems – various cohorts of citizens, with their of birth rates, death rates and paying capacities. Nobody challenges the applicability of probabilistic schemes for modeling receipts and payments in a pension system, however, all the history of the world stock market testifies to the inadequacy of the classical methods of probabilistic modeling of stock indices. Yielding to this uncertainty the actuaries usually transfer the research into the plane of scenario approaches, either just fixing the investments' rate of interest, or generating scenarios of the stock market on the basis of the pre-established probabilistic law.

The break in the theory of actuarial evaluation of accumulating pension systems will occur, when the adequate models of forecasting of stock indices is developed (the well-known models of classes ARCH/GARCH stop working, when the stock market system suffers an epistemological paradigm break, and the prehistory of the market indices dynamics becomes unsuitable for the forecasting of the future behavior of indices). In this connection the method of forecasting of the stock indices, described in chapter 7 may be applied to the actuarial calculations.

The model outputs the indices forecasts of the two possible types:

- a sequence of real random numbers distributed under the probabilistic law with triangular-fuzzy parameters of profitability and risk (hereafter – **type A**);
- a sequence of triangular fuzzy numbers describing the calculated corridor of profitability of an index (hereafter **– type B**).

There is a reason to reduce all other descriptions of actuarial model to one of these types. We can do it using the following algorithm:

The initial model that is a sequence of random numbers with classical probabilistic distributions is a special case of type **A**, when the triangular-fuzzy parameters of distributions become exact (real numbers).

It is possible to convert type **A** to type **B** as follows. Let the random variable have a distribution with triangular parameters $r^{\bullet}$ (the first initial moment of distribution) and $\sigma^{\bullet}$ (a square root of the second central moment of distribution). The dot after a symbol ($A^{\bullet}$) designates a triangular fuzzy number or a fuzzy function (set). Then we can convert type **A** to type **B** according to the formula:

$$\mathbf{R_{min} = r_{min} - \lambda\sigma_{max},}$$
$$\mathbf{R_{av} = r_{av},}$$
$$\mathbf{R_{max} = r_{max} + \lambda\sigma_{max}.} \qquad \mathbf{(9.1)}$$

Here λ is the Student factor (it is in the rational interval from 0.5 to 1.5). Then $R^{\bullet} = \{\mathbf{R_{min}, R_{av}, R_{max}}\}$ is a triangular fuzzy number, and the transition from type **A** to type **B** has been completed.

Notice that converting from type **A** to type **B**, we lose a certain part of information contained in distributions, but we greatly gain in simplicity of presentation and solution of the problem. Therefore, we shall further set forth the problem of a pension fund investments management in the elementary statement of type **B**.

9.1 Actuarial model of accumulating pension system

We shall consider the memory pension system, in which the investment of pension reserves is carried out on the stock market, at formation of the investment portfolio from N modeling classes (Fig. 9.1).

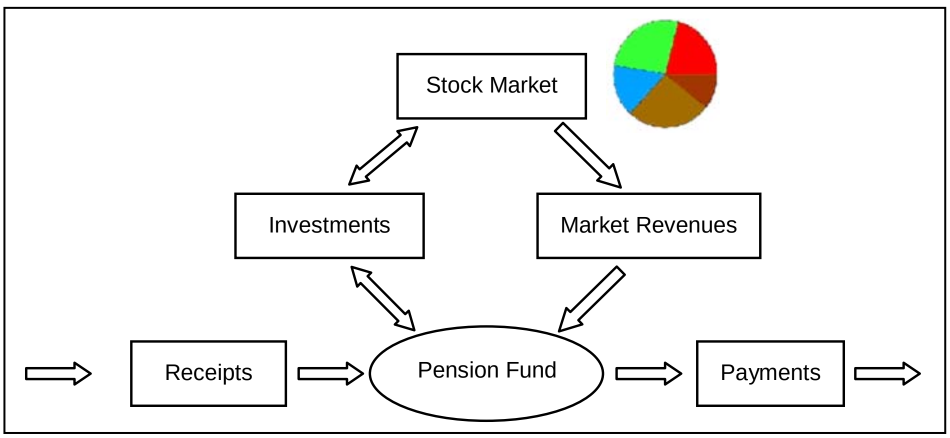

Figure 9.1 Accumulating pension system

Let's introduce the following designations:

- T – the horizon of planning, a certain number of years;
- T – the current time of a forecast (planning), the number of a year in the horizon of planning from 1 to T;
- $A^{\bullet}$ **(t)** – the receipts into a pension system, a fuzzy set;
- $L^{\bullet}$ **(t)** – the payments from a pension system, a fuzzy set;
- $I^{\bullet}$ **(t)** – the flows of investments of a pension system reserves, a fuzzy set;
- $R_i^{\bullet}$ **(t)** – a calculated corridor of profitability of i^{th} type of assets, i = 1... N;
- X **(t)** – the share distribution of investments between assets accepted by the beginning of the planning year **t** – a set of vectors of real numbers from 0 to 1 having the sum of 1;
- $B^{\bullet}$ **(t)** – the flow of profits resulting from the investments of the previous year, a fuzzy set;
- $Z^{\bullet}$ **(t)** – the reserve of a pension system at the beginning of the period of planning, a fuzzy set;
- **P (t)** – the plan of reservation of non-reducible balance of a pension system at the beginning of the period of planning, a set of real numbers.

The flows of receipts and payments $A^{\bullet}$ **(t)** and $L^{\bullet}$ **(t)** are the exogenous factors of the model**.** They are modeled on the basis of pension schemes used in a fund. Also, on the basis of a forecast, we know the profitability of assets $R_i^{\bullet}$ **(t)**.

The flow of investments $I^{\bullet}$ **(t)** is planned by the following rule. If the planned receipts exceed the payments, then some share of the difference between receipts and payments forms the flow of investments (this share is unknown, we will have to determine it while solving the problem). If the difference is negative, there is a flow of negative investments (the recalling of assets from the stock market).

The accumulated investments circulate on the market and bring profits, which can be estimated by the formula:

$$B^{\bullet}(t+1) = \sum_{j=1}^{t} I^{\bullet}(j) * \sum_{i=1}^{N} x_i(t) * R_i^{\bullet}(t) \qquad \textbf{(9.2)}$$

Thus, the balance of the reserve of a pension fund is calculated by the formula:

$$Z^{\bullet}(t+1) = Z^{\bullet}(t) + A^{\bullet}(t) + B^{\bullet}(t) - I^{\bullet}(t) - L^{\bullet}(t). \qquad \textbf{(9.3)}$$

The plans of reservation **P(t)** should be established on the basis of the specialized norms, on the assumption of the necessity of maintaining a failure-free functioning of pension systems under essential fluctuations of flows of payments and receipts (for example, 10% of average planned level of payments for the previous year):

$$\mathbf{P(t+1) = 0.1 * A_{av}(t).} \qquad \textbf{(9.4)}$$

In case the plan of reservation is not fulfilled, i.e. **Z (t) < P (t),** this event is considered adverse. The risk of such event (as the reserves are the triangular numbers) can be evaluated according to the formula (see section 4.3 of this monograph):

$$Risk(t) = \begin{cases} 0, \quad P(t) < Z_{min}(t) \\ R \times (1 + \dfrac{1-\alpha_1}{\alpha_1} \times \ln(1-\alpha_1)), \quad Z_{min}(t) \le P(t) < Z_{av}(t) \\ 1-(1-R) \times (1 + \dfrac{1-\alpha_1}{\alpha_1} \times \ln(1-\alpha_1)), \quad Z_{av}(t) \le P(t) < Z_{max}(t) \\ 1, \quad P(t) \ge Z_{max}(t) \end{cases} \qquad \textbf{(9.5)}$$

where

$$R = \begin{cases} \dfrac{P(t) - Z_{min}}{Z_{max} - Z_{min}}, \quad P(t) < Z_{max}(t) \\ 1, \quad P(t) \ge Z_{max}(t) \end{cases}, \qquad \textbf{(9.6)}$$

$$\alpha_1 = \begin{cases} 0, \; P(t) < Z_{min}(t) \\ \dfrac{P(t) - Z_{min}(t)}{Z_{av}(t) - Z_{min}(t)}, \quad Z_{min}(t) \le P(t) < Z_{av}(t) \\ 1, \quad P(t) = Z_{av}(t) \\ \dfrac{Z_{max}(t) - P(t)}{Z_{max}(t) - Z_{av}(t)}, \quad Z_{av}(t) < P(t) < Z_{max}(t) \\ 0, \quad P(t) \ge Z_{max}(t) \end{cases} . \qquad \textbf{(9.7)}$$

Now the problem of the optimum management of a fund's investment portfolio can be formulated as follows: to determine the sequences $I^{\bullet}(t)$ and the optimum distributions X(t) such that the following condition of minimum of criterion function is true:

$$\mathbf{\max_{(t)} Risk\ (t) \rightarrow min} \qquad \textbf{(9.8)}$$

So formulated, the problem of management (9.8) is a problem of search for the global minimum under the natural restrictions:

$$\mathbf{0 \le X\ (t) \le 1;}$$

$$\sum_{i=1}^{N} x_i(t) = 1;$$

$$I^{\bullet}(t) \le A^{\bullet}(t) - L^{\bullet}(t.) \qquad (9.9)$$

Let's consider a calculation example.

9.2 Example of actuarial calculation

Let the investments of assets of a fund be made in two classes of stock instruments: stocks and bonds. For simplicity, let's also fix the size of investment deductions at the level of difference between receipts and payments.

Let's assume that initially, before the optimization, only the stock investments are made. The parameters of investments, payments and receipts, profitability of stock instruments, and the results of calculations are shown in table 9.1.

Table 9.1 Cash flow forecast of a pension fund

Items of payments and receipts	Level	The beginning (end)	Forecast by years (number of the year – t)									
			2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Receipts and Profits												
Receipts in pension system A (t)	Min		95	95	95	95	95	95	95	105	114	114
	Average	100	100	100	100	100	100	100	100	110	120	120
	Max		105	105	105	105	105	105	105	116	126	126
Investments and Revenues												
Investments I(t)	Min		64	53	43	22	1	-31	-63	-64	-65	-75
	Average		70	60	50	30	10	-20	-50	-50	-50	-60
	Max		77	67	58	39	20	-9	-38	-37	-36	-45
Note: Investments with the increasing result	Min	0	64	117	159	181	181	150	88	24	-41	-116
	Average	0	70	130	180	210	220	200	150	100	50	-10
	Max	0	77	144	201	240	259	250	213	176	141	96
Portfolio distribution												
Stocks		1	1	1	1	1	1	1	1	1	1	1
Bonds		0	0	0	0	0	0	0	0	0	0	0
Total		1	1	1	1	1	1	1	1	1	1	1
Note: Calculation corridor of profitability												
Stocks	Annual Min	-10%										
	Annual Avrg	20%										
	Annual Max	30%										
Bonds	Annual Min	10%										
	Annual Avrg	12%										
	Annual Max	14%										
Revenues B(t)	Min		0	-6.4	-11.7	-15.9	-18.1	-18.1	-15.0	-8.8	-2.4	4.1
	Average		0	14.0	26.0	36.0	42.0	44.0	40.0	30.0	20.0	10.0
	Max		0	23.0	43.1	60.3	71.9	77.7	75.0	63.8	52.8	42.2
Payments												
Payments of pension system L(t)	Min		29	38	48	67	86	114	143	152	162	171
	Average		30	40	50	70	90	120	150	160	170	180

Items of payments and receipts	Level	The beginnin g (end)	Forecast by years (number of the year – t)									
			2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
	Max		32	42	53	74	95	126	158	168	179	189
Reserves												
Reserves of the pension fund Z(t)	Min	20	7	-13	-40	-73	-110	-150	-190	-226	-257	-283
	Average	20	20	34	60	96	138	182	222	252	272	282
	Max	20	33	70	128	205	296	396	496	587	668	741
Note: Norm of the pension system reserve												
		10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	12%
Risks			**1%**	**8%**	**9%**	**9%**	**9%**	**9%**	**9%**	**9%**	**9%**	**9%**
Aggregate risk		**9%**	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%

It is obvious, that there are the risks of insufficiency of pension reserves (at the same time we notice, that consecutive escalating of uncertainty year by year is reduced to the continuing widening of the intervals containing reserves).

Having received the optimum share distribution between stocks and bonds for each year of the forecast in the horizon of investment, let's now apply the optimization (9.8) restricted by (9.9). The result of the optimization is shown in table 9.2 (the software tool Solver of Excel was used). We see that the maximum possible risk has decreased from 9% to 4%.

Table 9.2 Cash flow forecast of the pension fund after optimization of investments

Item of payments and receipts	Level	The beginning (end)	Forecast by years (number of the year – t)									
			2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Receipts and revenues												
Receipts in pension system A(t)	Min		95	95	95	95	95	95	95	105	114	114
	Average	100	100	100	100	100	100	100	100	110	120	120
	Max		105	105	105	105	105	105	105	116	126	126
Investments and revenues												
Investments I (t)	Min		64	53	43	22	1	-31	-63	-64	-65	-75
	Average		70	60	50	30	10	-20	-50	-50	-50	-60
	Max		77	67	58	39	20	-9	-38	-37	-36	-45
Note: Investments with the increasing result	Min	0	64	117	159	181	181	150	88	24	-41	-116
	Average	0	70	130	180	210	220	200	150	100	50	-10
	Max	0	77	144	201	240	259	250	213	176	141	96
Portfolio distribution												
Stocks		1	0	0	0	0	0	0	0	1	1	1
Bonds		0	1	1	1	1	1	1	1	0	0	0
Total		1	1	1	1	1	1	1	1	1	1	1
Note: Calculation corridor of profitability												
Stocks	Annual Min	-10%										
	Annual Avrg	20%										
	Annual Max	30%										

Item of payments and receipts	Level	The beginning (end)	Forecast by years (number of the year – t)									
			2003	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Bonds	Annual Min	10%										
	Annual Avrg	12%										
	Annual Max	14%										
Revenues B(t)	Min		0	6.4	11.7	15.9	18.1	18.1	15.0	8.8	-2.4	4.1
	Average		0	8.4	15.6	21.6	25.2	26.4	24.0	18.0	20.0	10.0
	Max		0	10.7	20.1	28.1	33.5	36.3	35.0	29.8	52.8	42.2
Payments												
Payments of the pension system L(t)	Min		29	38	48	67	86	114	143	152	162	171
	Average		30	40	50	70	90	120	150	160	170	180
	Max		32	42	53	74	95	126	158	168	179	189
Reserves												
Reserves of the pension fund Z(t)	Min	20	7	-1	-4	-5	-6	-10	-20	-38	-70	-96
	Average	20	20	38	44	66	91	117	141	159	179	189
	Max	20	33	58	93	138	190	249	309	365	447	519
Note: Norm of the pension system reserve												
		10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	12%
Risks			**1%**	**4%**	**2%**	**1%**	**1%**	**1%**	**1%**	**2%**	**3%**	**4%**
Aggregate risk		4%										

Thus, by transition from an aggressive strategy of investment to a conservative one, it was possible to lower essentially the risks of insufficiency of pension reserves in the first years of the plan, and to narrow the planned interval of fluctuations of pension reserves practically by half. However, this strategy is subject to correction at the late stages – the reserves are up to the mark, and there is a room for risk, therefore, we can return to the stocks.

Certainly, the optimum distribution will change with the change of parameters of flows of receipts, investments and payments, and the problem of the optimization will have to be solved anew.

III. SOFTWARE SOLUTIONS FOR FUZZY FINANCIAL MANAGEMENT

Number of software applications for financial management is based on the scientific results of this monograph. Brief description of one of them is presented below.

10. THE SBS PORTFOILO OPTIMIZATION SYSTEM

The purpose of the "System of optimization of stock portfolio" program (hereafter referred to as SOSP), used by the Russia's Pension Fund (hereafter referred to as the PFR), is the optimization of the stock portfolio modeling by the corresponding stock indices on the basis of historical and forecast data. The programming language is Java. The program needs 20 megabytes of hard disk space.

The SOSP program was created under my own scientific guidance during 2002 – 2003.

Let's now describe the functionality of the program modules.

10.1 Investment profiles module

Figure 10.1 shows one of the screens of the investment portfolio module.

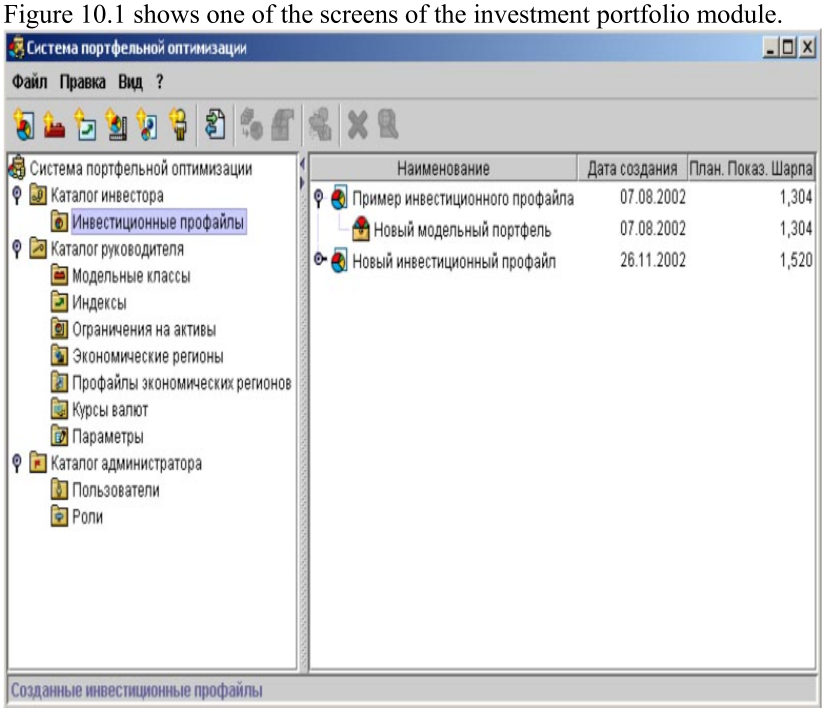

Figure 10.1 Screen of the investment portfolio module

The investment profile is a programming informational structure that contains all the history of the investment portfolio operations. In the PFR, the investment profile is understood as a managing company that controls the investment of the certain size. In the course of modification of the profile content the PFR employees can model the assets control operations of a managing company, and estimate the efficiency and risk of these operations.

The functionality of the module:

- provides the tabular mode of the summary of all investment profiles that displays the investment profile's name, date of creation, and the mean value of the target Sharpe index;
- allows to create a new investment profile, to re-balance the current modeling portfolio of the selected profile, to consolidate the investment profiles into a new investment profile, to delete a profile, and to install the current modeling portfolio in an investment profile;

- allows the viewing and printing of the final user's modeling portfolios reports, and saving them in XML, HTML, and PDF formats.

10.2 The investment profile and modeling portfolios creation module

Fig. 10.2 shows the sample screen of the module.

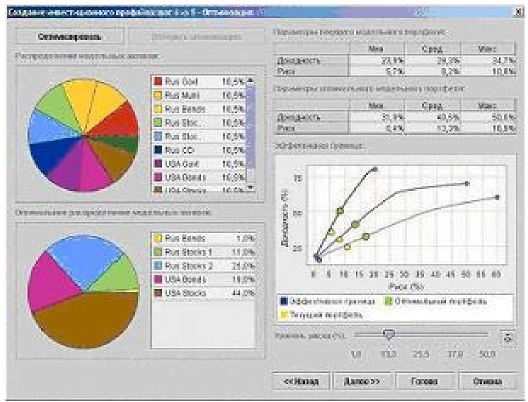

Figure 10.2 The sample screen of the investment profiles module

The functionality of the module:

- allows to create an investment profile with the indication of the investment horizon and monetary means subject to investment;
- allows to carry out the benchmarking for an investment profile, selecting the planned dates for the profitability checks and the corresponding values of profitability (no more than 1 benchmark a quarter);
- allows to choose modeling assets to invest in, and to specify the monetary volumes of investments into these assets. To mark the assets which will form the effective boundary. To present the distribution of assets as a pie chart;
- allows to control the pre-established limits for the size of modeling classes, with issuing a warning of restrictions violation;
- provides the mode of modeling portfolio re-balancing;
- provides the mode of investment profiles consolidation;
- gives a user an access to each of the modeling assets in a profile for evaluations of profitability and risk of a modeling index in the triangular-fuzzy form;
- provides the graphic and tabular representation of modeling indices performance, histograms of distribution of profitability, and flat cross-section of the function of plausibility;
- provides graphic results of the optimization in the form of fuzzy effective boundary shaped as a strip (the strip is formed by the method described in the chapter 6 of this monograph);
- allows to display on the diagram both the initial distribution of assets as a three-point, and the desirable distribution as a three-point on the effective boundary strip;
- enables a user to efficiently re-balance a modeling portfolio with setting the optimum values of shares (in a user called dialogue);
- allows to change a portfolio risk with the horizontal slider, with an option of returning to the initial risk;
- allows to estimate the profitability of a portfolio retrospectively (on the basis of historical performances) and prospectively (on the basis of triangular fuzzy functions) in three ways: in nominal prices (RuR), in real prices (RuR, allowing for the inflation), and in the preset currency (USD, GBP, EUR, JPY);
- allows to estimate the benchmark risk, re-calculating it with modified benchmark data, it also allows to redraw benchmark points on the diagram;

- lets compare a portfolio performance with the performance of the chosen modeling class, including the Russia's rate of inflation;
- allows to save the created investment profile/modeling portfolio;
- allows to generate and display reports when an investment profile is created or re-balancing of a modeling portfolio is finished.

10.3 The module of indices and modeling classes data

A sample screen of the module is shown on Fig. 10.3.

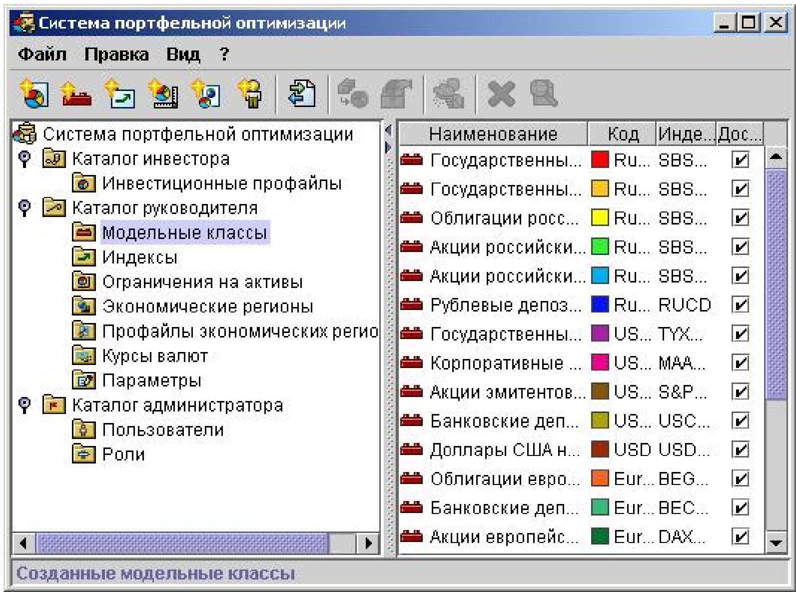

Figure 10.3 A screen of the module of indices and modeling classes data

The functionality of the module:

- allows a program manager to correct the number of modeling classes and to compare them with the new indices;
- allows a program manager to add new indices, and to update the indices' data using graphic user interface;
- allows a program manager to add new indices, and to update the indices' data by importing the necessary information from the corresponding files of the preset format;
- allows a program manager to correct current parameters of the program modules;
- allows a program manager to install and change the limits on the percentage of modeling assets in a portfolio.

10.4 The working with the economic region profiles module

A sample screen of the module is shown on Fig. 10.4.

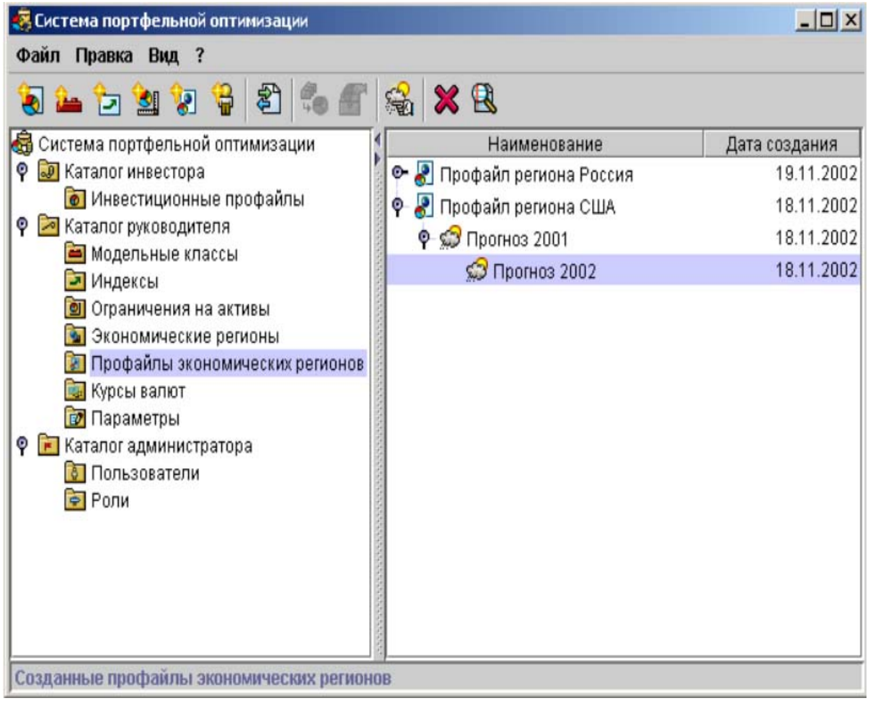

Figure 10.4 A screen of the working with the economic region profiles module

The profile of an economic region is the informational unit that allows a user to consolidate all the history of forecasting of stock and macro-economic indices for one country or for a group of countries.

The functionality of the module:

- provides the tabular mode of the summary of all created profiles of economic region containing the profiles and the dates of their generation;
- allows a program lead to correct a forecast in the structure of an economic region profile;
- allows an end user and a program lead to check the results of forecasting for all of the economic region profiles;
- allows an end user and a program lead to check and print the reports of each forecast, with an option to save them in XML, HTML, or PDF formats;
- allows a program lead to use the forecasts of profitability and risk of indices as expert evaluations;
- allows a program lead to maintain a reference book of economic regions.

10.5 The module of economic region profiles creation

A sample screen of the module is shown on Fig. 10.5.

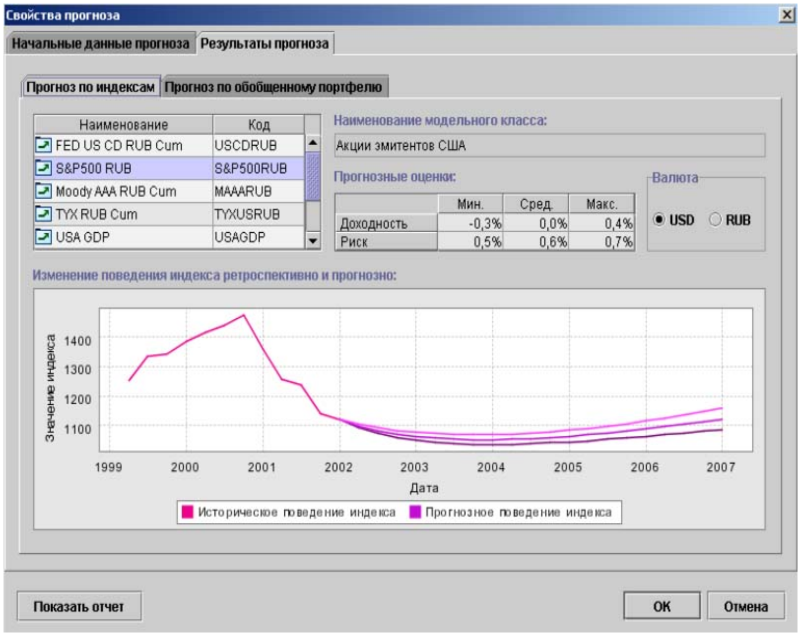

Figure 10.5 The creation of economic region profiles module

The functionality of the module:

- allows to create the profiles of an economic region, with control of availability of indices of macro-economic factors for the specified economic region and an option to assign the indices to groups;
- allows to set the necessary initial data required for forecasting;
- allows to conduct the forecasting according to the forecasting algorithm (on the basis of scientific results of the Chapter 7 of this book);
- allows to get the graphic representation of the forecasting results for the indices and the generalized portfolio;
- allows to save the created profile of the economic region and the forecast;
- allows to generate and display the report a profile of economic region is created or a forecast is changed.

CONCLUSION

My monograph is devoted to the research of operations of portfolio management conducted under the informational uncertainty. The stock market condition has always been and always will be uncertain (I hope, this premise does not require any special proof). Nevertheless, the stock market has existed until now and it will go on, the decisions are made today as they have been in the past. And the subject of this examination is what makes the basis of these decisions, and to what extent the intuitive framework of investment decisions can be rationalized and become the subject of scientific research.

Sometimes the portfolio decisions are forced as in the case of the investments of the Russian Federation Pension fund, which under the new Russian pension reform must invest the pension reserves into stocks and bonds of Russian corporations. The conservative character of pension accumulation demanding the increased safety, and the aggressive character of the stock market investments accompanied by the increased risk of losses seem incompatible not only to me, but also to the management of the PFR. This contradiction is especially obvious in a less developed country (such as Russia), where even the government bonds might default on liabilities (as it was the case in August, 1998).

However, the nature of pension savings is such that they just have to be invested in the stock market so that the national economy could get a low-interest source of money for its development. The result of such development is an additional gross domestic product, which should be subsequently redistributed between the future retirees. There is no other long-term investment mechanism providing the future pensions guaranteed from the inflationary depreciation in the capitalist society. That's why the pension reserves will still be invested in the stock market, and the task of the managers of all levels is to not lose assets or allow them to depreciate. My book is addressed to the advanced managers and their future successful decisions.

I believe, this book proves that fuzzy sets are a more preferable tool for modeling of financial systems behavior under uncertainty than the traditional probabilities. The subjective probabilities used in financial management, more likely by inertia than anything else, more and more often display that they are informationally limited, insufficient, and unreliable. Brainchild of XIX-XX centuries, the probabilistic models are getting less and less suitable to describe the realities of the XXI century. The scientific paradigm of financial management changes before our eyes, and the probabilistic methods cannot keep up with these changes.

The financial systems constantly get more complicated. It is caused by the technical progress giving additional opportunities for the growth and development to economic systems. The introduction of computer systems and networks allows the corporations to reach a qualitatively new level of financial organization. This objective complication of financial systems results in occurrence of new, possibly adverse, ways of the development, which are subject to study.

Unfortunately, the economic science often doesn't keep up with the events and cannot supply the practical financial management with adequate financial models. The scientific inadequacy in financial management results in defective practice of poor-quality management of financial assets, and ultimately in corporate bankruptcies and market crises. It was the self-conceit of financial analysts, the apologists of the so-called "new economy" that led to the expectations of boundless and indefinitely growing stock market and ended up in trillions of dollars of losses for corporations and households worldwide. The losses caused by the widespread unqualified advice generate full-scale mistrust in investment consultants and in the modeling premises they base their scientific analysis on.

Very often the practitioners of financial management, mistrusting the discredited theories, manage the assets entrusted to them by rough estimation based on the intuition, which for the most part can't even be verbalized at all. This intuitive activity multiplied by the experience of financial management is an invaluable research material. Those with the intuition and experience become the experts, whose activity becomes the object of scientific research. So the object of scientific research of financial systems has been updated: if earlier it only included the economic entity (corporation, industry, economic region, country), in modern financial management the object of scientific research is supplemented with the decision maker. Both the financial manager, and financial analyst, preparing the decisions for the manager are decision makers. The activity of both categories is the subject to detailed research, and the fuzzy sets are undoubtedly the best formalisms for modeling this activity.

This book's example of the method of complex financial analysis of a corporation shows how the expert notions of the level of factors can be included in the model of the bankruptcy risk evaluation, and how to convert the qualitative notions about the levels of factors to quantitative ones. We also used the expert evaluations of those parameters of the business plan that can have none other than a fuzzy form. The sales experts, just as any other person, can know nothing about future sales precisely; therefore, they tend to rely on fuzzy evaluations. The more skilled is the expert, the less fuzzy is the evaluation, and accordingly the lower is the risk of inefficiency of the decisions made. However, there is an ineradicable information uncertainty, which the professional expert should be able to feel and express at least in terms of a natural language. In turn, an expert's certainty or uncertainty in his or her evaluations can be easily described in quantitative terms, as shown both in [46] and the present work.

The stock market is an even more complex object of scientific research than a separate corporation, because tens of thousands of corporations and millions of individual and institutional investors operate on it. Joint activity of these market economic agents results in the investments into securities, recorded by the stock indices. Just as in the case of modeling of corporate financial systems, the expert notions and evaluations can be formalized and successfully applied to the modeling of the stock market and of its subjects' behavior. The evaluation of the investment attractiveness of securities (matrix methods of which are described in chapter 5) applied to the large number of issuers enables us to model the market as a whole, and the generalization of the results allows us to postulate the modern theories of a stock portfolio optimization and the stock indices forecasting (see chapters 6 and 7).

I believe I was able to develop a number of scientific theories and methods that are important for market researches and for practical financial management under substantial information uncertainty. The developed theories and methods had been implemented in the practice of the Pension fund of the Russian Federation in course of management of accumulation component of the Russian citizens pensions. I believe this is the best recommendation for my scientific research. Besides that, the developed models underlie number of financial management computer programs. It allows reproducing and using the results of my scientific works in the practice of financial management.

Appendices

APPENDIX 1. DETAILED STATEMENT OF THE METHOD OF FORECASTING OF STOCK INDICES ON THE BASIS OF A FUZZY MODEL

A1.1. Classification of the economic regions and indices. Terminology.

All indices to be forecasted and observed, are divided into three large groups:

- Bond indices, including government bonds, bonds of the regional subjects, bank deposits, corporate bonds and emissive mortgage securities;
- Stock indices, including high and low capitalization stocks (of the 1st and 2nd echelons accordingly), and the shares of mutual funds – the assets allowed for pension investments by the laws of the Russian Federation;
- Indices of macro-economic factors, including the gross domestic product, the consumer price index, the currency exchange rate in relation to the Russian rubles, and the P/E ratio.

We also assume that there is a one-to-one correspondence between the index and the economic region hereafter referred to as the index holder. We assume that all securities forming an index are issued on the geographical territory of the region – the index holder. Also all the tendencies affecting the index occur on the same territory. We select the following regions being of interest for researches:

- The USA and Canada (US);
- Russia (RU);
- The European Community (EC);
- England (GB);
- Japan (JAP);
- Developing countries (EMM).

The models and the techniques of forecasting vary depending on the type of the index. We shall describe these models and techniques step by step, from the phase to the phase of the forecasting process as they are listed in the section 7.5 of this book.

We shall use the following mathematical designations. The point after the symbol ($A^{\bullet}$) means that we are looking at a triangular fuzzy number or a fuzzy function (set). In all other cases real numbers, functions, and parameters are assumed by default. For a triangular number $A^{\bullet}$, A_{min}, A_{av}, and A_{max} are the minimum, average, and maximum values of the number.

We will also use the following naming convention:

- $\mathbf{t}$ is a discontinuous forecasting time (where each readout corresponds to the time interval), $\mathbf{t_{start}}$ is the starting point of the forecast, $\mathbf{t_{end}}$ is the final point of the forecast, $\mathbf{\Delta T}$ is the sampling interval (by default 1 quarter);
- $\mathbf{x_{A,B,N}}$ are the shares of assets of stocks, bonds and non-stock assets in the generalized investment portfolio correspondingly; $\mathbf{\Delta x}$ is the size of re-balancing of the corresponding assets shares at transition to the next forecasting time interval; $K_1^{\bullet}$, $K_2^{\bullet}$ are fuzzy parameters in the model of investment dynamics when the forecast is evaluated by $\mathbf{\Delta x}$;
- $r^{\bullet}$, $\sigma^{\bullet}$ are the final profitability by index and risk (mean square deviation), both are triangular fuzzy numbers; $r^{\bullet\backslash}$, $\sigma^{\bullet\backslash}$ are the same, but with the index recalculated in RuR rather than national currency;
- $R^{\bullet}(t)$ is a calculated corridor of profitability by the index – a triangular fuzzy set;
- $\mathbf{a_i}$, $\mathbf{b_{ij}}$ are the parameters of the rational dynamics of investments model (Tables 7.5 and 7.6);
- $\Delta r^{\bullet}_{ij}$ is a matrix of calculated risk premiums for all listed types of bonds – a triangular fuzzy numbers matrix;
- $P^{\bullet}(t+1)$ is a forecast value of the index – a triangular fuzzy function;
- $P^{\bullet\backslash}(t+1)$ is a forecast value of the index with the index recalculated in RuR rather than national currency;

- $E^{\bullet}(t+1)$ is a forecast value of the rates of growth of volumes of corporate profit per one average share comprising the index of stocks of the first echelon (for the USA – S&P500, for Russia – RTS), it is a triangular fuzzy function;
- $GDP^{\bullet}(t+1)$ is a forecast rate of growth of the gross domestic product – a triangular fuzzy function;
- $I^{\bullet}(t+1)$ is a forecast rate of inflation – a triangular fuzzy function;
- $J^{\bullet}(t+1)$ is a forecast rate of national currency in relation to RuR – a triangular fuzzy function;
- $PE^{\bullet}(t+1)$ is a forecast by the index of the P/E ratio – a triangular fuzzy function; $\Lambda^{\bullet}(t+1)$ is a forecast multiplier for the factor of the P/E ratio; $\mathbf{PE_{set}}$ is a preset (rational) value of the index, determined from the table 4.10;
- $\alpha^{\bullet}$ and $\beta^{\bullet}$ are fuzzy parameters in the equation of linear regression $\mathbf{f^{\bullet}(t) = \alpha^{\bullet} \times t + \beta^{\bullet}}$;
- $\gamma^{\bullet}$, $\delta^{\bullet}$ are fuzzy factors of elasticity of one parameter relative to the other;
- $Z^{\bullet}$ is a factor of the reduction of calculated profitability of the index of the first echelon stocks to that of the second echelon, it is a triangular fuzzy number;
- $Sh^{\bullet}(t+1)$ is a forecasted value of the modified Sharpe factor for the generalized stocks and bonds investment portfolio, it is a triangular fuzzy function.

A1.2 The phase 1 (start) model and methods

For this phase we set the start and the end of the forecast times (t_{start} and t_{end} accordingly), the known existing values $\mathbf{I(t_{start}), GDP(t_{start}), PE(t_{start})}$ are recorded, and the decision on the starting allocation of capital is made according to the Table 7.5:

$$\mathbf{x_A(t_{start}) = x_{A0},\ x_B(t_{start}) = x_{B0},\ x_N(t_{start}) = x_{N0}.} \qquad \textbf{(A1.1)}$$

In course of modeling, we found out that when the bearish tendencies dominate the market the starting allocation of assets is degenerated, and it is impossible to trace the dynamics of the portfolio, and the sensitivity of its shares to the fluctuations of exogenous factors. Therefore, it makes the model clearer to start from the portfolio point of reference (50% of stocks and 50% of bonds) in any case. If the bearish tendencies of capital flow remain, the portfolio will quickly degenerate, and it may be observed in dynamics.

The starting values $\mathbf{P\ (t_{start})}$ of all the indices corresponding to the given economic region are set.

The discontinuous time is tied to the continuous time so that the values of indices and parameters for the discontinuous time equal to the values of the last trading day of the corresponding quarter.

The current values of profitability and risks of modeling classes of stocks and bonds $\mathbf{r(t_{start})}$ and $\mathbf{\sigma(t_{start})}$ for the generalized investment portfolio, and the value of the modified Sharpe factor $\mathbf{Sh(t_{start})}$ are set based on the analysis of recent historical data (the history of the quarter preceding the forecast is sufficient; the evaluation $\mathbf{Sh(t_{start})}$ is then taken as an average of the three months of previous history of the generalized investment portfolio).

The current forecast time $\mathbf{t = t_{start}}$ is set, and the process passes to the second phase – the analysis of the macroeconomic tendencies.

A1.3 The phase 2 model and methods

By the virtue of substantial non-stationary nature of macroeconomic processes (the assumption of the expert model) we do not undertake to predict them by applying the known methods of auto-regression analysis, as for instance in the ALM models [Lattice Financial]. We suggest instead formulating them in the shape of a strip with the rectilinear boundaries.

$$\mathbf{f^{\bullet}(t) = \alpha^{\bullet} \times (t - t_{start})/4 + \beta^{\bullet},\ t \in [t_{start}+1,\ t_{end}]} \qquad \textbf{(A1.2)}$$

The $\alpha^{\bullet}$ and $\beta^{\bullet}$ are selected on the basis of additional considerations of the expert model. In particular, the growth of rate of inflation in the USA expected for the medium-term outlook means that $\beta^{\bullet} > (0, 0, 0)$. On the contrary, in Russia $\beta^{\bullet} = (0, 0, 0)$, since no growth of rate of inflation is expected, but the range of fluctuations of these rates is wide enough.

After finishing this phase of forecasting we have the estimations of $\mathbf{GDP^{\bullet}(t)}$ (gross domestic product), $\mathbf{I^{\bullet}(t)}$ (inflation), $\mathbf{J^{\bullet}(t)}$ (currency), $\mathbf{t \in [t_{start}, t_{end}]}$. We also forecast $\mathbf{E^{\bullet}(t)}$ (corporate revenues) by the well-known Fisher's formula for relation of interest rates:

$$\mathbf{1 + E^{\bullet}(t) = (1 + GDP^{\bullet}(t))\,(1 + I^{\bullet}(t)),} \qquad \textbf{(A1.3)}$$

and the process passes to the third phase– the analysis of the expected investment dynamics.

A1.4 The phase 3 model and methods

For the (t+1) step of forecasting we should evaluate on the (t) step the investment tendencies by the Table 7.6 to correctly determine the direction of capital flow in the time interval **[t, t+1]**. The inputs to the table are the $\mathbf{I_{av}(t)}$ and $\mathbf{PE_{av}(t)}$. Thus, we form anticipatory impact on the investment portfolio with one step lead relative to the planned macroeconomic dynamics.

So, for the input situation 4, which we recognize as attractive-intermediate when starting an investment and as attractive for capital flow, we forecast the increase of the capital invested in stocks and bonds, and the corresponding growth of the level of cumulative indices. Please note that the level of the bonds index is a **low elastic** factor relative to the volumes of operations, and the level of the stocks index is **highly elastic**. The reason is that the bonds interest rates fluctuation limits are rather narrow. They are restricted by the rate of inflation from below (or are extremely close to it), and by the level of profitability of the corporations that allows to maintain the accumulated debts reliably without essential worsening of financial status (with the minimum risk of bankruptcy) from above. Although to be perfectly honest let's note that a sudden fall of stock prices caused such a powerful outflow of money into the USA bonds that interest rates have been the lowest since 1960. But we consider this tendency temporary for the purpose of this discussion. Sooner or later the rates will even out because most of the capital now accumulated in the USA bonds will outflow abroad.

The process of forecasting passes now to the fourth phase – the forecast of calculated corridor of profitability by the index.

A1.5. Model and methods of calculated corridor of profitability of the bonds index evaluation (phase 4)

By the virtue of low elasticity of the bonds index relative to the market trade volumes we have decided to neglect this elasticity in our model and to construct the forecast of bonds profitability on the basis of the risk premiums matrix (Table A1.1). We determine the matrix values on the basis of additional macroeconomic considerations of the expert model.

Table A1.1 Premiums for bonds investment risk

Economic region	**Currency of the region**	**The risk premium to the rate of inflation (based on the national currency)**				
		govt	**muni**	**bank**	**corp**	**mortgage**
USA	USD	$\Delta r^{\bullet}_{11}$	$\Delta r^{\bullet}_{12}$	$\Delta r^{\bullet}_{13}$	$\Delta r^{\bullet}_{14}$	$\Delta r^{\bullet}_{15}$
RU	RUR	$\Delta r^{\bullet}_{21}$	$\Delta r^{\bullet}_{22}$	$\Delta r^{\bullet}_{23}$	$\Delta r^{\bullet}_{24}$	$\Delta r^{\bullet}_{25}$
EC	E	$\Delta r^{\bullet}_{31}$	$\Delta r^{\bullet}_{32}$	$\Delta r^{\bullet}_{33}$	$\Delta r^{\bullet}_{34}$	$\Delta r^{\bullet}_{35}$
GB	GBP	$\Delta r^{\bullet}_{41}$	$\Delta r^{\bullet}_{42}$	$\Delta r^{\bullet}_{43}$	$\Delta r^{\bullet}_{44}$	$\Delta r^{\bullet}_{45}$
JAP	JPY	$\Delta r^{\bullet}_{51}$	$\Delta r^{\bullet}_{52}$	$\Delta r^{\bullet}_{53}$	$\Delta r^{\bullet}_{54}$	$\Delta r^{\bullet}_{55}$
EMM	USD	$\Delta r^{\bullet}_{61}$	$\Delta r^{\bullet}_{62}$	$\Delta r^{\bullet}_{63}$	$\Delta r^{\bullet}_{64}$	$\Delta r^{\bullet}_{65}$

The model of risk premiums shown above is stationary and operates on the whole interval of forecasting.

And the calculated corridor of profitability of j^{th} type of bonds issued in i^{th} economic region is determined by the formula:

$$R_B{}^{\bullet}{}_{ij}(t) = I^{\bullet}{}_{ij}(t) + \Delta r^{\bullet}{}_{ij}. \quad \textbf{(A1.4)}$$

A1.6. Model and methods of evaluation of the calculated corridor of profitability by the index of the first echelon stocks (phase 4)

High elasticity of the factor of current profitability of stocks (relative to the level of a trading day, week, etc.) by the factor of the growth or the decline of the volume of tenders causes the essential price fluctuations of the index. However, considering the model of an investor's rational behavior we note that rapid dynamics of quotations in the medium-term prospect is eliminated by coming into effect of the factor of overvalue/undervalue of stocks. Thus in medium-term prospect a stock index forms a cyclic trend around its mean values determined by the rational level of the P/E ratio. Therefore, we decide not to model **three-dimensional** elasticity of profitability of the stocks index, but to account for it in the model indirectly at the level of elasticity by the P/E ratio.

The mentioned model of elasticity is:

$$R_A{}^{\bullet}(t) = \begin{cases} (PE_{set} - PE_{av}(t)) * \gamma_1{}^{\bullet}, & PE_{set} > PE_{av}; \\ (PE_{set} - PE_{av}(t)) * \gamma_2{}^{\bullet}, & PE_{set} < PE_{av}, \end{cases} \quad \textbf{(A1.5)}$$

where

$$PE_{set} = \begin{cases} (b_{11} + b_{12})/2, & \text{for the situations } 1, 2, 3 \\ (b_{21} + b_{22})/2, & \text{for the situations } 4, 5, 6 \\ (b_{31} + b_{32})/2, & \text{for the situations } 7, 8, 9 \end{cases} \quad \text{of tables 7.5 and 7.6,} \quad \textbf{(A1.6)}$$

$$\gamma_{1,2}{}^{\bullet} = \gamma_{1,2\ k}{}^{\bullet} \quad \text{for } k^{th} \text{ situations of tables 7.5 and 7.6,} \quad \textbf{(A1.7)}$$

and these parameters are determined on the basis of additional reasons of the expert model.

The fluctuation of factor of elasticity in spurts when the P/E ratio crosses the preset threshold reflects the asymmetry of the investment choice for the different type of an investor. So, conservative investors suspecting something wrong and trying to minimize the risk withdraw the assets faster than they increase the investments when the investment climate improves. Contrariwise, an aggressive investor will buy faster than sell to maximize the profit rather than minimize the risk. In the eyes of an investor of the intermediate type the rational rates of inflow and outflow of capital coincide, if the current value of the P/E ratio symmetrically settles to the right or to the left of the preset value he or she will move with the same speed along the effective boundary to the left or to the right from the control portfolio point, correspondingly.

The linear shape of the model (A1.5) by default assumes no deep fluctuations of current the P/E ratio from the preset value because using effective means of recognition of market situation (and all these means are described) an investor will operatively correct his or her investment strategy, and the fluctuation of the P/E ratio index will not be highly volatile.

That is, the model assumes detailed attuning to an investment situation (an investment tendency), because in reality rational investors follow the macroeconomic situation very closely, and their decisions on management of the stock capital are exact (**differentiated**) and operative (**alert**), which is reflected in the model.

The model (A1.5) assumes the mechanism of **negative feedback** self-control of the market. According to the ratios, the over-valuation of the index causes negative profitability and the reduction of the level that, in turn, results in under-valuation and the emergence of positive profitability. All these tendencies generate a cyclic behavior, a cyclic trend.

A1.7. Model and methods of evaluation of the calculated corridor of profitability by the index of the second echelon stocks (the phase 4)

There is a stock markets tendency of the low capitalization stocks being guided by the trends of high capitalization stocks. It is especially true for the technically weak stock markets, when the stocks circulating on it "have no say," that is, they are disconnected from their fundamental characteristics, and there are no market players who could match the fundamental parameters of a stock and its price. Thus, the

Russian stock market exists and will exist for some time looking up to the American market, tracking the American dynamics, and the stocks, issued in Russian provinces, will for a long time track the dynamics of stocks of the domestic industry giants.

Paradoxically, the correlation of the indices of stocks of the first and the second echelons in short-term prospect is close to zero. The reason is that the second echelon stocks circulate faster than the first echelon stocks, and their prices change quickly too. Considering the correlation of these stocks on the long-term basis, disregarding slow fluctuations of the indices, such correlation will asymptotically approach one.

Therefore, it will be correct to think that at the level of monotonous stock portfolio in the medium-term prospect there is a linear dependence between the calculated profitabilities of the stocks of the first and the second echelons:

$$R_{A2}^{\bullet}(t) = R_{A1}^{\bullet}(t) \times Z^{\bullet}. \qquad \textbf{(A1.8)}$$

Our conclusion is confirmed indirectly by the results of modeling with the **SBS Portfolio Optimization System** program (Fig. A1.1). It is clear that the curvature of the parabola of the effective boundary is slight (even with zero correlation), and with as the correlation increases this parabola will only straighten.

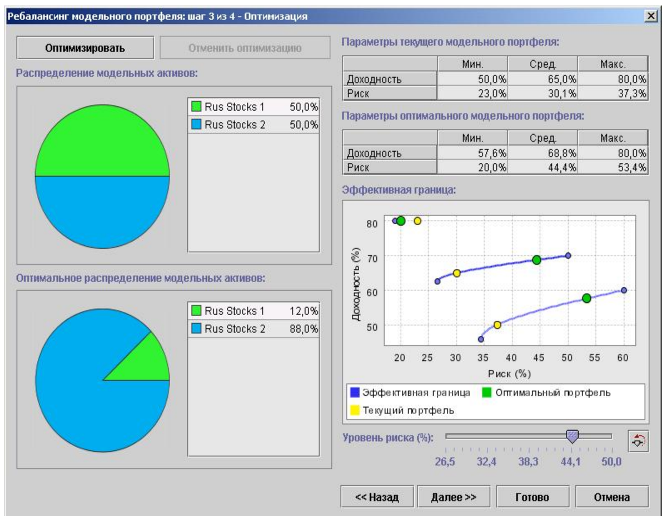

Figure A1.1 The first and the second echelon stocks modeling portfolio

So, we have obtained the forecast of the calculated corridor of profitability for all the types of stock indices, and now the process passes to the phase 5 – the evaluation of indices' profitability and risk and portfolio re-balancing.

A1.8. The phase 5 models and methods

We look for symmetric quasi-statistic evaluations of profitability and risk of stock indices because under the conditions of substantial uncertainty and the rational investment choice these evaluations are the most plausible (balanced). Such evaluations show that there is no **displacement** in the estimations of profitability and risk under the balanced investment choice, otherwise (for example, under asymmetrical risk) the possibility of over-valuation (under-valuation) of the index is assumed.

The calculated corridor of profitability in our model is related to fuzzy evaluations of profitability and risk by the following simple formula of **anticipation**:

$$\mathbf{R^{\bullet}(t) = r^{\bullet}(t+1) + \frac{\sigma^{\bullet}(t+1)}{2}}. \qquad \textbf{(A1.9)}$$

On the basis of the calculated corridor obtained on the current interval of forecasting we form the evaluations for the subsequent interval of forecasting, and that constitutes the anticipation here. The range of half mean square deviation in (A1.9) is the range of **rational confidence** in those evaluations which belong to the corresponding calculated corridor (assuming the normal disperse distribution with fuzzy parameters of distribution). If the level of the confidence is lower, the corridor is wider, and it covers actually improbable scenarios of the succession of events. On the contrary, if the confidence is higher, the corridor is narrower, and it misses quite plausible evaluations.

Transforming (A1.9) to the real numbers representation creates a system of three linear algebraic equations with three unknowns (temporarily, for the convenience of presentation, we shall remove the time dependence from the equations):

$$\begin{cases} \mathbf{r_{max}} + \boldsymbol{\sigma}_{\mathbf{max}}/2 = \mathbf{R_{max}} \\ \mathbf{r_{min}} - \boldsymbol{\sigma}_{\mathbf{max}}/2 = \mathbf{R_{min}} \\ \mathbf{r_{max}} + \mathbf{r_{min}} = 2\mathbf{R_{av}} \end{cases} \qquad \textbf{(A1.10)}$$

The system of equations (A1.10) is degenerated and it requires an additional condition to be solved. The equations of the estimated balancing can serve as such a condition:

$$\frac{\mathbf{r_{max}}}{\boldsymbol{\sigma}_{\mathbf{max}}} = \frac{\mathbf{r_{min}}}{\boldsymbol{\sigma}_{\mathbf{min}}} = \frac{\mathbf{R_{max}}}{\mathbf{R_{max}} - \mathbf{R_{min}}} \qquad \textbf{(A1.11)}$$

for

$\mathbf{R_{max} > 0, R_{min} > 0;}$

$$\frac{\mathbf{r_{max}}}{\boldsymbol{\sigma}_{\mathbf{min}}} = \frac{\mathbf{r_{min}}}{\boldsymbol{\sigma}_{\mathbf{max}}} = \frac{\mathbf{R_{min}}}{\mathbf{R_{max}} - \mathbf{R_{min}}} \qquad \textbf{(A1.12)}$$

for

$\mathbf{R_{max} < 0, R_{min} < 0;}$

and

$$\frac{\mathbf{r_{max}}}{\boldsymbol{\sigma}_{\mathbf{max}}} = -\frac{\mathbf{r_{min}}}{\boldsymbol{\sigma}_{\mathbf{min}}} = \frac{\mathbf{R_{max}}}{\mathbf{R_{max}} - \mathbf{R_{min}}} \qquad \textbf{(A1.13)}$$

for the mixed case

$\mathbf{R_{max} > 0, R_{min} < 0.}$

The equations (A1.11) - (A1.13) express that the relation of the profitability and the risk of indices in maximum and minimum cases depends only on the relation of the maximum and the minimum of profitability in the calculated corridor. Then, all the parameters of the model are found by the following equations:

for $\mathbf{R_{max}} < 0$ and $\mathbf{R_{min}} < 0$:

$$\mathbf{r_{min}} = \frac{\mathbf{2R_{min}}^2}{3\mathbf{R_{min}} - \mathbf{R_{max}}};. \qquad \textbf{(A1.14)}$$

$$\mathbf{r_{max}} = 2\mathbf{R_{av}} - \mathbf{r_{min}};$$

$$\mathbf{r_{av}} = \mathbf{R_{av}};$$

$$\boldsymbol{\sigma}_{\mathbf{max}} = \mathbf{r_{min}} \times \frac{\mathbf{R_{max}} - \mathbf{R_{min}}}{\mathbf{R_{min}}};$$

$$\boldsymbol{\sigma}_{\mathbf{min}} = \mathbf{r_{max}} \times \frac{\boldsymbol{\sigma}_{\mathbf{max}}}{\mathbf{r_{min}}};$$

$$\sigma_{av} = \frac{\sigma_{max} + \sigma_{min}}{2};$$

for $R_{max} > 0$ and $R_{min} > 0$:

$$r_{max} = \frac{2R_{max}^{2}}{3R_{max} - R_{min}}$$

$$r_{min} = 2R_{av} - r_{max}$$

$$r_{av} = R_{av} \quad , \qquad \textbf{(A1.15)}$$

$$\sigma_{max} = r_{max} \times \frac{R_{max} - R_{min}}{R_{max}}$$

$$\sigma_{min} = r_{min} \times \frac{\sigma_{max}}{r_{max}}$$

$$\sigma_{av} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

and for the mixed case ($R_{max} > 0$ and $R_{min} < 0$):

$$r_{max} = \frac{2R_{max}^{2}}{3R_{max} - R_{min}}$$

$$r_{min} = 2R_{av} - r_{max}$$

$$r_{av} = R_{av} \quad , \qquad \textbf{(A1.16)}$$

$$\sigma_{max} = r_{max} \times \frac{R_{max} - R_{min}}{R_{max}}$$

$$\sigma_{min} = - r_{min} \times \frac{\sigma_{max}}{r_{max}}$$

$$\sigma_{av} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Thus, we have obtained the evaluations $r^{\bullet}(t+1)$ and $\sigma^{\bullet}(t+1)$ of all the stock indices of an economic region. Practically it means that it is possible to solve the optimization problem for generalized investment stocks and bonds portfolio quarterly and to determine the rational trajectory of moving of the portfolio point from boundary to boundary in course of forecasting (the phase 6 of forecasting).

A1.9 Phase 6 models and methods

Let's consider the case of the generalized investment portfolio effective border sliding (only the middle line of the boundary is marked) step by step under the worsening of investment conditions (Fig. A1.2)

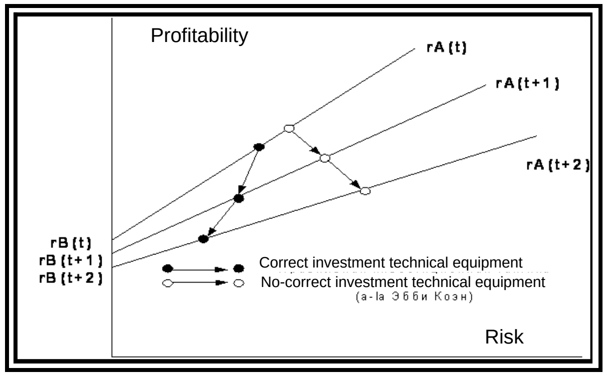

Figure A1.2 Management of stock portfolio in time

If you decide to follow the Abby Cohen's 2001 advice, you should do no more than maintain the fixed balance of assets. Such tactics on the bearish market causes only additional losses, the increased portfolio risk, and nothing else. **On the contrary**, it is necessary to get rid of stocks several times faster, than they fall, investing in bonds or leaving the market altogether. Thereby outstripping decrease of portfolio risk is achieved and the conservative investment choice is realized. The choice of Abby Cohen in this case appears unduly aggressive, **anti-optimum**; the **gradient** of her choice (the increment of profitability to the increment of risk ratio) is negative in all points of her investment trajectory. Our gradient is positive in all points, and moreover, it increases.

These operative reasons are recorded in the model by the modified Sharpe factor:

$$\mathbf{Sh^{\bullet}(t+1) = \frac{r_A{}^{\bullet}(t+1) - r_B{}^{\bullet}(t+1)}{\sigma_A{}^{\bullet}(t+1)}} \qquad \textbf{(A1.17)}$$

The expression (A1.17) is not a classical Sharpe factor, because in the numerator we subtract the average profitability of the whole class of bonds rather than state bonds only. But the meaning of this factor is very significant: it expresses the economic efficiency of investments in the generalized investment portfolio of all stocks and all bonds within the given economic region. We maintain that as the economic efficiency of a portfolio reduces (mainly due to the falling profitability of stocks) the share of stocks in the portfolio should decrease with the outstripping pace. That is, the condition of preservation of optimality when moving from the right to the left along the boundary is the condition of positive gradient (when moving from the left to the right the gradient may be either):

$$\mathbf{\frac{r_{av}(t) - r^{\bullet}(t+1)}{\sigma_{av}(t) - \sigma^{\bullet}(t+1)} > (0,0,0)} \qquad \textbf{(A1.18)}$$

where

$$\begin{aligned} &\mathbf{r^{\bullet}(t) = Sh^{\bullet}(t) \times \sigma^{\bullet}(t) + r_B{}^{\bullet}(t) = (r_A{}^{\bullet}(t) - r_B{}^{\bullet}(t)) \times x_A(t) + r_B{}^{\bullet}(t)} \\ &\mathbf{\sigma^{\bullet}(t) = x_A(t) \times \sigma_A{}^{\bullet}(t)} \end{aligned} \qquad \textbf{(A1.19)}$$

It follows directly from (A1.18) and (A1.19):

$$\mathbf{x_A(t+1) \le min\,(x_A(t)\frac{\sigma_A(t)}{\sigma_{Amax}(t+1)}, \frac{(r_{Aav}(t) - r_{Bav}(t)) * x_A(t) + (r_{Bav}(t) - r_{Bmax}(t))}{r_{Amax}(t+1) - r_{Bmin}(t+1)}, x_A(t) - \Delta x_{pl})} \qquad \textbf{(A1.20)}$$

for the scenarios of the outflow of capital from stocks for the recalling tendencies, and

$$\mathbf{x_A(t+1) \ge max\,(x_A(t)\frac{\sigma_A(t)}{\sigma_{Amin}(t+1)}, \frac{(r_{Aav}(t) - r_{Bav}(t)) * x_A(t) + (r_{Bav}(t) - r_{Bmin}(t))}{r_{Amin}(t+1) - r_{Bmax}(t+1)}, x_A(t) + \Delta x_{pl})} \qquad \textbf{(A1.21)}$$

for the scenarios of investment of capital in stocks for the attractive tendencies. There is no change of the stocks share in the portfolio for the waiting tendencies. In (A1.20) and (A1.21) $\Delta \mathbf{x_{pl}}$ is a planned inflow or outflow of capital which comes into effect if the rest of the calculated values in equations become nonoptimal or intolerable by boundary conditions.

Thus, we have obtained the target value of the share of stocks in a portfolio for the forecast period, determined by (A1.20) – (A1.21).

The rational sizes of shares of bonds (B) and the withdrawn capital (N) are determined on the basis of data of Table 1.2 (rational outflow of capital, where $|\Delta x_A(t)| = |x_A(t+1) - x_A(t)|$, $|\Delta x_B(t)| = |x_B(t+1) - x_B(t)|$). Table A1.2 makes it clear that in absence of stocks outflow, the next outflow is chosen on the basis of the bonds outflow values from the previous modeling step. To prevent the divergence of the portfolio formation process any new outflow in such cases is twice less than the previous (because the profitability of bonds is low no essential change of characteristics of the generalized investment portfolio is expected). This way of the organization of outflows is caused by instability of tendencies related to the waiting stocks choice, and unstable equilibrium of waiting states. And instability forbids sudden movements, because it is possible to bear unexpected substantial losses.

Table A1.2 Scenario of investment flows

Number of situation from Table 7.5	**Rational flows of capital: + inflow, - outflow, 0 – no movement**		
	A	**B**	**N**
1	$+\|\Delta x_A(t)\|$	$-\|\Delta x_A(t)\|$	0
2	0	0	0
3	$-\|\Delta x_A(t)\|$	0	$+\|\Delta x_A(t)\|$
4	$+\|\Delta x_A(t)\|$	$+\|\Delta x_B(t-1)\|/2$	$-\|\Delta x_A(t)\|/2-\|\Delta x_B(t-1)\|/2$
5	0	$+\|\Delta x_B(t-1)\|/2$	$-\|\Delta x_B(t-1)\|/2$
6	$-\|\Delta x_A(t)\|$	$+\|\Delta x_A(t)\|$	0
7	0	$+\|\Delta x_B(t-1)\|/2$	$-\|\Delta x_B(t-1)\|/2$
8	$-\|\Delta x_A(t)\|$	0	$+\|\Delta x_A(t)\|$
9	$-\|\Delta x_A(t)\|$	$-\|\Delta x_B(t-1)\|/2$	$+\|\Delta x_A(t)\|+ \|\Delta x_B(t-1)\|/2$

So, the phase 6 of the process is completed, and we move to the phase 7 – the forecasting of indices and the P/E ratio factor.

A1.10 Phase 7 model and methods

The forecast of the index is performed according to the formula

$$\mathbf{P^{\bullet}(t+1) = P_{av}(t) * (1 + R^{\bullet}(t) * \Delta T)}, \qquad \textbf{(A1.22)}$$

and the forecast of the P/E ratio factor – according to the formula (A1.3):

$$\mathbf{PE^{\bullet}(t+1) = PE_{av}(t) * \Lambda^{\bullet}(t)}, \qquad \textbf{(A1.23)}$$

where

$$\mathbf{\Lambda^{\bullet}(t) = \frac{(1 + R_A{}^{\bullet}(t) * \Delta T)}{(1 + GDP^{\bullet}(t)) * (1 + I^{\bullet}(t))}}, \qquad \textbf{(A1.24)}$$

$\mathbf{R_A{}^{\bullet}(t)}$ is the calculated corridor of profitability of the stock index.

What makes formulae (A1.22) – (A1.24) distinctive is the elimination of intermediate uncertainty at the time the forecast evaluation is formed because we believe that the forecast values are first and foremost influenced by the expected mean values of indices obtained in the previous time intervals of forecasting. That is, in our expert model the forecast uncertainty has the lifespan of one forecast quarter (and it influences the evaluations during that period). Were the elimination principle in evaluations not applied, our forecast would have appeared “noisy” due to the accumulated fuzzy evaluations.

Equation (A1.24) also expresses the very essence of our modeling assumptions about rational choice. When the current value of the P/E ratio equals to the preset one, the rational value $\Lambda^{\bullet}(t) = 1$ means that the system of the investment choice is in balance, and the growth of stock profits is backed by the corresponding growth of gross internal regional product. If the stock's gain is not fully backed by the real wealth (corporate profits) the stocks become over-valued, "overheated", and that starts up the mechanism of reduction of current profitability by the index (through the elasticity of the (A1.5) type).

After the phase 7 finishes the process passes to the technical phase 8 (branching of the procedure of forecasting).

A1.11 Phase 8 Model and methods

The forecast time is incremented and the condition $t > t_{end}$ is checked. If the condition is true, the forecast is completed, and we move to the phase 9, else we go back to the phase 3.

A1.12 Phase 9 Model and methods

At this phase the obtained forecast by the indices is corrected by the rate of national currency of economic region in relation to RuR. This correction is carried out according to the formula:

$$\mathbf{P^{\bullet\backslash}(t) = P^{\bullet}(t) \times J^{\bullet}(t).} \qquad \textbf{(A1.25)}$$

A1.13 Phase 10 Model and methods

At this phase we form the evaluation of calculated corridor of final profitability by the index corrected in the previous phase. The equation for the calculated corridor of final profitability is:

$$\mathbf{R^{\bullet\backslash} = \frac{P^{\bullet\backslash}(t_{end}) - P^{\bullet\backslash}(t_{start})}{P^{\bullet\backslash}(t_{end}) * (t_{end} - t_{start})}.} \qquad \textbf{(A1.26)}$$

A1.14 Phase 11 Model and methods

At this phase we get the final evaluation of profitability and risk of the stock index. This evaluation can be used as the basis of portfolio optimization, if the horizon of investment coincides with the period of the forecast. All the evaluations are obtained by replacing the calculated corridor $\boldsymbol{R^{\bullet}(t)}$ with the parameter $\boldsymbol{R^{\bullet\backslash}}$ in the formulae (A1.9) – (A1.16).

APPENDIX 2. GLOSSARY

Abby Cohen	is a modern American financial analyst. For some time she successfully predicted the market tendencies, earning popularity as a leading US forecaster. However, her advice of 2001 about balancing portfolios with falling assets by the principle following the market led to the billions of dollars in losses across America.
Active portfolio management	is the type of management characterized by the continuous re-balancing of portfolio on the basis of alerts (alert management of portfolio) or other reasons.
Aggressive investment choice	is the type of a rational investment choice with the raised ratio of stocks in the generalized investment portfolio.
Alert	is a warning signal testifying to the qualitative macroeconomic changes (a macro-alert) or the changes of factors of price, profitability, PE ratio, etc. (a technical alert).
Alert portfolio management	is an active management of portfolio on the basis of alerts.
Asymmetric investment choice	is a property of irrational investment choice. The asymmetry develops simultaneously in two planes. First of all, the bitterness of losses is more intense, than the pleasure of profit. Secondly, an irrational investor buys the securities which prices temporarily fall faster than he

or she sells the ones which prices temporarily rise. The asymmetry is based on **greed** and **fear**.

Average rational market line is a model used for long-term forecasting. It assumes the possibility of extrapolation of a medium-term interval of forecasting data for a long-term interval.

Bottom of the trend is a pronounced local or global minimum of an index. Usually the bottom is in the form of a triangular fuzzy number.

Calculated corridor of portfolio profitability is a triangular evaluation of a future value of an index at the end of an effective interval of forecasting (in our case, a quarter). It is related to an index profitability and risk by the simple equations.

Conservative investment choice is a type of rational investment choice with the increased share of bonds in the generalized investment portfolio.

Control portfolio point represents 50% of stocks, and 50% of bonds (50:50).

Cyclic trend is a trend that reflects the cyclic behavior of an index due to the continuously varying macroeconomic environment of the index and the over-valuations and under-valuations caused by it.

Disparity is an investment disbalance with the positive (under-valuation) or negative (over-valuation) nuance.

Economic region is a country or a number of countries indices of which can be estimated independently, on the basis of uniform currency with the comparable rates of inflation and rates of the gross domestic product growth. The USA and the Russia are the examples considered in the present work.

Effective boundary of a portfolio set is a continuous concave curve in the “risk-profitability” coordinates that characterizes the maximum of profitability of a portfolio with unknown weights of assets under the fixed portfolio risk. In fuzzy problem definition the effective boundary becomes the strip effective boundary.

Effective portfolio management is the portfolio management in the real time mode.

Equal preference – see **Indifference**.

Euphoria is an unreasonable buying of stocks causing their over-valuation. It is an accompanying attribute of the irrational investment choice.

Expert model is a set of qualitative descriptions of the current status of a research object (in our case, the stock market and its macroeconomic environment) and prospective tendencies of its development. Only the correctly formed expert model can lead us to the correct mathematical model and adequate methods. The forecasts not based on an expert model are meaningless.

Fear here is a type of irrational investment choice characterized by unmotivated under-valuation of index assets. It is accompanied by neurosis of an investor arising in course of the evaluation of the index zone of risk. Also see **Greed**.

Final profitability is a relative growth of an index price during the calculated year (annual percentage).

Fundamental analysis is a set of methods that enables one to evaluate the quality of a security on the basis of its fundamental characteristics, including the relation of the price and quality, and the data on the security issuer. Sometimes the conclusions of fundamental analysis contradict the conclusions of **technical analysis**. The present work is written based on the fundamental analysis.

Generalized modeling investment portfolio is a portfolio generated in the given economic region and consisting of stocks (A), bonds (B) and other assets (N). It is characterized by the list of the corresponding indices and share distribution (A, B, N). This is an abstract category used in the models of the indices forecasting.

Golden rule of investment (*GRI*) **–** "The greater profitability of assets corresponds to the greater expected risk." It meets the indifference criterion. The assets picked on the basis of GRI form a monotonous investment portfolio.

Gradient of a portfolio is the ratio of the increase of a portfolio profitability to the corresponding increase of the portfolio risk.

Greed – here is a type of irrational investment choice characterized by unmotivated over-valuation of index assets. It is accompanied by the neurosis of an investor arising in the course of evaluation of the risk zone of an index. Also see **Fear**.

Herd instinct is the term coined by George Soros. It expresses the property of mass investment processes when everybody is guided by each other and they simultaneously buy and sell the same assets. In the raising market the herd instinct causes the effect of rally. The herd instinct provokes the synchronous volatility. It prevents the reasonable diversification.

Heteroskedasty is a synonym of volatility of an index changing in time.

Highly elastic factor is a factor sensitive to the change of another one that has an effect on the behavior of the given factor. In our case this is an essential dependence of the calculated corridor of profitability of a stock index on the PE Ratio.

Homoskedasty is a synonym of volatility of an index constant in time.

Hysteria is an unreasonable sale of stocks causing their under-valuation, and market panic. It is an accompanying attribute of irrational investment choice.

Impossibility of forecasting is the absence of sufficient scientific base for a forecast formation. It is applicable to the long-term forecasts of indices.

Index is 1) a calculated object created by special rules, usually as a portfolio with the fixed distribution of shares; 2) quantitative values of the price of the index portfolio. One distinguishes the stock and macroeconomic indices (indicators).

Indifference is a property of rational investment choice. It means that in no situation a rational investor prefers one type of investment over another. Indifference is characteristic of intermediate investment choice. It is reached, in particular, in control portfolio point. It is synonym with **equal preference**.

Intermediate investment choice is a type of rational investment choice with the parity of stocks and bonds in the generalized investment portfolio. The control portfolio point belongs to this type of choice.

Investment balance is a state of indifference (equal preference) in the course of rational investment choice. It is typical when the bottom or the peak of an index is reached.

Investment tendency is a characteristic of the stock market macroeconomic environment in the given economic region. It usually depends on the level of key parameters (gross domestic product, inflation, currency exchange rates), and also on the preset PE ratio that characterizes the rational investment choice.

Irrational diversification is a scientifically unreasonable share distribution of an investment portfolio. For example, it is unreasonable to diversify the portfolio of bonds with the stocks falling in price.

Irrational investment choice means investments without a reasonable scientific basis that assumes the calculated losses. These investments are usually made under the influence of **greed**, **fear** and a **herd instinct**. The characteristic attributes of irrational investment choice are **euphoria** and **hysteria**.

Low elastic factor is the factor which is not substantially sensitive to a change of another factor that influences the behavior of the given one. In our case this is the virtual independence of calculated corridor of the bonds index profitability from the volume of tenders.

Monotonous investment portfolio is a generalized investment portfolio formed on the basis of the golden rule of investment. It is characterized by the absence of the “outsider” assets (assets with the worst profitability and risk factors simultaneously). It does not exist always and everywhere. For example, in July-August 2002 it didn’t exist in the USA because of global over-valuation of stocks, thus the segment of highly remunerative high risk investments was not formed, and the monotonous portfolio was not filled. When the profitability and risks of indices in the monotonous portfolio are hard to determined, the **ratios of order** are formed.

Negative feedback is a term of the automatic control theory. It reflects the ability of an automatic system to adjust an input signal so that it will pull the corresponding output in the direction opposite to the output’s current change. In our case, negative slope of the lines of elasticity of the factor of calculated profitability of an index to the factor of P/E Ratio causes such a sequence of forecast events in the prognostic model that over-valued assets start to depreciate, and under-valued securities start to appreciate.

Overvalue is the state of the market when the prices of assets are higher than the rational, previously evaluated level.

Passive management of portfolio – see **Principle of balancing**.

P/E Ratio is the ratio of a share (index) price to the net profit per share (average index share) calculated annually. Index the P/E ratio is calculated as weighted average over the stocks comprising the index, accounting for the market capitalization of the corresponding stocks and their shares in the index.

Peak of a trend is a local or the global maximum of a trend.

Principle of balancing (the principle of following the market, the Abby Cohen principle) is an unscientific principle of investment based on the balancing of a portfolio according to the tendencies of indices. The operation of so-called balance index funds is (currently bearing enormous losses) based on this principal. The last time it was defended by American financial analyst Abby Cohen in 2001.

Profitability – see **Final profitability**.

Rally is the term expressing rapid growth of assets after some price drop. A rally is usually provoked by the “bulls”.

Ratio of order is the mathematically expressed ratio of quantitative preference of some objects over the others. We consider the ratios of order of profitabilities and risks in the monotonous investment portfolio.

Rational investment choice (**rational investments**) is a choice of a **rational investor** based on scientifically sound market forecasts. It assumes positive **final profitability** of investments on the forecasting interval.

Rational investor is an investor motivating his or her investment choice by scientific reasons.

Reasonable diversification is a scientifically based inclusion of assets with various ratios of index profitability and risk in a portfolio. In particular, the **monotonous investment portfolio** is reasonably diversified.

Re-balancing portfolio is a modification of portfolio shares on the basis of some input conditions (for example, a change of an investment tendency, an **alert**).

Risk premium is an addition to the current profitability of stocks or bonds relative to the inflation rate. In the course of investment cash is withdrawn from circulation. That can lead to losses (depreciation), therefore there is an investment risk, for which an investor demands a compensation premium. An exotic kind of risk premium is the stock profitability premium for the incorrect bookkeeping (when the amount of corporate profit is exaggerated).

Risk Zone of an index includes the index values immediately close to the bottom or the peak characterized by the high risk of an investment tendency change.

Sharpe factor is a fraction that has a difference of profitabilities of an index and the state bonds (relatively risk-free assets) as its numerator and the index risk as its denominator. In our case, alongside with the classical Sharpe factor, the modified factor is used. The numerator of the modified Sharpe factor contains the profitability of the bonds index in the generalized investment portfolio, rather than the profitability of the state bonds.

Sharpe modified factor – see **Sharpe factor**.

Sobering up here is an intermediate state between the irrational and rational investment choice. The characteristic example is the August, 2002, in the USA.

Strip effective boundary – see **Effective boundary**.

Synchronous volatility happens when the assets of one index vary in phase due to full correlation of assets. It usually is a manifestation of herd instinct of investors.

Technical analysis is a set of methods that allows making index predictions on a limited interval of time (between one trading day and one quarter). Technical analysis cannot be applied to medium-term index forecasting because it assumes static tendencies that must manifest themselves in the near future. The counterbalance to the technical analysis is the **fundamental analysis**.

Trend is a medium line of an index price (in our case, a triangular fuzzy function or sequence). In technical analysis the trend is estimated by method of sliding average (with average summation of the price readouts of an index for a certain number of days). It is the index trend that is forecasted in this book.

Types of rational investment choice are aggressive, conservative, and intermediate.

Under-valuation is a state of the market when the prices of assets are lower than the rational, previously evaluated level.

Volumetric elasticity of the profitability factor is a dependence of the index profitability on the volumes of tenders.