The Stock Market Forecasting: An Application of the Interval Measurement and Computation

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Abstract: This study forecasts changes in the stock market by using the interval measured data and interval computing approach. Empirical results in this study strongly suggest that the interval forecasting is superior to the traditional point data based OLS forecasting, in terms of significantly smaller mean and standard deviation of forecast errors. Moreover, the accuracy ratio indicates that the out-of-sample interval forecasting has an actual forecasting accuracy over 58%. The ratio becomes more than 63% for the in-sample interval forecasting.

1. Introduction

1.1. Major issues in the stock market forecasting

One of modern capital asset pricing theories, the arbitrage pricing theory (APT) of Ross (1976) provides a theoretical framework to identify macroeconomic variables that can significantly or systematically influence stock prices. In an early attempt to discover economic driving forces for the stock market, Chen, Roll and Ross (1986) find five systematic factors that are still widely accepted in the literature: the spread between long- and short-term interest rates; expected and unexpected inflations; industrial production; and the spread between high- and low-grade bonds. All of them represent sources of risk that “are significantly priced” in the stock market. In other words, these macroeconomic variables have predicting power on the stock market. Therefore, they can be used to forecast changes in the stock market. This study adopts these five variables in the stock market forecasting.

There is a consensus in the literature that sensitivities of the stock market to macroeconomic factors vary over time. Many statistical methods have been introduced to deal with the time-varying issue. For instance, Brown, Durbin, and Evans (1975) developed the Cusum and Cusum of Squares tests to find switch points for a multi-factor regression model. Fama and French (1997) use the rolling regressions to produce time-varying regression coefficients and forecasts for excess stock returns of industrial portfolios. In a recent study, He (2005) suggests exponentially weighted rolling regressions to gradually discount old observations kept in the estimation period, under the assumption that earlier observations have less influence on coefficient estimates. He also uses the Flexible Least Squares (FLS) developed by Kalaba and Tesfatsion (1988, 1989, 1990) to generate rolling forecasts. The results are superior to those from the Cusum and Cusum of Squares, normal and exponentially weighted rolling regressions.

Nonetheless, the poor forecasting quality is a persistent problem. Fama and French (1997) report an average monthly forecast error of 2.72% for their out-of-sample rolling forecasts. The number for He’s (2005) sample is 1.88%. The FLS monthly rolling forecasts have a mean error of 1.57%. Apparently, multiplying by 12, these monthly numbers can be turned into big annual ones. A new feasible approach to improve the forecasting quality is to use a more effective data measurement and estimation technique.
The primary measurement in economic and financial data is to quantify points. Take the stock market as an example. A daily stock price index consists of daily closing prices and a monthly index is the average of daily index numbers. They simply reflect a particular number (level) or an average number at a particular time spot. When a growth rate is in need, a current point is then compared with a point at the previous time spot. Only the level change (a point) over the time interval (daily, monthly, or annual) is measured at the end of the period. The primary purposes of various forecasting methods based on the point measured data are in common, that is, to predict points that can match actual ones. A major shortcoming of the point data and point forecasting is that they are unable to use and forecast variability in a time period. In fact, not only regression coefficients are time-varying, macroeconomic variables are time-varying as well. For a given trading day (month or year), there are two price bounds, the highest and lowest. The two prices reflect the variability of the day (month or year). In addition, in the real world forecasts are often in the form of a range or interval. It is not unusual at all for an economist to make the next year’s inflation forecast to be, for example, between 2-4%. It is a forecasted inflation interval for the next year. Unfortunately, only the point data and point-based forecasting methods currently exist in the economic and financial forecasting literature.

1.2. Interval forecasting methods

In the past two decades some statisticians developed various point-based interval forecasting methods. However, they are fundamentally “semi” interval forecasts, because they are based on point data and point-based forecasting methods, such as Bayesian, Bootstrapping, Box-Jenkins, GARCH, and Holt-Winters methods. The general forecasting principle of these methods is that the forecasted interval is simply the sum of the point forecast and some percent of positive and negative variance of forecasts (Chatfield, 1993, 1998). There are many reasons why those “semi” interval forecasting methods are never widely used. The most important reason is the poor quality. Normally, those forecasted intervals are so narrow that there is only a 50% chance for a future point lying inside the interval (Gardner, 1988 and Granger, 1996). Evaluating “semi” interval forecasts is another major problem (Christoffersen, 1998 and Clements and Taylor, 2003). This problem stems from the methodology of “semi” interval forecasting. In the process of estimating the point forecast, the aim is to minimize estimation errors. However, some percent of estimation variance is added to and subtracted from the point forecast to form an interval forecast. This contradiction makes traditional forecast error measure impossible.

1.3 The interval computing approach

In the mathematical and computing fields, a new method, interval computing, has theoretically or computationally been developed since the late 1970s (Moore, 1979). In interval computation, both operands and computational results are intervals. By taking interval valued parameters (more information) into considerations, one may obtain computational results that can be reliable both computationally and mathematically with interval arithmetic. Numerous results have been obtained with interval computing especially for some otherwise very difficulty computational problems such as reliable non-linear global optimization and others (Kearfott, 1996; Hu, Xu and Yang, 2002; Kearfott and Hongthong, 2005). By applying interval computing, Stadtherr’s group discovered roots of equations never reported previously in chemical engineering (Gau and Stadtherr, 2000; Hua, Brennecne, and Stadtherr, 1996). These findings result in the 1998 Computing in Chemical Engineering Award of American Institute of Chemical Engineers. More recently, the interval computation has been extended to different computing areas such as data mining, decision making, game theory, and others (Korvin, Hu and Chen, 2002 and 2004; Collins, 2005).
The interval computation creates a new forecasting format and perhaps, more accurate forecasts. The major purpose of this study is to employ the interval method to forecast changes in the stock market.

In order to make interval forecasts, a different data measurement, interval measurement, must be used. A mathematical interval \([a, b]\) is the set \(\{x \in \mathbb{R} | a \leq x \leq b\}\) where \(a \leq b\). Mathematical intervals are represented and operated with machine intervals inside computers. In an interval data set, every observation is measured as an interval, a combination of an upper bound and a lower bound. Contrast to the point measurement, interval measurement automatically contains the (level) information measured by the point number as well as the variability in the time duration. Use stock price as an example. The closing price (Level) measured by the point number must locate in somewhere between the upper and lower bounds of the trading day. Moreover, the range between the upper and lower bounds represents the maximum variation of the day. A variation measurement is a risk measurement. Some traditional statistical measurements, such as range, variance, standard deviation, and coefficient of variation, are examples for risk measurements. In the same sense, intervals also measure risk. Given that more information is included in the interval measurement, it is reasonable to expect a higher quality for the interval forecasting.

The main objective of interval computing procedure is to match the center of two interval vectors, in order to measure their relationships which are essential to the interval forecasting. The predicted interval reveals future variation or risk which is important in determining required returns and capital asset value, there is no doubt that the output of the interval forecasting can provide useful information to investors and economic/financial policy-makers.

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The remainder of this paper is organized as follows. Section 2 describes the estimation model and data, Section 3 discusses interval computation procedures, Section 4 presents empirical results, and Section 5 contains concluding comments.

2 Model and data

According to Chen, Roll and Ross (1986), changes in the stock market \((SP_t)\) is linearly determined by the following five macroeconomics factors: growth rate variations of seasonally-adjusted Industrial Production Index \((IP)\), changes in expected inflation \((DEI_t)\) and unexpected inflation \((UI_t)\), default risk premiums \((DEF_t)\), and unexpected changes in interest rates \((TERM_t)\):

\[
SP_t = a + I_t(IP_t) + U_t(UI_t) + D_t(DEI_t) + F_t(DEF_t) + T_t(TERM_t) + e_t \quad (1)
\]

This study uses time series data that covers the period of January 1930-December 2004. There is no specific reason, other than the availability of data, for the selected sample period. The data set includes the following basic monthly series:

- **IP**: the growth rate of seasonally-adjusted Industrial Production Index at the beginning of the month. This study uses the one-month lead term of IP in the monthly data.
- **LONG**: the monthly returns on long-term U.S. government bonds.
- **CORP**: the monthly returns on long-term corporate.
• **SHORT**: the monthly returns on one-month U.S. Treasury Bills. According to Fama and French (1993), SHORT\(_{t-1}\) is the proxy for “the general level of expected returns on bonds.”

• **CPI**: the growth rate of the U.S. Consumer Price Index.

• **SP**: the growth rate of Standard and Poor’s 500 Stock Price Index.

For IP, CPI, and SP, the monthly growth rate is the first difference of natural logarithms in months of t and t-1; the annual growth rate is the first difference of natural logarithms in Decembers of t and t-1. The annual returns for LONG, CORP, and SHORT are compounded monthly returns of January through December. The following additional series are derived from the above basic series:

\[ DEF_t = CORP_t - LONG_t. \]  It represents the default risk premium (Fama and French, 1993).

\[ TERM_t = LONG_t - SHORT_{t-1}. \]  It measures unexpected changes in interest rates (Fama and French, 1993).

\[ UI_t; \] unexpected inflation. It is proxied by the residuals from the following regression model (Fama, 1981):

\[ CPI_t = \alpha_{t-1} + \beta_{SHORT_t-1} + \eta_t \]

\[ EIt_{t-1}; \] expected inflation at the end of month t-1. It is the difference of CPI\(_t\) and UI\(_t\).

\[ DEIt; \] the change in expected inflation. It measures the difference of EIt and EI\(_{t-1}\).

In addition to monthly inflation variables, this study also calculates quarterly expected and unexpected inflation variables. By multiplying 4, the quarterly inflation variables are converted into annual variables (Fama, 1981).

3. **Interval algorithm and estimation accuracy**

3.1. **An interval least squares algorithm**

In a given time period, for instance, a month or a year, economic data can be expressed as intervals. An interval measures the greatest variation for a given time period. In this study the maximum monthly number in a given year forms the upper bound of the year’s interval; while the minimum monthly number in the year becomes the lower bound of the interval. After construction of the annual interval data, it is necessary to minimize an objective function, in terms of the least squares principle, with the interval data. Different from the ordinary least squares algorithm, an interval valued linear system of equations needs to be solved to estimate the coefficients in (1). Therefore, an interval least squares algorithm has been developed and implemented in C++ in this study to solve the problem. Details of the algorithm and software will be reported in another paper to appear in the field of computer science.

Based on the least squares principle, the interval arithmetic (Moore, 1979) is used to construct an interval valued linear system of equations. In this process, the product of two intervals are used mostly. In interval arithmetic, the product of two intervals is defined as \( [a, b] * [c, d] = [\min \{ac, ad, bc, bd\}, \max \{ac, ad, bc, bd\}] \).

In order to estimate the coefficients, it is necessary to solve an interval linear systems of equations \( Ma = v \), where \( M \) is a 6 by 6 interval matrix; \( a \) is the vector of the coefficients to be determined; and \( v \) is an interval vector. It is assumed that the coefficients are scalars initially. By taking \( M_{\text{mid}} \), the mid-point matrix of \( M \), and \( v_{\text{mid}} \), the midpoint vector of \( v \), a classic linear system of equations about \( a \) is constructed. The numerical estimations of the coefficients are obtained by using Gaussian elimination with scaled partial pivoting. This initial approach has the intuition of
matching the center of two interval vectors. However, it has not yet taken the widths into considerations. Therefore, it is essential to adjust the width of the forecasted interval. Consistent with the rolling estimation period, the width is adjusted by a rolling scalar constant which is equivalent to the average width of previous ten years.

The coefficient estimates obtained from Equation (1) may be used to forecast changes of the stock market by estimating the following the out-of-sample forecasting model:

\[
SP_t = a_{t-1} + I_{t-1}(IP_t) + U_{t-1}(UI_t) + D_{t-1}(DEI_t) + F_{t-1}(DEF_t) + T_{t-1}(TERM_t) \quad (2)
\]

In order to better reflect time-varying relationships between the stock market and other macroeconomic variables, a rolling estimation period of ten consecutive years is used to establish the interval linear system of equations. Then Equation (2) is estimated to forecast the stock market, starting with the eleventh year.

3.2. Accuracy of interval forecasting

The interval forecasting makes an effective assessment measure feasible. Both the input and output of interval forecasting are intervals. The range covered by both the forecasted and actual intervals represents the accurate part of an interval forecast. The quality of forecasting can be easily measured by a ratio of the commonly covered range to the maximum distance reached by the forecasted and actual intervals. This ratio reflects the accuracy of the forecast. For example, if the predicted interval is [2, 5] and the actual interval is [1, 3], then the overlapped range is the interval [2, 3]. The width of the overlapped interval is 1 (3-2) and the maximum distance is the width of the interval [1, 5], that is 4 (5-1). Therefore, the accuracy ratio is 25%. If the actual interval is used as the denominator, the ratio increases to 50%, however, it is misleading. The accuracy ratio reflects the real quality of forecasting and does not need any confidence assumption. Consider the following example: the predicted interval is [1, 5] and the actual interval remains the same, [1, 3]. The accuracy ratio should be 40% (2/5). If the actual interval is used as the denominator, the value of the ratio becomes 100%. Obviously, it is wrong. In this study, \(SP_{est}\) represents the forecasted SP interval. Then, the concept of the estimation accuracy is defined as:

\[
\frac{w(SP \cap SP_{est})}{w(SP \cup SP_{est})},
\]

where \(w\) is the width function of an interval.

4. Empirical results

Table 1 provides summary statistics for the monthly and annual point data as well as the annual interval data. Compared with the interval data, the point data (monthly and annual) are more volatile, evidenced by higher standard deviations. Given the higher stability for both upper and lower bounds of the interval data, the range between the two bounds should be the major source for the higher variability of the point data, because most points locate somewhere between the upper and lower bounds. The point measurement only quantifies an item at a particular time spot; while an interval measurement reflects the greatest variation in the item between two time spots. Therefore, most of the volatility of the point measurements should be covered in the intervals. In other words, intervals primarily serve as the variability measurement or risk measurement.

The OLS estimates of Equation (1) indicate that the model has a higher explanatory power (\(R^2=26\%\)) with the annual data, rather than the monthly data (\(R^2=17\%\)). The better fitness of the data guarantees better forecasting results. In order to compare the forecasting results generated by the OLS and interval methods, this study uses the annual data.

Out-of-sample rolling forecasts of changes in the stock market based on Equation (2) are reported in Table 2. The OLS forecasting uses monthly, quarterly, and annual point data, respectively. The monthly average of forecast errors is about 2.65% with a
standard deviation of 2.44%. By multiplying 12, those two numbers can be converted into annual figures of 31.83% and 29.34% which are much larger than those for annual forecasts. The annual mean of forecast errors is 20.57% and the standard deviation of forecast errors is 19%. The results unequivocally suggests that annual forecasts have the best quality among the three types of OLS forecasts.

Nonetheless, forecast errors of the annual interval forecasting are even smaller and more stable. The mean of forecast errors is only 6.25% for intervals, the combinations of the upper and lower bounds. The interval forecasts are highly consistent over time. The standard deviation of forecast errors is as low as 3.73%. Equality tests provide evidence that the quality of interval forecasts is significantly higher than OLS forecasts. The T-statistic (5.96) suggests that the mean of forecast errors for interval forecasts is significantly lower than that for OLS forecasts. In addition, both the Newbold (1995) F-statistic (25.98) and Bartlett-statistic (123.6) reject the null hypotheses of equality and homogeneity of variances in favour of interval forecasts. All three statistics are significant at the 1% level. More importantly, there is a direct quality measure for the interval forecasting. The ratio of forecasting accuracy is defined as the overlapped range by both the forecasted interval and the actual interval divided by the maximum range stretched by both the predicted and actual intervals. The ratio is 58.21% in this study. It indicates that the interval approach can predict about 58% of changes in the stock market. This accuracy ratio measures the real preciseness of interval forecasts without any confidence assumption, therefore, is only feasible for the interval measurement. When the point measurement is used, it is impossible to find the overlapped range between the forecasted and actual points. As a result, the forecasting quality is traditionally assessed by a negative measure, the forecast error.

In-sample rolling forecasts tell the similar story (Table 3). Since the current coefficient estimates, rather than their lag terms, are used in in-sample forecasting, a better fitness of forecasts to the data is expected. Therefore, it is not surprising that forecast errors for all kinds of forecasts become smaller, compared to the out-of-sample forecasts. The biggest drop is in the annual OLS forecasts, from 20.57% (Table 2) to 6.52% (Table 3). Nevertheless, the interval forecasts still have significantly lower forecast errors and their standard deviation. The ratio of forecasting accuracy for interval forecasts also increases to 63.47%.

Figures 1 and 2 display the OLS and interval rolling forecasts of SP contrasted with the actual growth rates of SP. The scale in the interval forecast graph (Figure 2) ranges between -0.2 and 0.2, due to the stability of the forecasts. The graph of OLS forecasts has a range from -1 to 1.5. It is evident that OLS forecasts are unstable over time and have large forecast errors. In contrast to the OLS forecasts, interval forecasts have smaller errors in general. The lower bound forecasts have about 10 perfect or near perfect matches with the real lower bounds.

5. Concluding comments

Traditionally, forecasting in Economics and Finance uses point measured data and predicts a particular point in future. Point forecasts are normally imprecise. In order to improve forecasting quality, this study uses a different data measurement, interval, to forecast the range of future stock market changes. Several benefits of this new approach are apparent. First, intervals are an effective economic and financial information measurement. An interval contains information about the level as well as variability which is an important factor in asset pricing. Second, the output of the interval forecasting is more useful than the point forecasting. An interval forecast essentially consists of predicted levels and a predicted variability. Theoretically, the predicted variability can reduce uncertainty or risk, therefore, may influence required returns and capital asset prices. Third, interval forecasting can generate high quality results, because interval data contains more information than does point data. Finally, a unique quality assessment measure is feasible for interval forecasts. The ratio of forecasting accuracy compares the correctly predicted range to the maximum range
stretched by the forecasted and actual intervals. Unlike the mean and standard deviation of forecast errors, the accuracy ratio is a direct and simple assessment of forecasting quality. It does not require any statistical assumptions. Nevertheless, the interval forecasting does require interval data as inputs.

Empirical results in this study provide strong evidence that the interval forecasting is superior to the traditional point data based OLS forecasting. Both the mean and standard deviation of forecast errors for interval forecasts are statistically lower than that for the OLS forecasts. Moreover, the accuracy ratio indicates that the out-of-sample interval forecasting has an actual forecasting accuracy over 58%. The ratio becomes more than 63% for the in-sample interval forecasting.

This study is merely the first attempt to use the interval computing approach in financial forecasting. Future research in this area may be fruitful by further exploring the accuracy of interval forecasting by using higher frequency data (monthly or daily) and extending the interval forecasting into other important economic and financial aspects.

References:


**APPENDICES:**

<table>
<thead>
<tr>
<th>Table 1. Summary statistics of monthly and annual data (in percent) and regression coefficients</th>
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<tr>
<td>Monthly (1/1930-12/2004)</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
</tbody>
</table>

| OLS coefficients |
| With SP as dependent variable |
| IP | UI | DEI | DEF | TERM | Con | R² |
| 0.79 | 0.03 | -2.92 | 0.97 | 0.37 | 0.00 | 0.17 |
| 10.87 | 0.12 | -1.23 | 7.35 | 5.39 | 0.87 |

| Annual (1930-2004) | SP | IP | UI | DEI | DEF | TERM |
| Mean | 5.37 | 3.49 | 0.21 | -0.03 | 0.46 | 2.04 |
| Standard Deviation | 19.06 | 9.94 | 4.57 | 1.30 | 3.41 | 8.84 |

| OLS coefficients |
| With SP as dependent variable |
| IP | UI | DEI | DEF | TERM | Con | R² |
| 0.93 | -0.40 | 2.53 | 0.52 | 0.48 | 0.01 | 0.26 |
| 4.20 | -0.83 | 1.61 | 0.77 | 1.86 | 0.48 |

| Annual interval (1930-2004) | SP | IP | UI | DEI | DEF | TERM |
| Upper bound mean | 6.61 | 2.36 | 0.66 | 0.08 | 1.82 | 3.44 |
| Standard Deviation | 2.59 | 0.08 | 0.01 | 0.00 | 0.05 | 0.22 |

| Lower bound mean | -6.30 | -1.80 | -0.51 | -0.08 | -1.66 | -3.14 |
| Standard Deviation | 0.75 | 0.24 | 0.09 | 0.00 | 0.15 | 0.90 |

SP=Growth rate in S&P stock index.
IP=Growth rate in industrial production index.
UI=Unexpected inflation.
DEI=Changes in expected inflation.
DEF=Default risk premium.
TERM=Unexpected changes in interest rates.
Con=Constant term.
t-values in parentheses.
The 1% significant level represented by a.
The 10% significant level represented by c.

Table 2. Out-of-sample rolling forecasts of changes in the stock market: OLS vs. interval results

<table>
<thead>
<tr>
<th>Rolling window</th>
<th>Monthly</th>
<th>Quarterly</th>
<th>Annual</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecasting period</td>
<td>60 months</td>
<td>20 quarters</td>
<td>10 years</td>
<td>10 years</td>
</tr>
<tr>
<td>Forecast error mean(E)</td>
<td>0.026522</td>
<td>0.048455</td>
<td>0.20572</td>
<td></td>
</tr>
<tr>
<td>Std. dev. Of E</td>
<td>0.024446</td>
<td>0.044082</td>
<td>0.18996</td>
<td></td>
</tr>
</tbody>
</table>

Forecast error mean

Of Upper bound(UE) | 0.028631
Std. dev. Of UE | 0.024216

Forecast error mean

Of lower bound(LE) | 0.033874
Std. dev. Of LE | 0.031837

Forecast error mean

Of Interval (IE=UE+LE) | 0.062504
Std. dev. Of IE | 0.037266

Ratio of forecasting

Accuracy (FA) | 0.582072
Std. dev. Of FA | 0.189395

Equality tests for the following pairs

<table>
<thead>
<tr>
<th>Forecast error mean</th>
<th>Annual</th>
<th>Interval</th>
<th>Test result</th>
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<tr>
<td>Std dev. Of FE</td>
<td>0.20572</td>
<td>0.062504</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.18996</td>
<td>0.037266</td>
<td></td>
</tr>
</tbody>
</table>

T-statistic | 5.96 a
F-statistic | 25.98 a
Bartlett-statistic | 123.60 a

Forecast error=Absolute value of the difference between the forecasted and actual values.
The T-statistic tests the null hypothesis of equality of means without the assumption of equal population variance.
The F-statistic tests the null hypothesis of equality of variances.
The Bartlett-statistic tests the null hypothesis of homogeneity of variances.
The 1% significant level represented by a.

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<tr>
<td>Forecast error mean(E)</td>
<td>0.023503</td>
<td>0.031501</td>
<td>0.065210</td>
<td></td>
</tr>
<tr>
<td>Std. dev. Of E</td>
<td>0.021330</td>
<td>0.028751</td>
<td>0.054414</td>
<td></td>
</tr>
</tbody>
</table>

Forecast error mean

Of Upper bound(UE) | 0.024819
Std. dev. Of UE | 0.020353

Forecast error mean

Of lower bound(LE) | 0.027135
Std. dev. Of LE | 0.025885

Forecast error mean

Of Interval (IE=UE+LE) | 0.051729
Std. dev. Of IE 0.033623
Ratio of forecasting
Accuracy (FA) 0.634659
Std. dev. Of FA 0.195348

Equality tests for the following pairs

<table>
<thead>
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<th>Annual</th>
<th>Interval</th>
<th>Test result</th>
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</thead>
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<tr>
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<td>0.065210</td>
<td>0.051729</td>
<td></td>
</tr>
<tr>
<td>Std dev. Of FE</td>
<td>0.054414</td>
<td>0.033623</td>
<td></td>
</tr>
</tbody>
</table>

T-statistic 1.71^c
F-statistic 2.62^a
Bartlett-statistic 14.41^a

Forecast error = Absolute value of the difference between the forecasted and actual values.
The T-statistic tests the null hypothesis of equality of means without the assumption of equal population variance.
The F-statistic tests the null hypothesis of equality of variances.
The Bartlett-statistic tests the null hypothesis of homogeneity of variances.
The 1% significant level represented by a.
The 10% significant level represented by c.

Figure 1. Out-of-sample 10-year rolling OLS forecasts

Figure 2. Out-of-sample 10-year rolling interval forecasts